A
TEXT-BOOK OF LIGHT

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PREFACE

THIS book has been written for the use of Sixth Form students who are specializing in science. While covering the Light sections of the syllabuses of the principal examining bodies for Advanced and Scholarship levels, it also contains all that should be needed by candidates for university scholarships. The large number of students working for first M.B. at school has also been borne in mind in the preparation of the book.

By confining myself to the needs of this type of student it has been possible to give a sufficiently complete treatment in a comparatively small volume. Descriptions of stock demonstration experiments, of apparatus, and of experimental work have been kept as brief as possible, and the space saved used for a thorough explanation of the principles involved. No attempt has been made to exceed the limits of the subject by dealing with points such as Simple Harmonic Motion and the Rutherford-Bohr atom, which receive adequate treatment in mechanics and chemistry. In some places, as in explaining astigmatism and distortion, an illustrative method has been adopted, as a perfectly strict treatment would have been too advanced. The calculus has been used throughout. Polarization, the Ultra-violet, and X-rays have been dealt with rather fully; and here, as throughout the book, brief accounts of modern applications, particularly in medicine, have been given.

Many of the suggestions of the 1934 Report of the Physical Society's Committee on the Teaching of Geometrical Optics have been incorporated. Photometry and Optical Instruments have been treated as thoroughly as the importance and practical interest of these topics deserve, and the terms and sign convention used are those of current technical practice, so that the student

who proceeds further with the subject will at least have nothing to unlearn.

The sign convention used is one of the two alternatives recommended by the Report. All real distances are reckoned positive, and all virtual distances negative ; the focal length of a converging lens is thus positive. Both in the text and in the worked examples the algebraic nature of the formulae has been emphasized, and also the importance of stating clearly the sign of each quantity before substituting it in any formula. If this is fully understood, there should be none of that confusion over simple calculations which causes so many students to regard the whole subject of optics with distaste. My experience with this sign convention has so far been that pupils consider it much less artificial than the Cartesian system employed in most older text-books, and that the average careful worker makes fewer slips.

The questions at the end of each chapter are from papers of "A" and "S" standard set in former Higher School Certificate examinations of the following bodies: Oxford and Cambridge Joint Board, Northern Universities Joint Matriculation Board, Central Welsh Board, Oxford, and Cambridge. In the later chapters many of the questions are from Oxford and Cambridge Scholarship papers, and some from the examinations for Royal Scholarships. The source of each example is indicated. My thanks are due to the various authorities for their kind permission to use the questions. In addition, many examples of similiar standard have been worked out in the text.

My indebtedness to the standard advanced treatises and practical text-books needs no acknowledgment ; much of the material on Photometry and Ultra-violet Spectrographs is drawn from literature kindly supplied by the General Electric Co., Ltd. and by Adam Hilger, Ltd. respectively. Many firms and individuals have given permission to reproduce diagrams ; the source of each is acknowledged in its place, and I express my thanks for them here.

Dr. G. T. P. Tarrant gave me much friendly advice in the early stages of the book, and kindly read and criticized the manuscript.

Mr. B. O. Wheel checked the calculations in the text and the answers to the examples. I am very grateful to both for pointing out errors and obscurities.

My warmest thanks are due to Sir Richard Gregory for his kindly advice and interest in all stages of the work, for his generous assistance with material, and for his many suggestions for its improvement.

G. R. NOAKES

PREFACE TO SECOND ISSUE

FOUR pages on Bohr's calculation of the Balmer series wavelengths have been added, in response to a suggestion of Professor C. R. Sundarachar, Central College, Bangalore. Some other small additions, and necessary corrections, have been made.

I am very grateful to all those users of the book who have written to me with criticisms and suggestions.

G. R. N.

May, 1940.

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CHAPTER I

INTRODUCTION

Sources of light.—Light is a form of radiant energy to which our eyes happen to be sensitive. We appreciate differences in **brightness**; we are also conscious of differences in quality, to which we give the name of **colours.**

Self-luminous bodies emit light generated by the transformation of some other form of energy. All other bodies are visible because of the light they return from self-luminous bodies, and their appearance will depend on the quantity and quality of the light they receive. Each point on a visible body can be regarded as a source from which the eye receives all the energy reaching it within a cone with the point as vertex and the pupil of the eye as base.

All natural light comes from the sun, with the exception of infinitesimal quantities which reach us from other suns and from such sources as glow-worms. The sun emits light because it is hot. Most artificial sources also emit light because they are at high temperatures, though this is an inefficient way of producing light as the greater part of the emission of a hot body is in the form of invisible radiation. The efficiency increases as the temperature rises, and so the old luminous gas flame was replaced by a much hotter flame of the Bunsen type, in which a mantle of thorium and cerium oxides was suspended. Part of the increase was due to the increased temperature, part to the choice of substances. Early electric lamps used carbon filaments in an evacuated globe. Modern ones use filaments of tungsten, coiled in the form of spirals in order to reduce energy losses by other means than radiation and enclosed in an atmosphere of inert gas to reduce evaporation of the metal. About $\frac{1}{50}$ of the electrical

energy consumed in heating the filament is emitted as light, that is, a 100 watt lamp is actually producing light energy at the rate of 20,000,000 ergs per second or about $\frac{1}{370}$ horse-power.

A hot body has a **continuous visible spectrum**, that is, emits light of all possible colours though their relative proportions depend on the temperature, and the hotter the body the less red it appears.

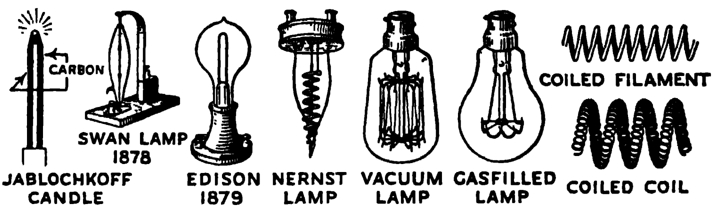


FIG. 1.—Development of electric filament lamp.
(By courtesy of H.M. Stationery Office.)

In the Nernst lamp of 1897 the filament was of rare earths which became electrically conducting themselves after first being heated to incandescence. No vacuum was required. The filament in Vacuum Lamps is a long open grid and although little heat can be lost by conduction it is not possible to run it at a very high temperature as it would evaporate and blacken the bulb. This is unfortunate as the efficiency increases with temperature. Evaporation may be greatly reduced by filling the bulb with an inert gas (*e.g.* argon), but the gas filling so increases heat losses by conduction that high temperatures still could not be reached until Langmuir (1913) introduced the coiled filament for Gas-filled Lamps. More recently (1934) the efficiency of the smaller sizes of lamp has been increased by up to 20 per cent. by again coiling the coiled filament upon itself.

Efficient sources are obtained in certain discharge tubes, in which a current is maintained in a gas at low pressure by means of a high voltage. But with a single gas the light given does not include all possible colours, and objects illuminated by it appear strangely transfigured. Such sources are used chiefly for ornamental and advertising purposes. They are now known as "cold-cathode" lamps. The G.E.C. "Osira" lamp, containing a mixture of gases which includes mercury vapour, is a recent development of this type. Although not giving all possible colours, this lamp provides a sufficiently representative selection to supersede older methods of illumination for such purposes as

street lighting. The negative electrode contains a heated filament, hence this is called a " hot-cathode " lamp. The tube is surrounded by an evacuated jacket to conserve heat. Fig. 2 shows such a lamp in an experimental stage.

FIG. 2.—" Osira " gas-discharge lamp.
(By courtesy of the General Electric Co., Ltd.)

The " Pointolite " lamp, a permanent arc maintained between tungsten electrodes *in vacuo* or in gas at a low pressure, is started by an auxiliary thermionic current from a heated filament which is switched out when the arc is established. The mercury arc, a quartz discharge tube in which the gas is mercury vapour and the electrodes two pools of mercury, is started by momentarily joining the pools. The light given is the characteristic spectrum of mercury vapour, and while the chief use of the lamp in practice is as a source of invisible ultra-violet radiation, it is an important source of light, as an intense radiation of a single colour, green, is easily isolated.

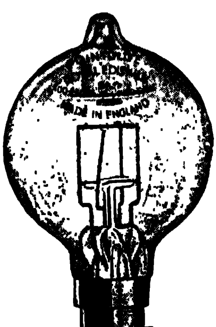


FIG. 3.—" Pointolite."
(By courtesy of the Edison Swan Electric Co., Ltd.)

Those substances that can be volatilized in an ordinary Bunsen flame impart a characteristic colour to it, the chemists' " flame test." The sodium flame is the easiest way of obtaining light of a single colour, yellow. Various ways of introducing sodium compounds into the flame are used with effect, but one of the simplest is to bore a hole about 2 cm. in diameter in a piece of asbestos board about 12 cm. square, soak the board in saturated brine and dry it, and place it on a tripod with the hole over the flame.

The **carbon arc** is really a combination of both hot body and gaseous discharge types and is more efficient than either. It is started by lightly touching the carbon electrodes together, and maintained by the discharge through the hot gases between them when they are separated. The chief source of light is the incandescent crater of the positive electrode, which is at a temperature of about 3700° C. The " flame " between the electrodes gives a characteristic light the so-called " Swan spectrum," which may

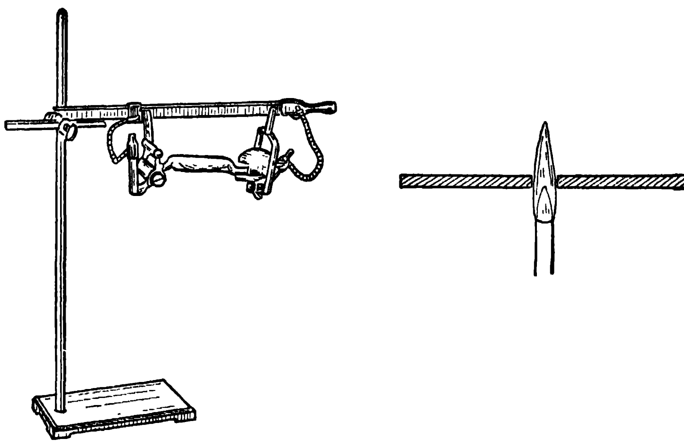


FIG. 3*a*.—Mercury-vapour lamp and sodium flame.

be altered by volatilizing substances in the hot crater or by substituting metallic electrodes for carbon. The carbon arc is used where the source of light must necessarily be small, as in cinematograph projectors. The carbons are consumed in use and have to be adjusted automatically or by hand. The Jablochkoff " candle ", shown in Fig. 1, used alternating current, and both carbons burned down together.

Absorption of light.—When light travelling in air strikes the surface of a **transparent** substance, part is *reflected* and part *transmitted*. At the surface of an **opaque** substance part will be *reflected* and part *absorbed*. The absorption is very high with a black surface. When light is absorbed it is converted to other forms of energy, of which the commonest is heat. The heat generated may be used for its detection and measurement by delicate platinum-resistance

or thermoelectric methods, or may be partially converted into mechanical kinetic energy in the Crookes radiometer. Two distinct *electrical effects* are of importance: selenium changes in electrical resistance when exposed to light, and some substances, notably the alkali metals, show what is properly known as the **photoelectric effect**, emitting electrons in numbers proportional to the light energy absorbed. Modern **photoelectric cells** used for

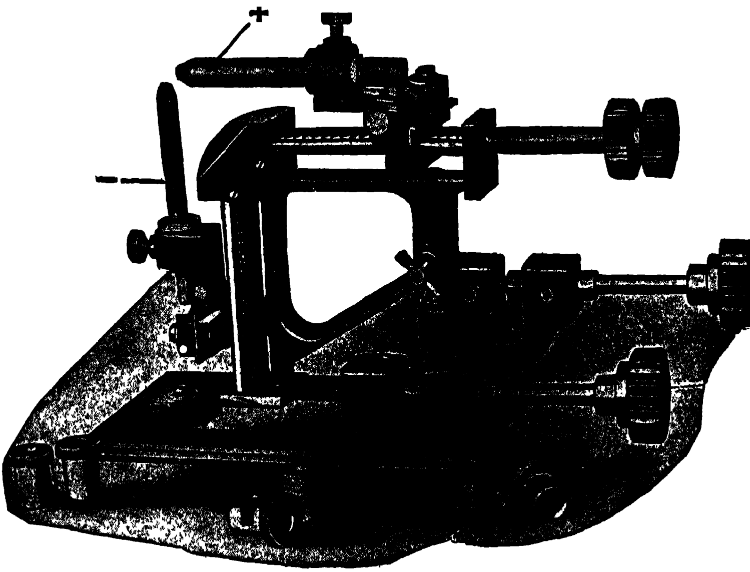


FIG. 4.—Carbon arc with adjustments.
(By courtesy of Messrs. F. E. Becker & Co. and Messrs. A. Kershaw & Son.)

illumination measurements consist of a thin layer of cuprous oxide on a copper plate, covered with a transparent thin layer of metal. Probably the simplest way of regarding them is as a combination of photocell and "metal rectifier." While an alkali-metal cell is designed to magnify the emitted electron-stream and needs a considerable applied voltage for its action, the copper oxide type acts as a real "cell" and can give an E.M.F. up to $\frac{1}{20}$ volt, which may be registered directly. A cell of this type, the "green sensitive" Weston cell, has a sensitivity whose variation with colour approximates to that of the eye.

Photochemical reactions initiated or maintained by the absorption of light are numerous and include the processes by which starch is formed in green plants, those occurring on the photographic plate, and probably vision.

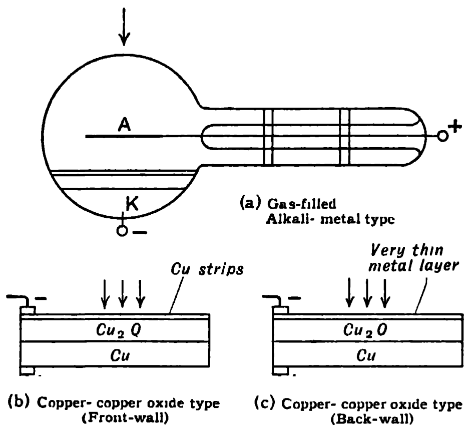


FIG. 5.—Alkali-metal and copper-copper oxide photocells.

Propagation of light. Rays.—Light is propagated with a finite velocity, which is constant for a given kind of light in a given medium. The paths along which the energy travels can for most purposes be taken as straight lines in a homogeneous medium. **Rays** are lines drawn to show the direction in which the energy travels ; a pencil of rays diverging from one point is called a beam of rays. Since we shall be dealing almost entirely with homogeneous media, rays will generally be straight lines and all light-path calculations will be reduced to geometry. This is the simplest treatment, and has the advantage that no assumption as to the manner of propagation is demanded until the student has mastered the elementary facts sufficiently to be able to judge for himself what the wave theory of light has to explain.

Shadows.—A luminous point gives a sharp shadow when its light is intercepted by an opaque object. An extended source gives a region of total shadow (**umbra**) into which no light from any point of it penetrates, surrounded by a gradually fading region of partial shadow (**penumbra**) in which all the light from

some parts of it is received. Fig. 6, drawn for a source larger than the object, illustrates this. Considering the extreme points of the source S, A and B, no light from A ever reaches the portion between the two OP lines, no light from Q the portion between the OQ's. The region OXO is thus total shadow, but above it A will be visible and below it B. A screen moved outwards from the object towards X will show a decrease in umbra and an increase in penumbra. Beyond X

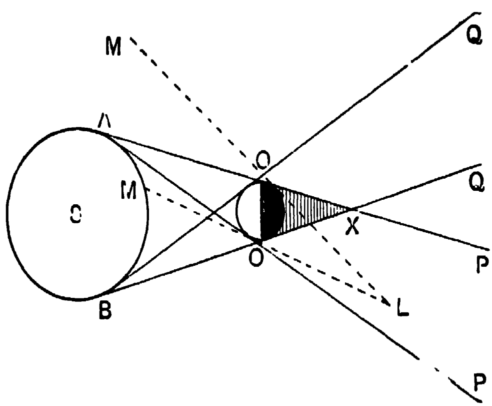


Fig. 6.—Shadow with a large source.

the umbra disappears, for both A and B are visible. The portions of S that are actually visible at any point L can be found by drawing two lines LM, shown dotted in the figure. Regarding the figure as representing an eclipse of the sun, it can thus be seen that in the penumbra proper a lune-shaped portion of the sun's disc will be seen (**partial eclipse**), and in the region bounded by PXQ beyond the umbra the edges of the sun are seen (**annular eclipse**). Only if the observer is within the umbra is the eclipse total.

CHAPTER II

PHOTOMETRY

Definitions.—A line through a point in a plane perpendicular to every line in the plane is called the **normal** at that point. The angle between a ray of light striking the plane and the normal at the point of incidence is the **angle of incidence.**

Unit solid angle is the angle subtended at the centre of a sphere of radius r by a portion of its surface of area r^2. The solid angle ω subtended at the centre by a portion of its surface of area A is thus A/r^2, and if A is the whole surface of the sphere the total solid angle subtended at its centre is $4\pi r^2/r^2$ or 4π.

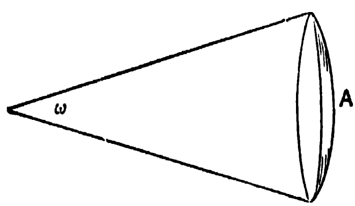


FIG. 7.—Solid angle.

The quantity of light energy which flows through a given surface in unit time is called the **luminous flux through the surface.**

The whole energy radiated by a source of light in all directions in unit time is called the **total luminous flux from the source.**

The luminous flux emitted per unit solid angle is called the luminous intensity of the source.

The luminous flux falling on unit area of a given surface is called its illumination.

The **luminous intensity of unit area** of a surface, whether self-luminous or illuminated, in a given direction is called its **brightness**

in that direction. It is important to distinguish between *brightness* and *illumination*. This page is nearly uniformly *illuminated*; you are able to read it because the printer's ink, which reflects little of the incident light, is much less *bright* than the white paper.

Units.—The unit of **intensity** is the **international candle.** Standards of this intensity are preserved at the National Physical Laboratory at Teddington and at corresponding institutions abroad, in the form of electric lamps run under standard conditions. It is one-tenth of the intensity in a direction at right angles to the flame of a Vernon Harcourt pentane lamp burning under definite conditions of humidity and pressure.

A source which has an intensity n times that of this unit in a given direction is said to be of n candle-power in that direction.

The unit of **luminous flux** is the **lumen.** This is the flux emitted per unit solid angle by a uniform point source whose intensity is one candle-power.

The total luminous flux from a uniform point source of one candle-power fills a solid angle of 4π and is thus 4π lumens.

The unit of **illumination** will be **one lumen per unit area,** the flux being everywhere normal to the area. One lumen per sq. cm., an impracticably large unit, is called one phot. One lumen per sq. m. is called one lux or one metre candle; this is the illumination at one metre distance from a source of unit intensity, for at that distance 1 sq. m. subtends unit solid angle at a point source. One lumen per sq. ft. is an illumination of one foot candle; this again can be seen to be the illumination at a distance of one foot from a source of unit intensity. One metre candle is $\frac{1}{(100)^2}$ phots, or $\frac{(12 \times 2{\cdot}54)^2}{(100)^2}$ foot candles.

Illumination of a small surface due to a uniform point source.—This will depend on the intensity of the source, the distance of the surface from it, and the angle of incidence.

The terms "intensity of illumination" for illumination and "illuminating power" for luminous intensity are in common use and occur frequently in the examination questions at the end of this chapter. They have been avoided in the text to prevent confusion; the student who meets them in problems will have no difficulty in seeing what is asked for, as invariably "the illuminating power *of a source*" and "the intensity of illumination *of a surface*" are specified.

1. *Effect of distance from source.*—Consider unit point source at A (Fig. 8). To get surfaces with the angle of incidence everywhere the same, consider spheres of radii r_1 and r_2 metres described with A as centre. The flux from A is 4π lumens. The illumination of the two surfaces will be $\frac{4\pi}{4\pi r_1^2}$ and $\frac{4\pi}{4\pi r_2^2}$, or and $\frac{1}{r_2^2}$ metre candles. The relative illuminations, whatever the intensity of the source, are $\frac{1}{r_1^2}:\frac{1}{r_2^2}$. That is, other things being equal, the illumination varies inversely as the square of the distance. This is the **Law of Inverse Squares.**

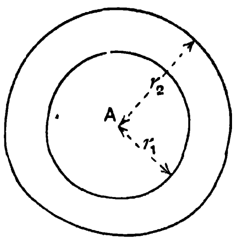


FIG. 8.—Law of inverse squares.

2. *Effect of intensity of source.*—A source of intensity P candle-power at A will emit $4\pi P$ lumens. The illumination at distance r metres is then $\frac{P}{r^2}$ metre candles. So the illumination is proportional to the intensity of the source.

3. *Effect of angle of incidence.*—The flux which falls normally on the small area dA will, incident at an angle θ, cover an area

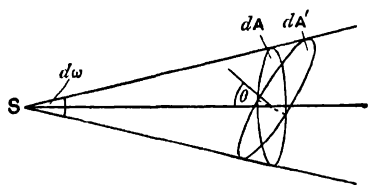


FIG. 9.—Cosine law.

$dA'=\frac{dA}{\cos\theta}$ (Fig. 9). That is, $\frac{\text{illumination of } dA'}{\text{illumination of } dA}=\frac{\cos\theta}{1}$. This is **Lambert's Cosine Law.**

To sum up, the illumination due to a source of P candle-power at a distance r feet or metres for an angle of incidence θ is

$$I=\frac{P\cos\theta}{r^2}$$

foot candles or metre candles. *Note that this is only strictly true for a point source and a small surface.*

Illumination of a large plane surface by a uniform point source.—A point source of intensity P c.p. is at A, vertically above a plane surface at distance h metres from it. The illumination over a small area at B, vertically below A, will be $\frac{P}{r^2}$ metre candles. The illumination over a small area at C, such that $B\widehat{A}C = \theta$, will be $\frac{P}{AC^2} \cdot \cos\theta$, or, since $AC = \frac{h}{\cos\theta}$, $\frac{P}{h^2} \cdot \cos^3\theta$ metre candles.

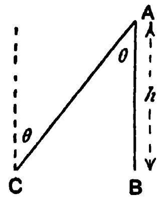


FIG. 10.—Illumination of a large surface.

EXAMPLE.—A 20 c.p. lamp is suspended 4 feet above the centre of a square table of side 6 feet. Find the illumination at the centre of the table and at the corners.

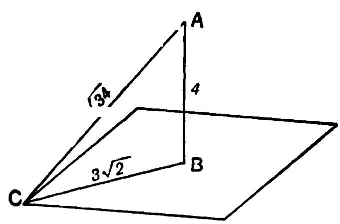


FIG. 11.—Problem.

Let A be the lamp's position, B the centre of the table, C one corner. $AB = 4$ ft., $BC = 3\sqrt{2}$ ft., $AC = \sqrt{18+16} = \sqrt{34}$ ft.

For B, $\cos\theta = 1$, $r = 4$, so $I = \frac{20 \times 1}{16} = \frac{5}{4}$ ft. candles.

For C, $\cos\theta = \frac{4}{\sqrt{34}}$, $r = \sqrt{34}$, so $I = \frac{20 \times 4}{34\sqrt{34}} = 0{\cdot}40$ ft. candles.

Brightness of a source radiating light equally in all directions.—An extended source can be regarded as an aggregation of point sources, each point of which is emitting light equally in all directions. An unpolished white surface, such as is provided by plaster of Paris or magnesium oxide, will reflect light nearly equally in all directions. This is called a **diffusing surface.** An ideal surface giving equality in all directions would be a **perfectly diffusing surface.**

Consider such a source or surface of area A. Let I_0 be the intensity in lumens normal to the surface. The brightness, viewed normally, is then I_0/A lumens/sq. cm. As emission is uniform, each square centimetre has an intensity of I_0/A lumens in all directions. Viewed from an angle θ to the normal the effective

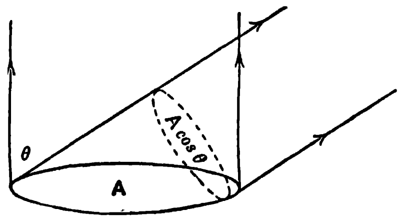


Fig. 12.—Brightness of uniform source or diffusing surface.

area is $A \cos \theta$, so that the intensity of the whole source in this direction is $I = A \cos \theta \times \frac{I_0}{A} = I_0 \cos \theta$. This is Lambert's Cosine Law for emission. The brightness in this direction will be

$$\frac{I}{A \,.\cos \theta} = \frac{I_0 \cos \theta}{A \,.\cos \theta} = \frac{I_0}{A}.$$

That is, while the intensity is proportional to the cosine of θ, the brightness is the same in all directions.

To compare the intensities of two sources.—We cannot compare the intensities of two sources even roughly by looking at them. We should inevitably select the brighter as the more intense, for we can only detect *differences of brightness*. The general principles of visual photometry rest on this fact, and are as follows.

Two similar diffusing surfaces, A_1 and A_2, one illuminated by each source, will have brightnesses in the same ratio as their illuminations. No quantitative estimation of brightness difference can be made, but we can tell when the difference disappears. If the surfaces are illuminated by small sources S_1 and S_2 of intensities P_1 and P_2 at distances r_1 and r_2 normal to them, their illuminations will be $\frac{P_1}{r_1^2}$ and $\frac{P_2}{r_2^2}$. As the surfaces are similar, equal brightness means equal illumination, so that when A and B appear

equally bright then $\frac{P_1}{r_1^2}=\frac{P_2}{r_2^2}$, or $\frac{P_1}{P_2}=\frac{r_1^2}{r_2^2}$. The chief uncertainty is not in measuring r_1 and r_2, but in estimating equal brightness. This is best performed where the two surfaces are viewed side by side with a sharp separating line, where the field of view is small, and the illumination not too low. Differences in colour, and fatigue of the eye, increase the uncertainty. Both surfaces must be as nearly as possible the same, and stray light (unless it happens from the design of the apparatus to affect both equally) must be avoided. We must remember that we are using a formula which only holds strictly in an ideal case, and also that by a single measurement we are comparing the intensities of the sources in one direction only.

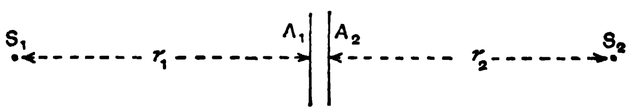


FIG. 13.—Principle of visual photometry.

In performing an experiment the two sources and the photometer head carrying the surfaces are mounted on a bar, the photometer bench, several metres long, on which they can slide. The screens are set normal to this, the axis of the system. When equal brightness is observed the distances r_1 and r_2 are measured. Several sets of readings should be taken, and their mean given as the final result. As low illumination reduces the sensitivity the best results should be obtained with r_1 and r_2 as small as is consistent with use of the inverse square law. How small these distances can be made without serious error can be tested by plotting P_1/P_2 against r_1 or r_2, and as long as the graph is a straight line parallel to the r axis the law is satisfactory.

In accurate work S_1 and S_2 are compared with a third source kept permanently illuminating one screen, the intensity of the third source not necessarily being known. This is analogous with the substitution method of weighing with an untrue balance, and compensates for inequality of the surfaces or any optical devices used for viewing them. Several observers take readings where the results are of importance.

To measure the fraction transmitted by a transparent plate.—S_1, the source whose light we are considering, gives equality at distance r_1 with another source S_2 at distance r_2. When the plate F is placed between S_1 and A, S_1 has to be moved to a distance r_1' to restore equality. The effective intensity of S_1 has thus been reduced from P_1 to P_1'.

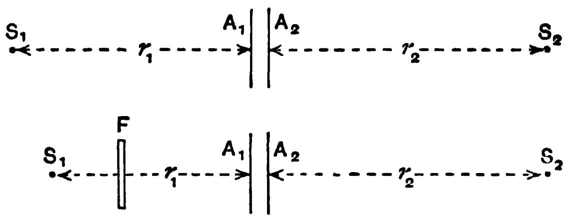


FIG. 14.—Measurement of fraction transmitted.

Originally, $$\frac{P_1}{r_1^2}=\frac{P_2}{r_2^2}.$$

With F in, $$\frac{P_1'}{r_1'^2}=\frac{P_2}{r_2^2}.$$

So that, $$\frac{P_1}{r_1^2}=\frac{P_1'}{r_1'^2} \quad \text{and} \quad \frac{P_1'}{P_1}=\frac{r_1'^2}{r_1^2}.$$

The effective intensity of S has been reduced to $\frac{r_1'^2}{r_1^2}$ of its former value, so the plate is transmitting $\frac{r_1'^2}{r_1^2}$ of the luminous flux it receives.

The fraction cut off is $\left(1-\frac{r_1'^2}{r_1^2}\right)$. Such an experiment measures the transmission but not the absorption, for some of the lost light is scattered and reflected at the plate's surfaces. The effect of a little dirt on the surfaces, and of inclining the plate to the axis of the bench, can be studied with this point in mind. To find the fraction actually absorbed by a given thickness, the difference in normal transmission of two plates with similar surfaces differing in thickness by the given amount would be measured.

A correction has to be made for the refraction of the plate, which effectively reduces r_1'. For a glass plate of refractive index 1·5 the amount to be subtracted from r_1' is $\frac{1}{3}$ the thickness of the plate.

To measure the fraction reflected by a polished surface at a given angle.—This problem is simply the comparison of the in-

tensity of a source S_1 with the effective intensity of its reflection S_1', in the surface. With S_1 on the axis at distance r_1, equal brightness is obtained. Then the mirror is set up so that the image S_1' is on the bench axis, and equality obtained again by moving S_1. If $S_1M = a$, $MA_1 = b$ (Fig. 15), then (p. 34)

$$S_1'A_1 = (a + b).$$

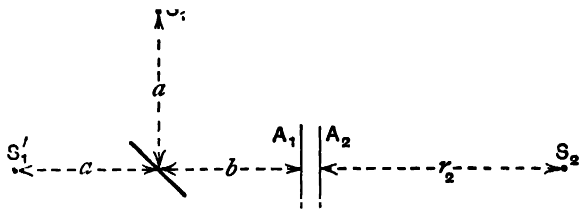


FIG. 15.—Measurement of fraction reflected at 45°.

At first, $$\frac{S_1}{r_1^2} = \frac{S_2}{r_2^2}.$$

With the mirror in, $$\frac{S_1'}{(a+b)^2} = \frac{S_2}{r_2^2}.$$

So, $$\frac{S_1'}{S_1} = \frac{(a+b)^2}{r_1^2}.$$

This is the fraction reflected. A_1 must, of course, be screened from direct light from S_1. For angles near 45° this presents no difficulties, but for angles near 0° and 90° it would be less easy. The following example suggests a method of dealing with *normal* reflection.

EXAMPLE.—Two lamps, S_1 and S_2, illuminate a photometer

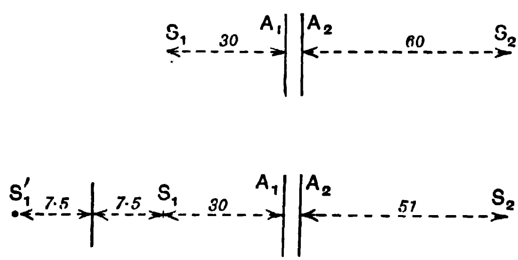


FIG. 16.—Problem.

screen equally when their distances are 30 cm. and 60 cm. respectively. A plane mirror, with its reflecting face at right angles to the photometer bench, is placed 7·5 cm. from S_1 on the side remote

from the screen, and S_2 has then to be moved 9·0 cm. to restore equality of brightness. Find the fraction of the incident light reflected normally from the mirror.

Let P_1 be the C.P. of S_1; P_2 be that of S_2.

Then at first, $\frac{P_1}{30^2}=\frac{P_2}{60^2}$, so $P_2=4P_1$.

Let $P_1{}^1$ be the C.P. of $S_1{}^1$, the reflection of S_1 in the mirror. The illumination at A_1 is the same as that due to S_1 and $S_1{}^1$.

So,
$$\frac{P_1}{30^2}+\frac{P_1{}^1}{45^2}=\frac{P_2}{51^2}.$$

Substituting for P_2,
$$\frac{P_1}{30^2}+\frac{P^1{}_1}{45^2}=\frac{4P_1}{51^2}, \text{ whence } \frac{P_1{}^1}{P_1}=0{\cdot}86.$$

So the fraction reflected by the mirror is 0·86.

Types of photometer head.—1. *Rumford's shadow photometer.* The shadows of an upright object O are thrown on a screen; the

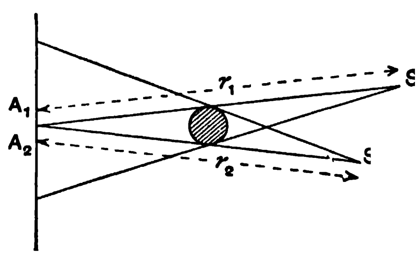


FIG. 17.—Rumford's shadow photometer.

shadow cast by S_2 is A_1, that by S_1 is A_2. A_1 and A_2 are equally bright when the two shadows appear equally dark against the background illuminated by both sources. Stray light will not vitiate readings as both A_1 and A_2 receive it, but it will reduce the accuracy of setting as it will reduce contrast. The two shadows are necessarily of different sizes, will not be sharply bounded with an extended source unless O is close to the screen, when they will not be separated unless the illumination is inconveniently oblique; and the field of view is large and changes with each reading. The cos θ term in the expression for the illumination ought really to be taken into account if S_1 and S_2 shine on A_1 and A_2 respectively at very different angles of

incidence, so it is best to make these angles of incidence as nearly equal as possible.

2. *Bunsen's grease-spot photometer.* A_1 and A_2 are here the two sides of a paper screen with a grease spot in the middle, which transmits a high proportion of the incident light instead of reflecting it as the paper does. If the sum of the fractions reflected and transmitted by the spot equalled the fraction reflected by the paper, we should see spot and paper equally bright when both A_1 and A_2 are equally illuminated. Actually, the brightness of spot and paper differ when equality of illumination is obtained, and the difference depends on the angle of view as the spot does not diffuse as perfectly as the paper, so a further uncertainty is added to that of judgment of equality. But if both sides are viewed simultaneously at equal angles, and equal contrast between spot and paper on both sides looked for, this difficulty is overcome.

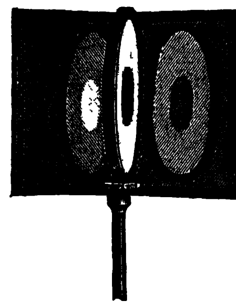


FIG. 18.—Bunsen's grease-spot photometer showing mirrors to view both sides of screen.

3. *The Lummer-Brodhun photometer.* Using right-angled 45° prisms (p. 102) it is possible to make the equivalent of a "grease-spot" screen which shall be *totally reflecting* at the reflecting parts and *totally transmitting* where it should transmit. Two prisms, C_1 and C_2, are placed with their hypotenuse faces together and separated by a thin air film except at the centre, where they are cemented with Canada balsam. Rays striking the hypotenuse face at 45° at the air film are totally reflected (rays 1 and 3, and 4 and 6), while those striking the cemented part are totally transmitted (except for the entirely negligible absorption of the thin film of cement).

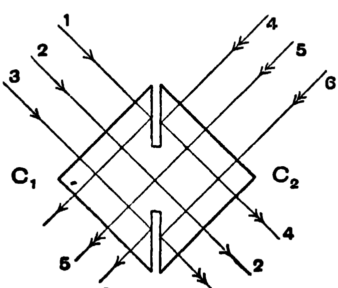


FIG. 19.—" Cube " of Lummer-Brodhun photometer.

A_1 and A_2, two sides of a white diffusing screen, are illuminated by S_1 and S_2 normally. Light from A_1 (single arrow) is reflected by the 45° prism B_1 and strikes the hypotenuse face of C_1 where only the centre of the beam is transmitted, the rest being reflected to the blackened side of the box D where it is absorbed. Light from A_2 (double arrow) receives similar treatment, the outer part

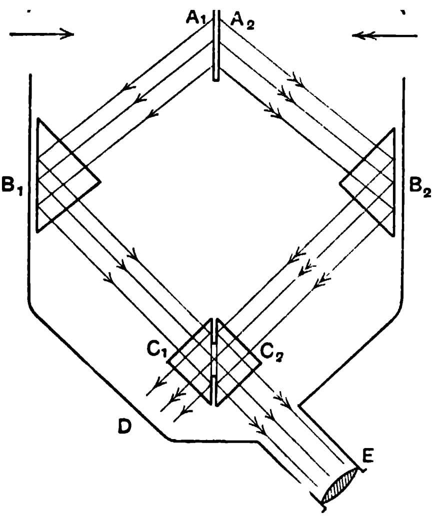


FIG. 20.—Lummer-Brodhun photometer.

of the beam being totally reflected and the rest totally transmitted. The eyepiece at E is focussed on the hypotenuse face of C_2. The central part of the field of view will be lit by A_1, the outer part

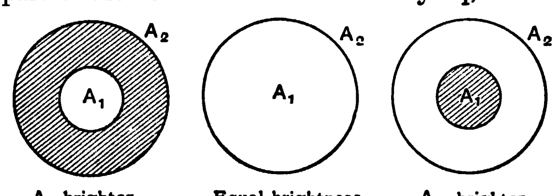


FIG. 21.—Appearance of field in Lummer-Brodhun photometer.

by A_2, and the boundary will be sharp. The appearance of the field will be as shown (Fig. 21) and the disappearance of the boundary indicates equal brightness.

In a more sensitive form, the estimation to be made is that of equal contrast as well as equal brightness; a portion of A_1's light is cut down by about 10% by a strip of glass in its path and superimposed on A_2's field; the same is done for A_2 The two prisms, C_1 and C_2, are cemented at several places on their hypotenuses. The field viewed is as shown.

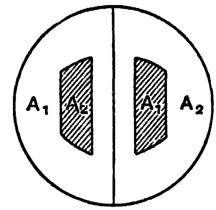


FIG. 22.—Field seen in "contrast" type of Lummer-Brodhun photometer.

Then, at equality, the boundary between the main patches disappears and the darker central portions stand out with equal contrast from their background. In both forms the field is kept small, not more than 6° in angular width.

One disadvantage is the loss of light in transmission through the prisms. As this is equal on both sides it will not affect the result, but it means that the sources will have to be nearer the screen to give optimum illumination. The Lummer-Brodhun photometer in the "contrast" form is the most accurate of all visual photometers.

4. *Sources of different colours. The flicker photometer.* Photometers depending on the judgment of equal brightness or equal contrast are unreliable when the sources differ much in colour, the contrast method less so for most observers. One method which suggests itself in this case is to reduce the colour difference in the sources to be compared. In the substitution method, if the third comparison source is intermediate in colour between S_1 and S_2 the colour difference of a single observation is halved, and with a sufficient number of intermediate sources this method would work with ordinary sources of widely different colours. This is the **cascade method** used at the National Physical Laboratory for establishing standards of the same colour as various electric lamps by comparison with the much redder pentane lamp. The colour difference between the sources to be compared can be reduced by suitable filters, whose transmissions have been determined by another method.

An accurate though tedious method is that of spectrophotometry, beyond the scope of this book; the intensities of each part of the spectrum are compared separately. Crova

suggested that the relative intensities for a narrow strip of the spectrum in the yellow region (the "Crova wave-length") represented accurately enough their true relative intensities.

The only visual instrument which compares sources of widely different colours directly is the **flicker photometer.** Fig. 23 shows the principle.

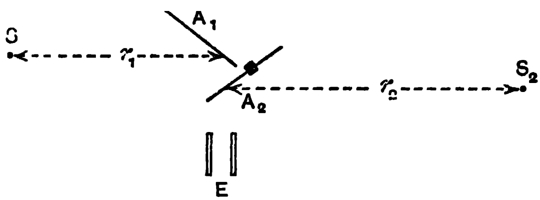


FIG. 23.—Principle of flicker photometer.

A_1 is a fixed diffusing screen illuminated by S_1. A_2 is a rotating disc, also with a diffusing surface, with half its area cut away in equal sectors. The planes of A_1 and A_2 are equally inclined to the line of sight. The eye at E will see A_1 and A_2 alternately, and if they are of equal brightness no flickering at *low speeds of rotation* will be observed even with large colour differences.

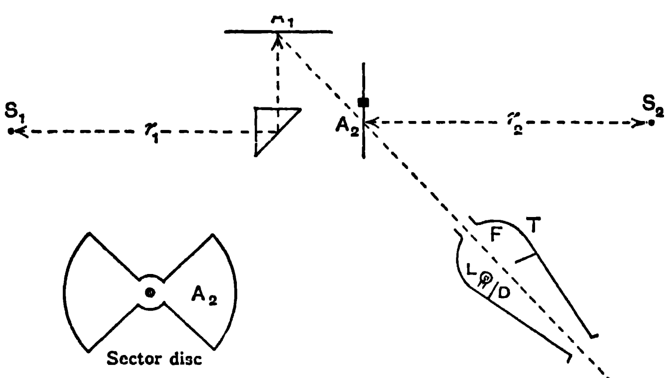


FIG. 24.—Guild's flicker photometer.

With suitable precautions this can be made a very accurate instrument. The essentials are a small field of view, very high

illumination, uniform rotation of the sector disc and variable speed control, and the avoidance of surface imperfections which would produce flicker on their own account, even if the surfaces were equally illuminated with light of the same colour. A large bright background surrounding the photometric field is an advantage.

In the form devised by Guild (Fig. 24), A_1 and A_2 are coated with magnesium oxide, an easily renewable diffusing surface made by "smoking" with magnesium ribbon. The sectors of A_2 are accurately 90°, and this rotates so that S_2 illuminates it normally. Normal illumination of A_1 by S_1 is achieved with a reflecting prism R. The two surfaces are viewed at an angle of 45° through the tube T, which cuts down the field of view; the front part F is a diffusing hemisphere lit by a small lamp L, which provides a uniform bright background, a diaphragm D screening the eye from L. A_2 is rotated by belt drive from an electric motor giving steady rotation at different speeds. Attention is paid to the perfection of all surfaces, the edges of the sector, and uniform rotation. All measurements are made under a standard illumination of 25 metre candles and with a field of view of 2° angular diameter. The substitution method is used. This ensures that when S_1 has once been set to give 25 metre candles on A_1 no further trouble is taken over illumination, and that no measurements to A_1, which would involve correction for the refraction of the prism as well as its losses, have to be made.

For each measurement, the speed of alternation is reduced to the minimum at which no flicker can be found, about 10-25 alternations per second. This critical speed is higher for large colour differences. The accuracy of a single observation with ordinary lighting sources is 0·75%, and with sources of widely different colours within 1%. The Lummer-Brodhun used with calibrated filters gives accuracy slightly less good but of comparable order.

Correction has to be made in all colour measurements for the colour vision of the observer, and the correction factor is based on his interpretation of the relative brightness of equal sources seen through blue and yellow filters transmitting equal energy.

Light distribution from an actual source.—The intensity of a lamp in different directions can be found by turning it in its stand on the photometer bench, or if the lamp can only be used in one position, by rotating the bench about axes passing through the lamp as in the apparatus of Fig. 25.

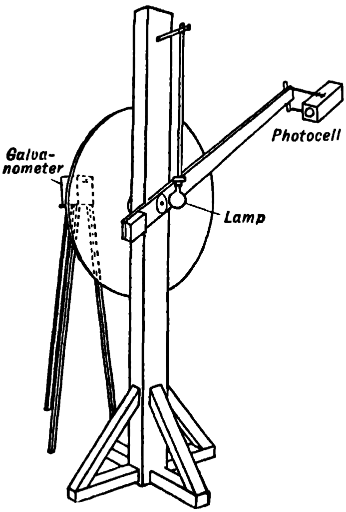
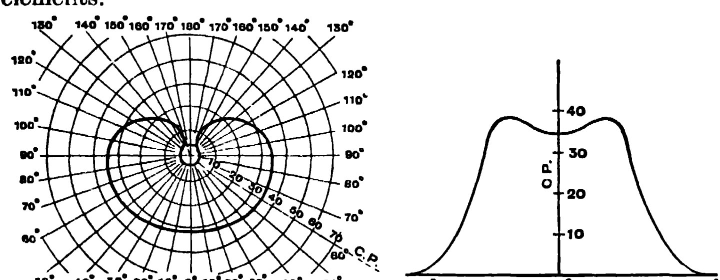


FIG. 25.—Measurement of intensity of a lamp in different directions. (*J. Sci. Instr.*)

For a lamp symmetrical about an axis the distribution in one plane only through the axis need be found. It is usual to plot a curve in polar coordinates with source as origin and intensity as radius vector. This is called a **polar curve** (Fig. 26*a*). The information could, of course, be shown equally well in rectangular coordinates, plotting intensity against angle made with the axis (Fig. 26*b*).

The average intensity in this plane can be found from either diagram by splitting it up into a number of equal sectors or strips, summing the intensities, and dividing by the number of elements.



(*a*) (*b*)

FIG. 26.—Intensity distribution curves in polar and rectangular coordinates.

The distribution in three dimensions may be exhibited on an **isocandle diagram,** on which the two coordinates are directions measured with respect to horizontal and vertical planes passing

through the lamp, and the curve joins directions of equal intensity. Fig. 27 shows such a diagram for a modern street lamp.

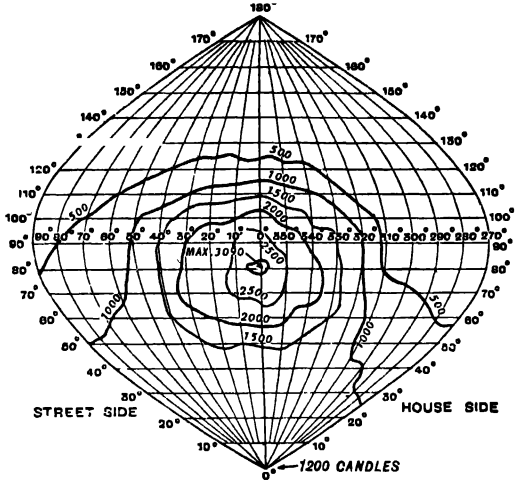


Fig. 27.—Isocandle diagram.
(By courtesy of the General Electric Co., Ltd.)

With unsymmetrical sources, the polar curves in important planes are found. Fig. 28 shows, for a floodlight used as in (*a*),

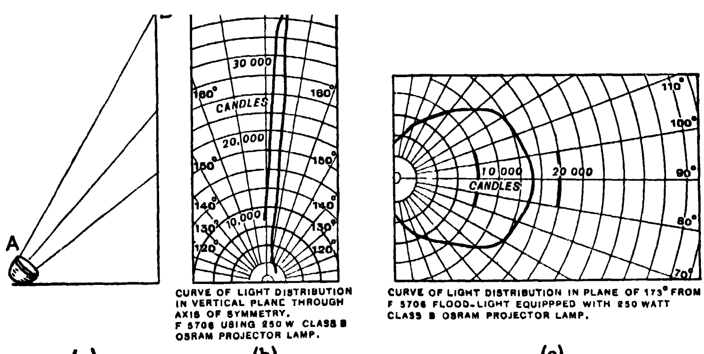


Fig. 28.—Distribution curves for a floodlight.
(By courtesy of the General Electric Co., Ltd.)

the curve for the plane of the diagram (*b*), and for a plane through *AB* at right angles to it (*c*).

Mean spherical candle-power.—The **mean spherical candle-power** of a source is its average intensity, or the intensity of a uniform point source giving the same luminous flux. For a source symmetrical about an axis this will be the same as the average candle-power in any plane through the axis given by the polar curve. For an unsymmetrical source this could in theory be found if a sufficient number of polar curves were obtained and combined to form a "polar surface." We could then split the space surrounding the source into small solid angles $d\omega$ and average according to the ordinary rule. The mean spherical candle-power would be $\frac{\Sigma_0^{4\pi} P d\omega}{\Sigma_0^{4\pi} d\omega}$, or the $\frac{\text{total luminous flux}}{4\pi}$. In practice this method would be tedious. But the total luminous flux can be measured directly by enclosing the source in a suitable *integrating surface*, a diffusing cube or sphere. One form of integrator is the **Ulbricht globe**, a hollow sphere painted to give a white diffusing surface inside. The surface of the sphere is uniformly illuminated just as if a uniform point source of intensity equal to the mean spherical candle-power of the lamp were placed at its centre. Observations are made through a small hole O screened from direct light from the lamp L by a screen S with a diffusing surface. The illumination at O is proportional to the mean spherical candle-power of L; the relation can be calculated if the reflecting power of the surface is known, but it is usual to standardize the globe with a known lamp. The diameter of the globe may be from 1 to 3 metres.

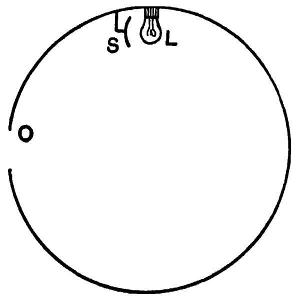


FIG. 29.—Ulbricht globe.

Measurement of illumination.—This is of great practical importance. Instruments calibrated to give a direct reading of illumination are called **illumination photometers.**

The Benjamin foot-candle meter. A translucent screen is illuminated from below by a lamp run at a standard voltage. The illumination at different parts of the screen is marked on the adjacent scale. A Bunsen grease spot slides over the screen, and

when the spot disappears the illumination on its upper surface equals that of the screen beneath. A dry cell and voltmeter are incorporated in the apparatus. The instrument requires frequent testing as the lamp ages.

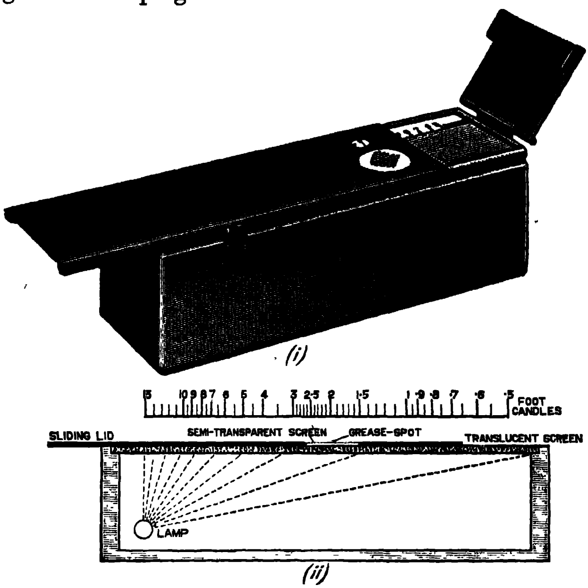
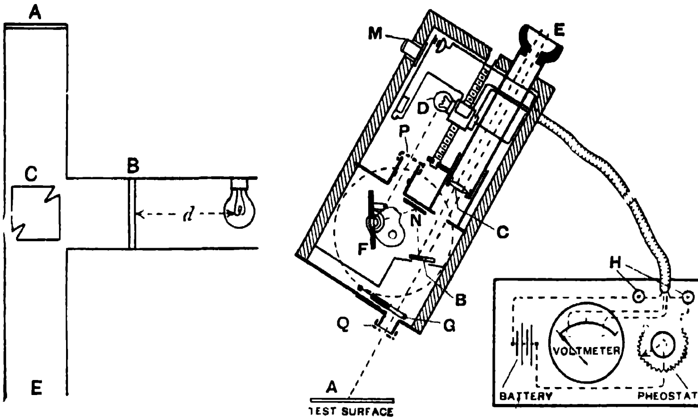


FIG. 30.—Foot-candle meter.

The Weber illumination meter. A is a ground glass screen receiving the unknown illumination, and B another screen whose illumination can be varied by altering its distance d from a standard lamp; the carriage of B is calibrated to give B's illumination directly. When A and B viewed through the Lummer-Brodhun cube C appear equally bright they are equally illuminated.

The luxometer. This varies the illumination of a comparison surface by varying θ. Fig. 31*b* shows the arrangement. The test surface A, which receives the illumination to be measured, is viewed by the observer at E through the clear portion of the half-silvered mirror B and the focussing lens C. At the same time, the light from the internal comparison lamp D falls upon the

adjustable comparison surface F, which is likewise viewed from E by reflection in the mirrors N and B. The brightness of F can be varied by turning a milled knob until it exactly matches the brightness of the test surface A, as evidenced by the dis-



(*a*) Weber illumination meter. (*b*) Luxometer. (*By courtesy of Griffin & Tatlock, Ltd.*)

FIG. 31.

appearance of the line of demarcation between the two when viewed from E. The scale is so graduated that the illumination at the surface A can be read directly in foot candles.

Photoelectric illumination meters. A modern copper-copper oxide photocell connected to a galvanometer will give a deflection proportional to the illumination on its face, and the galvanometer scale can be made to read foot candles directly. Such an instrument is portable, robust, and very rapid, needs no external battery, and has a colour sensitivity approximating to that of the eye. A disadvantage is that its face reflects considerably, particularly at large angles of incidence. Instruments of this type have superseded visual photometers for routine work ; it is as if a world using silver voltameters for measuring electric currents had suddenly evolved calibrated ammeters.

Practical considerations.—The chief aim of lighting is to enable the eye to distinguish the brightness contrasts it looks for without strain. In general, the perception of great detail and subtle contrasts, and all work with poorly reflecting materials, require great illumination. Recommended illuminations are 6-10 foot candles for an office, 10-15 foot candles for sewing light materials and 15-25 foot candles for dark, and 100-500 foot candles on the table of an operating theatre. The illumination obtained will depend on the intensity and number of the lamps, their position, and the reflecting power of the surfaces of the room. Depreciation due to dirt and ageing of the lamps has to be allowed for in designing an installation.

Also important is the avoidance of **glare**, either directly from the source or reflected from surfaces. " Glare " may be defined as dazzling or annoying brightness, just as " noise " is deafening or annoying loudness. Discomfort is produced, not when looking steadily at a lamp or its reflection in, say, a wet road or glazed paper, but when the attention is directed to some less bright object and the lamp or reflection intrudes on the edge of the field of view; just as the noise of a tram is endurable except to two passengers conversing. The eye is easily irritated by intense light coming from below its level, but probably the chief annoyance of glare is that it distracts attention, making concentration fatiguing, and that it reduces contrast in the object viewed. The possibility of glare will be reduced by reducing the brightness of the source, which can be done with little reduction in intensity by surrounding it with a diffusing globe or reflecting its light from a diffusing surface. The intensity may be cut down by only 25%, but the intensity per unit area by a factor of hundreds of thousands. Another advantage of diffuse lighting is that shadows will be much less harsh, and distraction produced by strong shadows is frequently as great as that due to glare.

A concise summary of the principles of modern illumination engineering is given in the Science Museum handbook on Electric Illumination (H.M.S.O., 6d.).

A practical problem.—The problems to be solved in floodlighting illustrate the principles of photometry well. The aim in floodlighting a building is to produce sufficient brightness over the main part of the surface for the observation of detail. Suppose it is required to produce a brightness of 1 c.p. per square foot on a diffusing surface of Portland stone which reflects 0·6 of the incident light. The solid angle to be filled by the stone is that of a hemisphere, or 2π. Each square foot emits at 1 c.p., or 1 lumen per unit solid angle, and so has a total emission of 2π lumens. It

9. If light from a lamp falls on the surface of a clean piece of plaster of Paris and on a plane mirror, a considerable fraction of the incident light will be reflected in each case, but the character of the reflected light will be different. In what does this difference consist, and why is it that an observer looking at the plaster of Paris would see the object itself, but if he looked at the mirror (in the right direction) he would see an image of the lamp, and the surface of the mirror would be comparatively indistinct ?

Describe in detail how you would arrange a photometric experiment to determine the percentage of light reflected from a plane mirror for an angle of incidence of about 45°. (O.H.S.C.)

10. Define "intensity of illumination."

State and derive the law which gives the relation between intensity of illumination and distance, when the dimensions of the source of light are small compared with the distance.

Describe an apparatus suitable for the comparison of the candle-powers of two small lamps, and explain how the comparison is made. (C.W.B.H.S.C.)

11. Distinguish between *illuminating power* and *intensity of illumination at a point*, defining the units in which each is measured.

Explain how you would compare the illuminating powers of two sources of light, pointing out the difficulties that arise in practice and indicating how far these difficulties are overcome in the method you describe. (C.H.S.C.)

12. How does the illumination of a screen vary with (*a*) its distance from the source of light, (*b*) its angle of inclination to the rays ? A 30-c.p. lamp (which may be regarded as a point source radiating equally in all directions) is suspended 3 ft. above a horizontal table. Calculate the illumination of the table at a point P immediately beneath the lamp, and also at points Q and R distant 3 feet and 6 feet from P respectively. (N.U.J.M.B.H.S.C.)

13. Two small surfaces, equally bright, one a square and the other a circle, are placed on opposite sides of a point at which they are to produce equal illumination. The distances from the point are in the ratio 3 : 1. If the length of a side of the square is twice the length of the radius of the circle, find how the circle must be placed if the square is at right angles to the line joining the centres of the surfaces and passing through the point. (R.S.)

14. Define the terms "intensity of illumination at a surface" and "illuminating power of a source of light." Describe a good type of photometer, and mention any practical difficulties which may arise in using it to compare two sources of light.

An electric lamp hangs 3 feet above the centre of a circular table 8 feet in diameter. Compare the intensities of illumination of the surface of the table at the centre and at the edge respectively. (Assume the lamp to emit light uniformly in all directions.) (N.U.J.M.B.H.S.C.)

15. Describe the grease-spot photometer, and explain how you would measure the light loss which results from enclosing a light source by a glass globe. Two 16-candle-power lamps are placed on the same side of a screen at distances of 2 and 5 metres from it. Calculate the distance at which a single 32-candle-power lamp must be placed in order to give the same intensity of illumination on the screen. (N.U.J.M.B.H.S.C.)

16. How would you demonstrate that the intensity of illumination of a surface varies inversely as the square of the distance from the source ? What arrangements would you suggest in order to determine what candle-power at a given distance would give the same illumination as that of the full moon ? (N.U.J.M.B.H.S.C.)

17. Describe in detail how you would determine the candle-power of an electric lamp, and point out any difficulties which may arise. What would be the illumination, measured in foot candles, produced directly on the page of a small book by a 16-c.p. lamp 5 feet away in a direction making an angle of 30° with the surface of the page ? Explain why the actual illumination would probably be greater than this if the lamp and book were in an ordinary room.

(O.H.S.C.)

CHAPTER III

REFLECTION AT PLANE SURFACES

Regular reflection.—A perfectly diffusing surface equally illuminated by a source appears equally bright in all directions. A highly polished surface will appear bright in one direction only

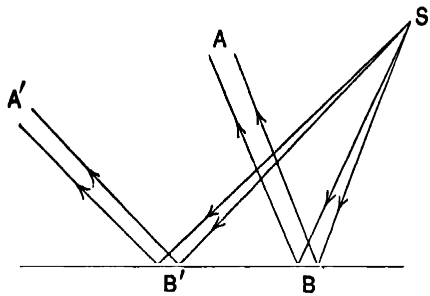


FIG. 32.—Regular reflection.

for a given position of the eye. In the diagram an eye at A will see a bright patch at B, and at A' a bright patch only at B'. Light entering the eye at A has come along the path BA, and can only have reached B along SB; similarly, only light incident along SB' is reflected along $B'A'$. Thus there is an unique relation between the directions of the incident and reflected light. This is called **regular reflection.** Most surfaces reflect both diffusely and regularly to a certain extent. A surface which reflects regularly nearly all the light falling on it is termed a mirror.

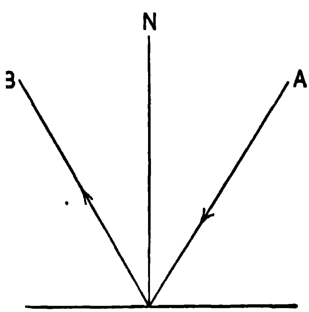


FIG. 33.—Regular reflection of a single ray.

Laws of regular reflection.—Consider a single ray of light AP

incident on a plane mirror at P. The angle between AP and the normal PN is called the **angle of incidence,** and that between the reflected ray PB and the normal the **angle of reflection.** Experiments show that in all cases :

(1) **The incident ray, the reflected ray, and the normal at the point of incidence all lie in one plane.**

(2) **The angles of incidence and reflection are equal.**

These are the two laws of regular reflection. They can be tested directly sufficiently closely for demonstration purposes using the **optical disc** (Fig. 34). A small plane mirror P is mounted with

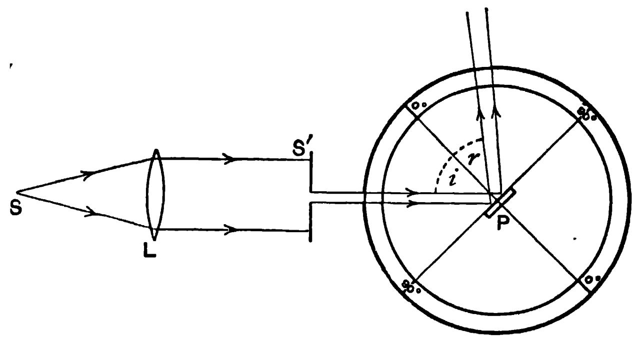


FIG. 34.—Optical disc used to test laws of regular reflection.

its plane perpendicular to a disc whose white diffusing surface is divided into four quadrants each graduated from 0° to 90°. The mirror is set with its reflecting surface along a 90°-90° line and passing through the axis of rotation of the disc.

A narrow parallel beam from a small source S at the principal focus of the lens L passes through the slit S' and strikes the mirror at P, grazing the disc so that its incident path can be seen. The reflected beam also grazes the disc, which agrees with the first law. As the disc is rotated the angle of incidence is varied and $\angle i$ and $\angle r$ can be read directly. A graph of i against r is a straight line through the origin at 45° to the axes, which agrees with the second law. More accurate direct tests can be performed with the spectrometer (p. 248), which enables a better parallel beam of light to be obtained and more refined angle measurements to be taken.

Image of a point in a plane mirror.—A is a point emitting light in all directions ; XY is the trace of the mirror. AO is the normal from A to the surface. A ray of light AP making an angle of incidence θ is reflected along PB at an angle of reflection θ, in the plane of the paper. The reflected ray produced backwards cuts AO produced in A'.

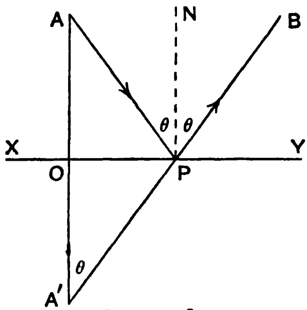


Fig. 35.—Image of a point in a plane mirror.

PN and OA are parallel, so $O\widehat{A}P = A\widehat{P}M = \theta$ (alternate angles) and $O\widehat{A'}P = N\widehat{P}B = \theta$ (corresponding angles). In the triangles POA and POA', $A\widehat{O}P$ and $A'\widehat{O}P$ are right angles, $O\widehat{A}P = O\widehat{A'}P$, and OP is common. So the triangles are congruent and $OA' = OA$.

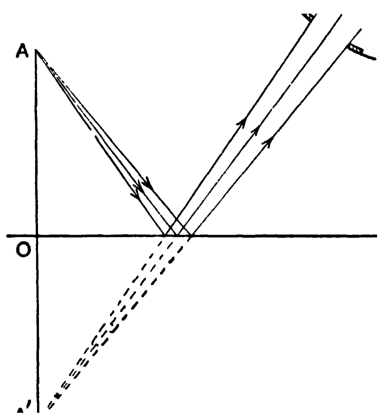


Fig. 36.—Rays reaching eye from image of a point in a plane mirror.

The same conclusion would be reached whatever the value of θ, so it follows that the paths of all reflected rays produced back cut AO produced in one point A'. An eye looking into the mirror will receive a cone of rays, apparently diverging from A' (Fig. 36). So A' is the image of the point A.

To sum up, **the image of a point in a plane mirror lies behind the mirror along the normal from the object produced, and is as far behind the mirror as the object is in front.** It is a **virtual image,** as the rays of light by which it is seen do not come from it but *only appear to do so.*

Image of an extended object in a plane mirror.—Each point on an actual object will thus give an image point in a definite position, so that an image of the whole will be produced. It will differ from the object in two respects. It will be inverted in one direction, and it will be slightly less bright owing to loss of light at the surface.

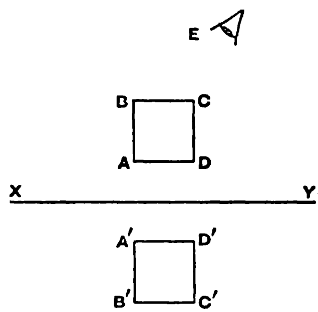


FIG. 37.—Lateral inversion.

Consider a mirror XY perpendicular to the plane of the paper, and a square object $ABCD$ in this plane. Using the argument of the last paragraph for the points A, B, C, D, the image formed is the square $D'C'B'A'$. An eye at E will see the sides $A'B'$ and $D'C'$ in the same relative positions as AB and CD, but $A'D'$ and $B'C'$ will have appeared to change sides as compared with AD and BC. That is, there has been an inversion in a direction normal to the mirror but not parallel to its plane, those parts of the object nearest to the mirror and most distant from the eye giving the parts of the image nearest to the mirror and nearest to the eye. As mirrors are usually employed in a vertical plane the fact of common observation is a **lateral inversion** in a horizontal plane; hence this is the usual name for this effect.

Two plane mirrors can be arranged to produce lateral inversion

(Fig. 38*a*), complete inversion (Fig. 38*b*), or no inversion at all (Fig. 38*c*). In Fig. 38*b*, M_2 has been rotated through 90° about

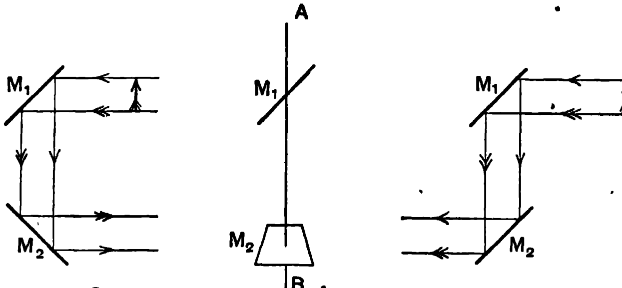


FIG. 38.—Various arrangements of two plane mirrors.

the axis AB in the plane of the paper from the position of Fig. 38*a*, and the final image would be seen looking into the paper.

Location of the image in a plane mirror. Method of no parallax.—Two objects at different distances from the observer change their apparent relative positions as the observer moves; the nearer object appears to move the faster and in the opposite direction to the observer. This relative motion with change of viewpoint is called **parallax.** Only when two objects are adjacent will there be no parallax between them. This is, of course, true both for tangible objects and for optical images formed by plane mirrors or by any other means

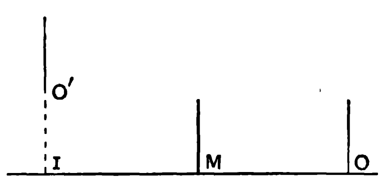


FIG. 39.—Method of no parallax.

In Fig. 39, I is the image of the pin O in the mirror M. A second pin O', mounted above the mirror, is moved until there is no parallax between O' and I. Then both O' and I are equidistant from the eye. By measuring the distances of O and O'

from the mirror we can verify that I is as far behind it as O is in front. The experiment is best performed on the optical bench (p. 140). With the usual glass mirror silvered on its back surface a correction for the refraction of the glass must be made ; and as on p. 77, the reflecting surface can be taken to be $\frac{1}{3}$ the thickness of the glass in front of the silvered one.

The no-parallax method of locating images is most important, and can be made very accurate. Remember that its use is not confined to the virtual image formed in a plane mirror.

Test for a plane mirror. From the position of the virtual image formed we can tell if we have a plane mirror. A telescope is focussed on an object at least 100 metres away. It is then turned and pointed at the image of the object in the mirror, and if this is still in focus the mirror is satisfactorily plane. The reason for choosing a distant object is to make negligible the distance between the telescope objective and the surface.

Rotating mirror.—A ray of light striking the mirror at an angle of incidence θ is reflected at the same angle. The **deviation**, the angle between incident and reflected rays, is thus $180° - 2\theta$. If the direction of the incident ray is unchanged and the mirror is rotated through an angle ϕ, the angles of incidence and reflection each become $(\theta + \phi)$, the deviation $[180° - 2(\theta + \phi)]$, and the change in deviation 2ϕ. Thus the reflected ray is turned through twice the angle described by the mirror.

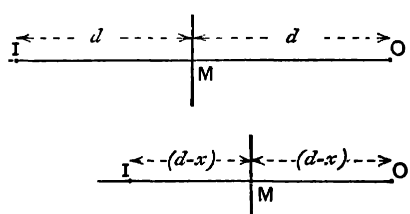


FIG. 40.—Moving mirror.

Moving mirror.—An object at O, at distance d from the mirror, gives an image at I, distant d behind it. The distance OI is thus $2d$. If the mirror moves a distance x towards O, distance of object and image each become $(d - x)$ and the distance OI will be $2(d - x)$. The image has thus moved twice the distance moved

by the mirror in the same direction. If the mirror is kept still and the object moved a distance x towards it, the distance of object and image are again each $(d-x)$ and OI is $2(d-x)$. But the distance moved by the image is now x, the same as that moved by the object but in the opposite direction.

Inclined mirrors.—If a lamp is placed in front of two mirrors, M_1, M_2, whose planes are perpendicular to the plane of the paper (Fig. 41), one image will be seen while they are inclined at 180°. If they are gradually closed like a book, pivoting about an axis through O, first two and then three, four, and an increasing number of images are seen as the angle between the mirrors is reduced. These are formed by successive reflections, the virtual image formed in one mirror being reflected in the other.

FIG. 41.

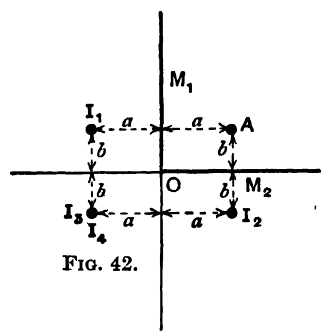


FIG. 42.

We will consider a simple case because the figure is easier to draw clearly. If the mirrors are inclined at 90° the object A will give an image I_1 in M_1 and I_2 in M_2. Now I_1 is in front of the surface of M_2 produced, and so rays appearing to come from it can be reflected in M_2; and a third image I_3 is then produced. The reflection of I_2 in M_1 gives another image I_4 which coincides with I_3; this coincidence is because we have chosen a particular case in which the angle between the mirrors is an aliquot part of 360° (and not because it happens to be a right angle), as can be seen by drawing other cases. At 90°, which in circular measure is $2\pi/4$, three images are seen. If the angle is greater

than 90° there will still be three; according to the position of the eye either I_3 or I_4, but not both, will be visible. If the angle is less than 90°, I_3 and I_4 separate and four images are seen. By drawing or by experiment it can be seen that

for an angle between $\frac{2\pi}{2}$ and $\frac{2\pi}{3}$, 2 images are seen;

" " " $\frac{2\pi}{3}$ " $\frac{2\pi}{4}$, 3 " "

" " " $\frac{2\pi}{4}$ " $\frac{2\pi}{5}$, 4 " "

so for an angle between $\frac{2\pi}{n}$ and $\frac{2\pi}{n+1}$, n images are seen.

For nearly parallel mirrors n will be very large indeed, and for parallel ones infinite. But as light is lost at each reflection the number actually seen in these cases is limited.

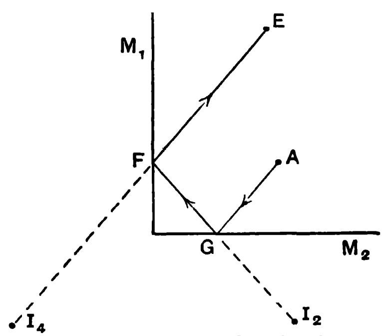


FIG. 43.—Construction for rays forming image after two reflections.

To trace the rays of light by which the eye sees a certain image, say I_4, it is best to put in a point E representing the position of a point on the pupil first. Join I_4E, and let this cut M_1 in F; then FE is the path of a ray reaching the eye from M_1. Now I_4 is the image of I_2 in M_1. Join I_2F, cutting M_2 in G. GF is the path of FE before M_1 reflects it. Join AG. Then $AGFE$ is the path of one ray. By choosing other points on the pupil a pencil of rays can be drawn in.

Deviation produced by two inclined mirrors.—A ray strikes M_1 at A, making an angle of incidence α, and then M_2 at B at an angle

of incidence β. The deviation is $(180° - 2\alpha)$ at the first reflection and $(180° - 2\beta)$ at the second, so the total deviation is

$$[360 - 2(\alpha + \beta)]$$

and the angle between the initial and final directions of the ray is $2(\alpha + \beta)$. In the triangle AOB, $B\widehat{A}O = 90 - \alpha$, $O\widehat{B}A = 90 - \beta$; so

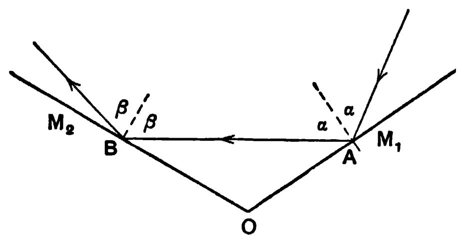


FIG. 44.—Deviation of a ray by two mirrors.

their sum is $180 - (\alpha + \beta)$ and $B\widehat{O}A$ is $(\alpha + \beta)$. The angle between the initial and final directions of the ray is thus *constant for all angles of incidence* and *equal to twice the angle between the mirrors.*

Some uses of plane mirrors.—The optician's Snellen's Chart, to be placed at a distance of, say, 20 feet from the reader, is usually placed just behind him and viewed in a plane mirror of half its linear dimensions placed 10 feet away, the letters on the chart being drawn laterally inverted. Plane mirrors are used for **driving mirrors** inside saloon cars; the large field of view of the convex mirror (p. 59) is an advantage *outside*, but the driver wants to use the small field of view through the rear window and not observe his car's interior. For any position of the eye the field of view can be found by drawing lines joining the eye to the extremities of the mirror; all points whose images lie within the space bounded by these lines produced will be visible.

The **optical lever** is used to measure small displacements, which rotate a plane mirror and cause a reflected beam to rotate through twice the mirror's twist. By observing the reflected rays from a great enough distance this can be made very sensitive; either the image of a scale is viewed in a fixed telescope or a spot of light is reflected on to a white scale. The latter method is used in reflecting galvanometers to observe small rotations of the suspended system, though sometimes concave mirrors are used for this purpose, and always arrangements for focussing the spot sharply are added.

The height and bearings of pilot balloons and aircraft can be found by two observers, each looking through a sighting tube on to a plane mirror, a device due to A. V. Hill. The simultaneous images are covered with ink marks on the mirror surface; the two lines from sight-hole to ink-spot are then treated as theodolite observations to find the depth of the image below the mirror and its position, from which the height and position of the object above the mirror can at once be found. This has the advantage of speed, necessary for moving objects.

The **sextant** depends on the constant deviation of rays striking two mirrors inclined at a fixed angle. It is used to measure the elevation of celestial bodies above the horizon.

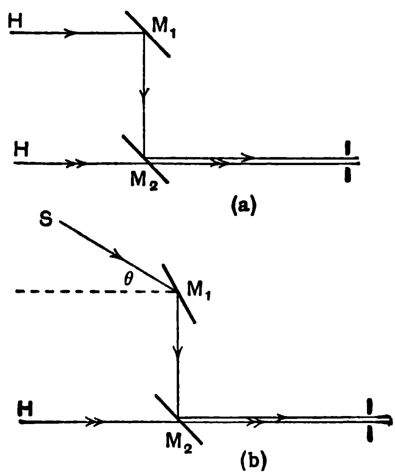


FIG. 45.—Principle of sextant.

M_1 (Fig. 45*a*) is pivoted and attached to a pointer to measure its rotation. M_2 is divided vertically into two halves, one clear and one silvered. To find the elevation θ of an object S above the horizon, M_1 is first set so that the eye at E sees the horizon by reflection at both mirrors (single arrow) and directly through the clear glass (double arrow), the two images appearing continuous. M_1 and M_2 are then parallel and the reading of M_1's position is taken. M_1 is then turned until the image of S (Fig. 45*b*, single arrow) coincides with the horizon seen directly; the position of M_1 is observed, and the angle it has been turned through is half the altitude θ, for light from S has been deviated through an angle θ. It seems curious to employ an instrument

which reduces the angle to be observed, for this surely increases the difficulty of accurate measurement. But the advantage of the sextant lies in the principle of the *two inclined mirrors*; for if the angle of incidence varies, as it will do if the instrument is held in the hand on a rolling ship, the deviation remains constant. Its chief use is then at sea.

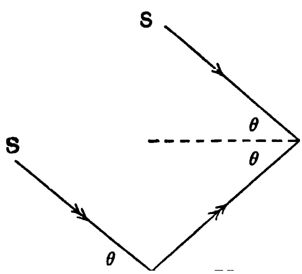


FIG. 46.—Artificial horizon.

The instrument may be used with an artificial horizon, the angle between the object seen directly and its reflection in a pool of mercury being observed (Fig. 46). This is clearly 2θ. If measurements of the altitude of near objects are made by this method, a correction must be made for the fact that rays striking M_1 and those striking the mercury surface are not parallel, so that the angle measured will be greater than 2θ.

Plane sheets of clear glass inclined at 45° to source and object are used to secure strong illumination without glare from the lamp. The objects are viewed through the glass (Fig. 47).

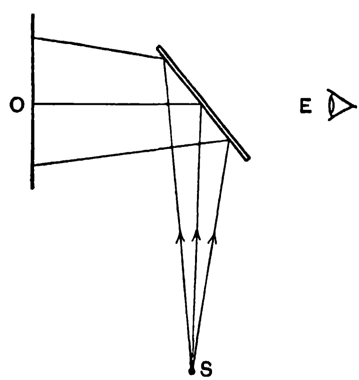


FIG. 47.—Illumination using plane glass sheet.

Two plane mirrors arranged as in Fig. 38*c* form the essential part of the **periscope**, which in all practical types incorporates a terrestrial telescope as well and uses reflecting prisms (p. 102). The inverting reflecting surfaces in prism binoculars are arranged as Fig. 38*b*.

QUESTIONS ON CHAPTER III

1. State the laws of reflection for a parallel beam of light incident upon a plane mirror.

Indicate clearly by means of diagrams (*a*) how the position and size of the image of an extended object may be determined by geometrical construction, in the case of reflection in a plane; (*b*) how the positions of the images of a small lamp, placed unsymmetrically between parallel reflecting planes, may be graphically determined.
(C.W.B.H.S.C.)

2. For what purposes would you use a sextant? Explain exactly how you would use it.

When you look at the junction of two plane mirrors set exactly at right angles you can see your own image. Explain how this image is formed, and how it differs from the image seen in a single plane mirror. (O.H.S.C.)

3. A ray of light is reflected in succession from two plane surfaces inclined to one another at a fixed angle. Show that the deviation of the ray is independent of the angle of incidence on the first surface.
(C.H.S.C.)

4. Explain the construction of the sextant, and how it is used to measure the angle between the sun and the horizon. For what purpose is this measurement made? (O.H.S.C.)

5. Explain the principle of the optical lever, and describe briefly *one* experiment to illustrate its use.

Two plane mirrors are fixed together so that the angle between their reflecting surfaces is constant, and a ray of light is reflected at both mirrors in succession, the plane of incidence being perpendicular to the line along which the reflecting surfaces meet. Prove that if the mirrors are turned about this line as an axis, the direction of the twice-reflected ray is not altered. (C.W.B.H.S.C.)

6. Describe the construction of the *sextant* and the *periscope*. Illustrate your answer by clear diagrams and indicate the optical principles involved. (L.H.S.C.)

7. Two plane mirrors are inclined at an angle α with each other. A ray of light travelling in a plane at right angles to the line of intersection is incident on one of the mirrors at an angle β and is then reflected backwards and forwards from one mirror to the other, finally emerging in a parallel but opposite direction to that of incidence. Show that if n reflections occur,

$$2\beta + (n-1)\alpha = \pi \text{ when } n \text{ is odd,}$$
$$n\alpha = \pi \text{ when } n \text{ is even.}$$
(R.S.)

8. A, B are two points on the surface of a plane mirror, and a ray of light OA is reflected along AP. Show that the path OAP is less than OBP.

Two plane mirrors are inclined at an angle of 45°. A ray of light is incident on one of them in a plane at right angles to their line of intersection and in a direction parallel to the plane which bisects the angle between them. Draw an accurate diagram showing the ray reflected twice at each mirror. (R.S.)

9. Two vertical plane mirrors A and B are inclined to one another at an angle α. A ray of light, travelling horizontally, is reflected first from A and then from B. Find the resultant deviation and show that it is independent of the original direction of the ray. Describe an optical instrument that depends on the above proposition.

(N.U.J.M.B.H.S.C.)

CHAPTER IV

CURVED MIRRORS

Definitions.—Our chief consideration will be convex or concave portions of a spherical surface

The centre of the sphere, of which the mirror is part, is the **centre of curvature** C (Fig. 48). The circular face presented to the incident light is called the **aperture,** the trace of which, cut by the paper, is shown as AB. The line through C and X, the centre of the aperture, is called the **principal axis** and all other lines parallel to it simply axes. The point P, at which the principal axis meets the surface, is called the **pole** of the mirror. The distance PX is called the sagitta of the arc APB. It is seen that $PX(2PC - PX) = AX \cdot XB$, or, if the aperture AB is small compared with the radius, PX^2 will be negligible, so that we may write $PC = \dfrac{AX \cdot XB}{2PX}$, a result we shall use later.

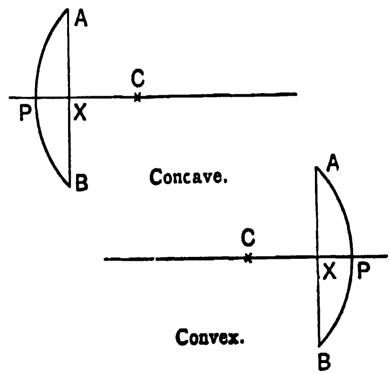


FIG. 48.—Spherical mirrors.

Experiments with the optical disc.—Using strips of cylindrical mirror on the optical disc we can study the behaviour of a section of a spherical mirror in the plane of the disc. A better qualitative demonstration is made with an actual mirror in a smoke box, but by imagining the disc rotated about the principal axis of the mirror we can visualise what is happening in three dimensions.

A wide beam of parallel rays is made to diverge by the convex mirror and converge by the concave. The concave mirror does not bring reflected rays to a point. A bright curve called the **caustic curve** is observed. This is the envelope of the reflected rays, that is, the curve to which they are all tangential. A beam filling the aperture of an actual spherical mirror would give a **caustic surface** shaped somewhat like a trumpet flare. Tracing back the paths of rays reflected from the convex mirror we can see that they do not appear to diverge from a single point, but that their directions envelop a similar curve, a virtual caustic.

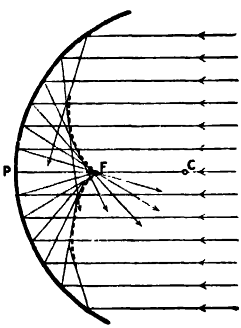


FIG. 49.—Caustic curve.

For a narrow beam of parallel rays parallel to the axis and close to it, the caustic is reduced to a point at the cusp or tip, and if the beam is not close to the axis, to some other small element of the caustic.

The point on the principal axis to which *a narrow beam of parallel rays parallel to the axis and close to it* is made to converge in the case of a concave mirror, or from which it is made to appear to diverge in the case of a convex mirror, is called the **principal focus.** It is a **real principal focus** for a concave mirror as rays actually pass through it, and a **virtual principal focus** for a convex mirror.

Similar caustics are obtained with the wide beams converging to or diverging from points on the axis ; and also for *any kind of beam* inclined to the axis, though in this case the caustic will not be symmetrical about the axis. A point image will thus only be formed of a bright point if the light striking the mirror is confined to a single very narrow pencil incident axially.

Graphical construction.—The radius drawn from C to a point L on the surface of the sphere is normal to a small element of the surface surrounding L, which may, if sufficiently small, be considered plane. We can thus use the laws of reflection at plane surfaces for small elements of a spherical surface.

Consider a ray parallel to the axis striking the surface at L *very close to the pole* P at an angle of incidence θ (Fig. 50). The angle of reflection will be θ, and after reflection it cuts the axis in F. The angle LCF is also θ, so that the triangle LFC is isosceles and $LF = FC$. But if LP is very small $LF = PF$, so $PF = FC$ or $PF = \frac{1}{2}PC$. The distance PF is called the **focal length** and PC the **radius of curvature. The focal length is half the radius of curvature.**

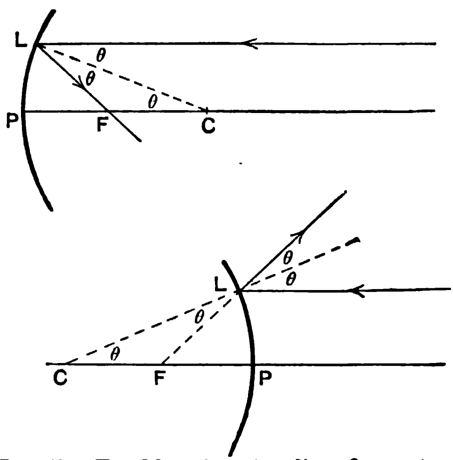


Fig. 50.—Focal length and radius of curvature.

The image of a point close to the principal axis can be found by tracing the paths of rays from it. Two rays only need be taken, but we have a choice of four which are easy to trace. We know that an image is formed, and that the paths of all the rays meet there. The rays are straight lines ; the point at which two rays intersect will be where all intersect, and so must be the image point.

The ray OL (Fig. 51) parallel to the axis is reflected so that it goes through (or appears to proceed from) the principal focus. The ray OF is reflected parallel to the axis. The ray OC strikes the mirror normally and is reflected normally, while OP is reflected making an angle equal to OPC on the other side of the axis. The paths of these rays after reflection are found to pass through I, so that I is the image of O. If the rays actually pass through I it is a **real image**, while if they only appear to diverge from I it is

a virtual image. We could proceed to build up the image of a small extended object close to the axis point by point in this way, but it is sufficient to take one point only to locate it, and that most distant from the axis is chosen. Thus if OO' represents a small object at right angles to the principal axis, II' represents its image.

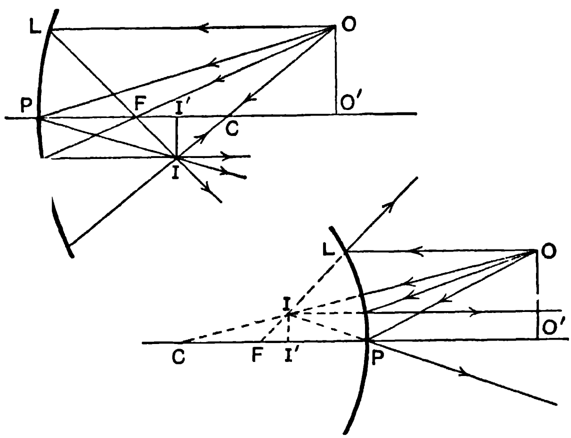


FIG. 51.—Graphical construction.

Before going further, an important deduction can be made from these diagrams. In the similar triangles OPO', IPI', $\frac{II'}{OO'}=\frac{PI'}{PO'}$, so that $\frac{\text{size of image}}{\text{size of object}}=\frac{\text{distance of image}}{\text{distance of object}}$.

In solving problems graphically the first thing to do is to choose scales for distances along the axis and at right angles to it. Draw a line to represent the axis and put in P, F, C; draw a *straight line* perpendicular to this through P to represent the mirror surface. (*It must be remembered that although this line is made as long as is convenient, it really represents the very small region of the mirror in the neighbourhood of the pole.*) Draw a line OO' to scale at the right distance from P to represent the object. Trace the paths of two rays from O and so find the tip I of the image, and then draw in II' perpendicular to the axis to represent the complete image. Interpret II' and PI' in terms of the scale.

EXAMPLE.—An object 5 cm. high is placed on the axis of a concave mirror of 60 cm. radius of curvature at a distance of 80 cm. Find the position, size, and nature of the image.

Scale: 1 cm. represents 5 cm. for both distances and object.

$PF=6$ cm.; $PC=12$ cm.; $PO'=16$ cm.; $OO'=1$ cm.

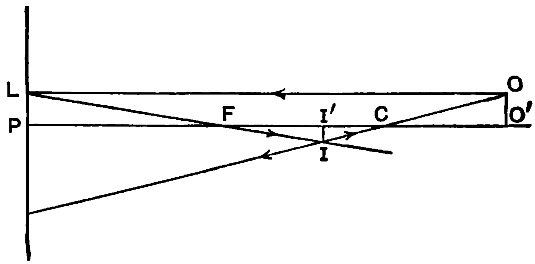


FIG. 52.—Example. (Reduced in scale.)

From O draw OL parallel to the axis; its path after reflection is LF. Draw OC; this ray is reflected after striking the mirror along its own path. These intersect at I. Draw II'. PI' is 9·6 cm., II' is 0·6 cm. So the distance of the image is 48 cm., and it is 3 cm. high, real, and inverted.

Conjugate foci.—A set of diagrams will enable a table of the following type to be drawn up. The results shown below are for a concave mirror of 60 cm. radius of curvature.

Distance of object PO'	Position referred to mirror	Distance of image PI'	Position referred to mirror	Kind of image
1. Very great	∞	30 cm.	At F	Point, real
2. 80 cm.	Beyond C	48 cm.	C and F	Real, inverted, diminished
3. 60 cm.	At C	60 cm.	At C	Real, inverted, same size
4. 48 cm.	C and F	80 cm.	Beyond C	Real, inverted, magnified
5. 30 cm.	At F	Very great	∞	Parallel beam
6. 20 cm.	Between P and F	40 cm.	Behind mirror	Virtual, upright, enlarged
7. 0 cm.	At P	0 cm.	At P	Virtual, upright, same size

Notice the pairs of results 1 and 5 and 2 and 4. Object and image are seen to be interchangeable. We can state that if an object placed at distance p gives an image at distance q from the

mirror, then an object at distance q will give an image at distance p. The two points at distances p and q from the mirror are called a pair of **conjugate points** or **conjugate foci.**

In cases 3 and 7, object and image are in the same position, so that pole and centre of curvature are two cases in which *both conjugate points coincide.* They are called **self-conjugate points.**

Case 6 appears isolated. At first sight it looks as if the conjugate property has broken down, for we cannot get an object at I behind the mirror, that is, a real object; but a beam of rays converging towards the point I serves as a virtual object (which would have been at I if the mirror had not intercepted the light) giving a real image at O. So that if one of the conjugate points is behind the mirror, either a virtual image or a virtual object may be there.

For a convex mirror, parallel rays from a very distant object give a virtual image at F, and images of all other real objects are virtual and formed between P and F. At a greater distance than PF behind the mirror is a region in which both conjugate points will be virtual, a case of little practical interest.

Formulae.—The information derived laboriously in the last paragraph and tabulated at length can be summarized in simple algebraic formulae connecting the distances of O and I from the pole. It is necessary before proceeding to this to establish a sign convention, as we shall be dealing with distances on either side of the pole and must distinguish between the two sides in the formulae.

There are two ways of doing this. The first is to take the pole as the origin of a Cartesian coordinate system and call all distances measured outwards from the pole on one side positive, and on the other side negative. It has been customary in school text-books to take the direction of the incident light as negative, perhaps drawing all diagrams with the light coming from the right-hand side so that the sign arrangement of ordinary graphical work is really used. The *Report of the Committee of the Physical Society on the teaching of Geometrical Optics*, published in 1934, recommended the abandonment of this usage and suggested that if a Cartesian system were employed, the direction of the incident light should be taken as positive.

The other method recommended in the Physical Society's Report has several advantages over any Cartesian system. In this, all distances actually traversed by light in coming from a real object or to a real image are positive, and all distances traversed by light virtually in appearing to converge to a virtual object or diverge from a virtual image are negative.

All real distances are positive. All virtual distances are negative.

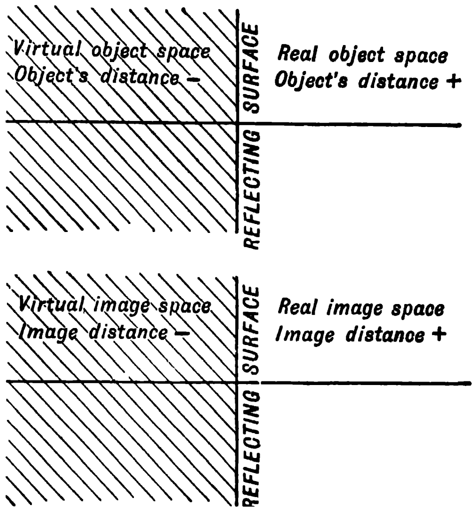


FIG. 53.—Object and image spaces for a reflecting surfaco.

The rule for focal lengths follows at once. **If the principal focus is real, the focal length is positive, while for a virtual principal focus the focal length is negative. The radius of curvature of a surface has the same sign as its focal length.** This is the system we shall use. Fig. 53 shows all that has to be remembered in applying it to mirrors. Real objects and real images only can occupy the space in front of the mirror surface; virtual objects and virtual images must both be somewhere in the space behind the mirror.

So, for a *concave mirror* with **real principal focus,**

the focal length is

the radius of curvature is +.

For a *convex mirror* with **virtual principal focus,**
the **focal length is** $-$,
the **radius of curvature is** $-$.

Object and image distances for mirror of small aperture.—Consider a bright point O on the axis. OL is a ray striking the mirror at L very close to P, which is reflected to cut the axis in I. Call the angle LOP α, LCP β, LIP γ, and let the angles of incidence and reflection at L both be θ.

(i) The **concave mirror.**

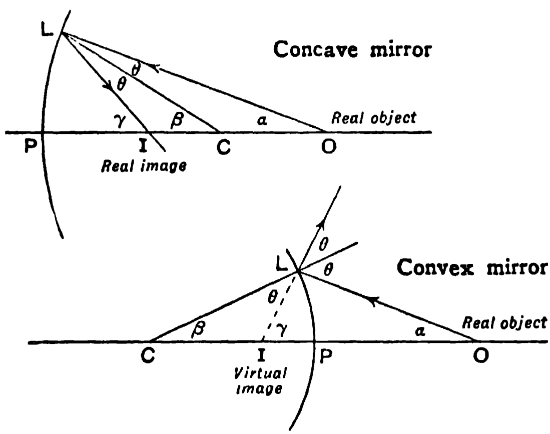


FIG. 54.

In Fig. 54, $\gamma = \theta + \beta$ (**exterior** angle and **interior opp.**),
$\beta = \theta + \alpha$.

So, $\gamma - \beta = \beta - \alpha$ or $\alpha + \gamma = 2\beta$.

Now if PL is very small we have in radians approximately

$$\alpha = \frac{PL}{PO}, \quad \beta = \frac{PL}{PC}, \quad \gamma = \frac{PL}{PI}.$$

So,
$$\frac{1}{PO} + \frac{1}{PI} = \frac{2}{PC}.$$

Now we have agreed that with the sign convention chosen, PO, PI and PC shall all be positive algebraically. The formula as it stands gives the *numerical relation* between the lengths. To convert it to the *algebraic* form.

Let u stand for the quantity $(+PO)$ (real object);
„ v „ „ „ $(+PI)$ (real image);
„ r „ „ „ $(+PC)$.

Then substituting,

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}.$$

And as we have seen $r=2f$, we can also write $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

Note carefully again what the quantities u, v, r *represent. They are the distances with their right algebraic signs.*

(ii) The **convex mirror.**

In Fig. 54,

$$2\theta=\alpha+\gamma,$$
$$\theta=\alpha+\beta.$$

So,

$$\gamma-\alpha=2\beta.$$

As before,

$$\alpha=\frac{PL}{PO},\quad \beta=\frac{PL}{PC},\quad \gamma=\frac{PL}{PI}.$$

So,

$$\frac{1}{PI}-\frac{1}{PO}=\frac{2}{PC}.$$

This is the *numerical equation.*

As

$$u=(+PO) \text{ (real object)},$$
$$v=(-PI) \text{ (virtual image)},$$
$$r=(-PC),$$

the algebraical equation is again

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}.$$

As to the meaning of this equation now we have it. The first step in the argument was to eliminate θ, so that the position of I will be the same for a given O for all values of θ provided the approximation made for small angles holds. So all rays from O pass through, or appear to pass through, I and I is the image of the point O. For points very close to O corresponding points very close to I will be found. So the image formed by the small region of the mirror near the pole of a small object on the principal axis will be sharp, and in the equation $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}$, u and v represent the algebraic distances of object and image from the pole.

The linear magnification, $m = \dfrac{\text{height of image}}{\text{height of object}}$, has been shown to equal $\dfrac{\text{distance of image}}{\text{distance of object}}$, so $m = \dfrac{v}{u}$. This will be positive if v and u have the same sign as for a real inverted image of a real object, and negative for a virtual image of a real object.

(A negative sign may also be used to indicate *inversion*, and the formula is then $m = -v/u$. At this stage, when the all-important thing to remember is "real positive, virtual negative," it is not proposed to adopt this custom, but to use $m = v/u$).

We have obtained the same equation for both concave and convex mirrors ; if it is true in each case when r is large it should hold for a plane mirror, which is simply a curved mirror of infinite radius of curvature, r. The equation $1/v + 1/u = 0$ gives $v = -u$ and $m = -1$, which is what is found independently for a plane mirror.

The student is recommended to write all problems out at length as in the following example.

EXAMPLE.—Find the position, size, and nature of the image of an object 5 cm. long placed upright on the principal axis of a concave mirror of 60 cm. radius of curvature at a distance of 80 cm. from the pole.

Here PF, PC, PO are all real distances and positive.

So u is $+80$ cm. : r is $+60$ cm.

Using the equation, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{2}{r}$, $\dfrac{1}{v} + \dfrac{1}{80} = +\dfrac{2}{60}$.

So, $\dfrac{1}{v} = +\dfrac{1}{30} - \dfrac{1}{80} = +\dfrac{1}{48}$. $\therefore v = +48$ cm. ;

that is, the image is 48 cm. in front of P. The magnification $m = \dfrac{v}{u} = \dfrac{+48}{+80} = 0{\cdot}6$. So the image is real and inverted and $5 \times 0{\cdot}6$ or 3 cm. high.

Deductions from the equation $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{2}{r} = \dfrac{1}{f}$.—From this formula all the information tabulated on p. 49 can be extracted, and much more besides.

The form of it shows the conjugate property at once ; for given two algebraic distances p and q, if the equation is satisfied when $u = p$ and $v = q$ it must also be satisfied when $u = q$ and $v = p$. Self-conjugate points will occur when $u = v = r$, or when $u = v = 0$.

To show the variation of v with u, graphs of $\frac{1}{v}$ against $\frac{1}{u}$ or v against u may be drawn, but these are discussed later. We can obtain all that a graph could give us from the formula. Differentiating $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$-\frac{dv}{v^2}-\frac{du}{u^2}=0, \quad \text{or} \quad \frac{dv}{du}=-\frac{v^2}{u^2}.$$

That is, if the object is moved in towards the pole the image moves out in the opposite direction, and moves most rapidly when u is small.

If we choose to measure distances x and y out from the principal focus, writing

$$u=f+x, \quad v=f+y,$$

the formula becomes

$$\frac{1}{f+y}+\frac{1}{f+x}=\frac{1}{f},$$

which reduces to $xy=f^2$, the **Newton formula.**

The magnification $m=\frac{v}{u}$ becomes $\frac{f+y}{f+x}$ or $\frac{f+\frac{f^2}{x}}{f+x}$ or $\frac{\frac{fx+f^2}{x}}{f+x}$, which is $\frac{f}{x}$ or, since $xy=f^2$, $\frac{y}{f}$.

Always x and y have the same sign, since f^2 must be positive. For a convex mirror with a real object x must always be greater than f and positive, so y is less than f and the image always lies between F and P. For a concave mirror, if x is greater than f and positive, y is less than f and positive, so that an object further from the mirror than C gives an image between F and C, and conjugately; if x is negative and between 0 and f, y is negative and between ∞ and 0, so that with the object between F and P we get a virtual image somewhere behind the mirror between the pole and infinity.

As to the magnification, if $y=f$, $m=1$, which we already know. If $y=2f, 3f, 4f, \ldots$, $m=2, 3, 4, \ldots$. If $x=y=-f$, then $m=-1$. We can thus graduate the axis in a series of equal steps at distances f apart and make a scale from which we can read at once the magnification at any point of an image received there

(Fig. 55). Note that there are *two* points at which the magnification is unity, the two self-conjugate points.

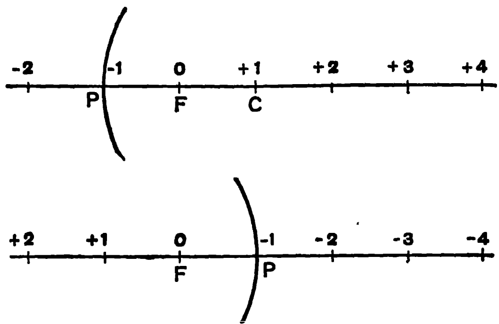


FIG. 55.—Image positions for a given magnification.

Superficial and longitudinal magnification.—We have seen that the lateral linear magnification is $\frac{v}{u}$. A small area normal to the axis has its length and breadth both magnified in this ratio in the image and the **superficial magnification** is thus $\frac{v^2}{u^2}$.

The **longitudinal magnification** for a small object lying along the axis will be $-\frac{v^2}{u^2}$. For if it lies between u and $u+\delta u$, and the image lies between v and $v-\delta v$, the size of object and image are respectively δu and δv and the magnification $\frac{\delta v}{\delta u}$ is nearly $\frac{dv}{du}$, which equals $-\frac{v^2}{u^2}$.

The image of an extended object.—We have considered so far the image of a small object on the axis formed by narrow pencil incident near the pole. Oblique incidence or a wide pencil will produce blurring in general; while a large object, even if a clear image is obtained by selecting one narrow pencil from each point, will give a curved image which is also distorted, as different parts are differently magnified.

A ray of light from O, parallel to the axis which strikes the mirror at M, will intersect the axis at I', which is nearer to P than the corresponding point for OL, which is closer to the pole.

The effective focal length of the mirror is less for light striking near the edge of the aperture than the true focal length for rays near the pole. This effect is called **spherical aberration** (Fig. 56*a*).

Almost inseparable from this is the behaviour of a pencil of rays from O striking a small area of the surface at M (Fig. 56*b*).

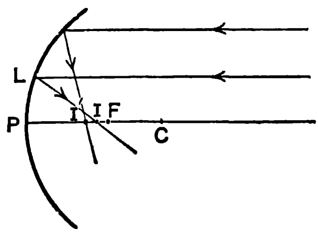


FIG. 56*a*.—Spherical aberration.

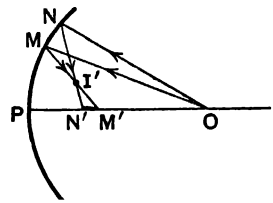


FIG. 56*b*.—Astigmatism.

In the plane of the paper the rays ON and OM converge after reflection to intersect at I', and cut the axis at M' and N'; all rays striking the arc MN will pass through I and cut the axis somewhere between M' and N'. If the diagram is now rotated about the axis OP through a small angle, I' describes a small arc, but $M'N'$ remains a line. The pencil striking the small area described by MN is thus brought not to *one point*, but to *two lines*. This effect is called **astigmatism.** The lines formed by I' and $M'N'$ are called the first and second focal lines respectively. The nearest approach to a point focus is where the area of the pencil between I' and $M'N'$ is least. This is the **circle of least confusion.**

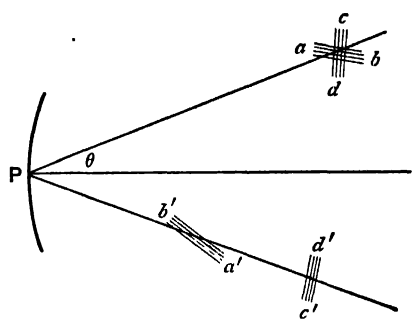
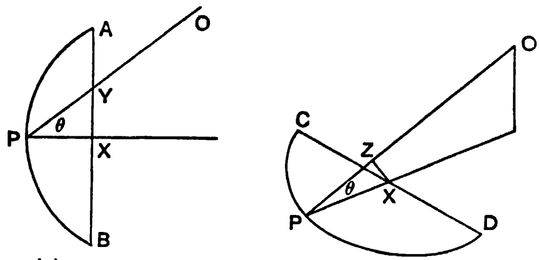


FIG. 57.—Astigmatism alone.

We attribute spherical aberration to a wide angle pencil and astigmatism to oblique incidence. It is clear that where we have spherical aberration we shall always have astigmatism too. But we can study astigmatism alone by having narrow pencils incident near the pole from a small object well above the axis (Fig. 57), at an

angle θ. Horizontal lines will appear sharp at the first focal line and vertical lines at the second. From the point of view of the light, the mirror has different curvatures in vertical and nearly horizontal planes. Consider the figures. The radius of curvature



(a) Vertical plane (plane of incidence) (b) Horizontal plane

FIG. 58.—Astigmatism.

r is proportional to $\frac{1}{PX}$ (p. 45). In a vertical plane (Fig. 58*a*) PX should be replaced by PY, where $PY = \frac{PX}{\cos\theta}$, so that r is effectively $r\cos\theta$. In the plane perpendicular to the paper containing OP, PX should be replaced by $PZ = PX\cos\theta$, so r is effectively $\frac{r}{\cos\theta}$. If our measurements of u and v are made *along the incident and reflected beams* the equations giving v for the first and second focal lines will then be

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r\cos\theta},$$

$$\frac{1}{v}+\frac{1}{u}=\frac{2\cos\theta}{r}.$$

These equations will be found derived properly in more advanced books.

Use of stops.—A simple aperture stop will improve definition in the centre of the field but leave the edges still blurred. But if the stop is placed at the centre of curvature, each point on the object can send only a narrow beam to the surface, which will strike its own part of the surface nearly normally, so that both defects are minimized.

With such a stop we shall get clear definition, point for point. But a point at O will use the pole P and give an image at I on the axis ; a point at O_1 will make P_1 its pole and give an image

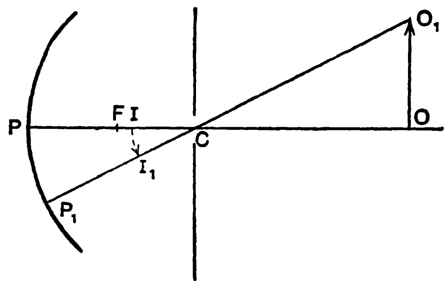


FIG. 59.—Stop at centre of curvature.

at I_1, and as $P_1O_1 > PO$, $P_1I_1 < PI$, so the image will be curved. The magnification of the edge, $\frac{P_1I_1}{P_1O_1}$, will be less than that of the middle, $\frac{PI}{PO}$, and so it will also be distorted.

Ellipsoidal and paraboloidal mirrors.—It is a property of the ellipse that the angles made by the lines SP, $S'P$ drawn from the foci to any point P on the curve make equal angles with the normal at P. All rays diverging from S are thus made to pass through S' (Fig. 60). The same will be true of an ellipsoid of revolution whose axis is SS'. S and S' are called aplanatic points for the surface. The parabola may be regarded as an ellipse with one focus at infinity. Hence in the paraboloid, which is obtained by rotating a parabola about its axis, all rays parallel to the axis are brought to a point at its focus without any aberration, and a point source at its focus will give a parallel beam parallel to the axis.

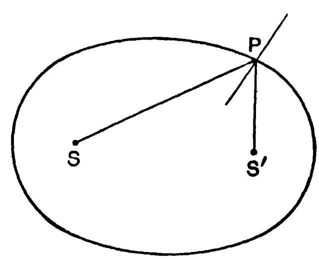


FIG. 60.

Uses of curved mirrors.—Concave mirrors, usually paraboloidal in form, are used as the objectives of large telescopes and as reflectors in motor headlamps and searchlights. Spherical concave mirrors are used as shaving mirrors, and also find a place in the dentist's armoury.

Convex mirrors are used for external driving mirrors on cars and for many precautionary and ornamental purposes.

Measurements on spherical mirrors.—*To find the radius of curvature. Method I.—The spherometer.* Three pointed legs, L, M, N, are set at the corners of an equilateral triangle. The point P of a micrometer screw is, when in the same plane as the tips of the legs, at the centre of the circle circumscribing the triangle LMN. If $LM = MN = NL = l$, the radius of the circle is then $\frac{2}{3}NQ = \frac{2}{3}\frac{l}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = \frac{l}{\sqrt{3}}$ (Fig. 61). The spherometer is placed on a plane sheet of glass and the zero reading of the micrometer taken when all four points just touch the surface. It is then placed on the mirror, P adjusted until all four points lie in the curved surface, and the micrometer read. The difference b between the two readings gives the height of P above the centre X of the circular section AB (Fig. 62) on which L, M and N lie. In the diagram,

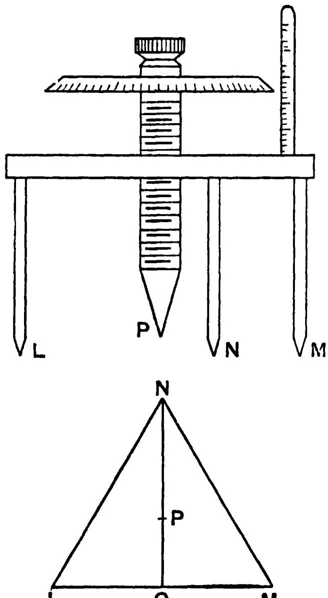


FIG. 61.- -Spherometer.

$$AX \, . \, XB = PX \, (PP' - PX),$$

or

$$\frac{l}{\sqrt{3}} \cdot \frac{l}{\sqrt{3}} = b\,(2r - b).$$

So,

$$r = \frac{b}{2} + \frac{l^2}{6b}.$$

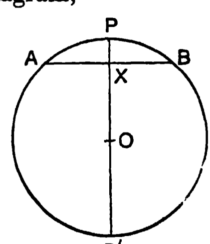


FIG. 62.

With ordinary mirrors this is not a very satisfactory method, as it measures the radius of curvature of the front face of the glass shell on whose back the mirror is deposited.

Method II.—The revolving table. The mirror is mounted on a

horizontal slide on a table which rotates about a vertical axis, which the principal axis of the mirror always intersects. A small vertical object is held in a stand not connected with the table. The mirror is moved parallel to its principal axis until a position is found at which rotation of the table produces no displacement of the image. The axis of rotation is then at the mirror's centre of curvature (Fig. 63a). The mirror is then moved until rotation produces no motion of its pole, when the axis of rotation passes through the pole (Fig. 63b). The distance the mirror is moved between these positions is the radius of curvature.

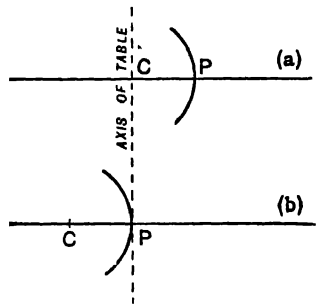


FIG. 63.—Revolving table.

Method III.—Self-conjugate point. The centre of curvature is a self-conjugate point, so a small object placed at C will give an image in the same position. With a concave mirror we can use a small lamp slightly off the principal axis and receive its real image on a screen, or we can use as object the tip of a pin on the principal axis and find when there is no parallax between object

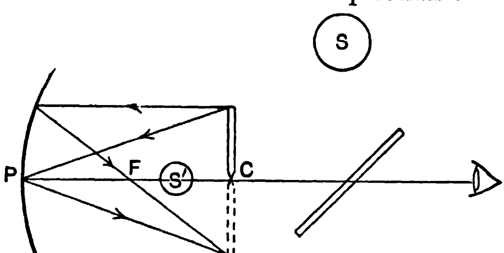


FIG. 64.—Locating centre of curvature by "no parallax."

and real image. If the object has to be illuminated on the side nearer the mirror the source will probably have to be inconveniently near the line of sight. The best way of proceeding is to form a real image S' between F and C of a lamp S with the help of an inclined sheet of glass, so that both object and image stand out as sharp shadows on a bright background.

In the case of a convex mirror the centre of curvature is virtual. A convex lens L and the mirror are set up with their axes parallel at a distance apart LP, which should be about the focal length of the lens. The lens gives an image I of the object O on its axis; I acts as a virtual object, of which, if it falls beyond the mirror's principal focus, a virtual image I' will be

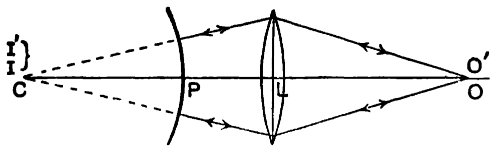


FIG. 65.—Locating centre of curvature of convex mirror.

formed. The lens will now give a real image O' of I'. Now I and O are conjugate points for the lens, so that if I and I' both fall at C, O and O' coincide. The distance LP is measured, the mirror removed, and LI measured. The radius of curvature PC is then $LI - LP$.

With the concave mirror there could be no confusion between the two self-conjugate points, but with this arrangement it is possible, for O and O' will again coincide when I and I' are both at P. *The two cases are easily distinguishable as for the "object" I' at C the real image O' of I' will be inverted, while if I' is at P, O' will be erect.*

Method IV.—Conjugate foci. With a concave mirror an object O outside the principal focus gives a real image I. If O is a pin with its tip on the axis, I can be located by the no-parallax

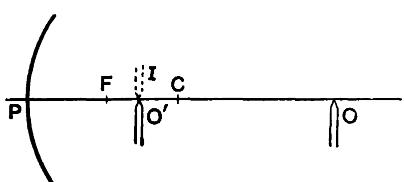


FIG. 66.—Conjugate foci.

method, using a finder-pin O'; PO and PO' are measured and substituted for u and v in the formula $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$. Great accuracy is not gained by taking the readings over a large range of u and

v. It is best to make several settings with O always somewhere near C, for then, as u and v are of the same order, the accuracy to which they can be measured is about the same. For example, suppose that $r = +20$ cm. and u were to be taken $+100$ cm.; then v would be $+11$ cm., and for comparable accuracy in the single measurements v must be measured in millimetres where u is measured in centimetres. Even when this is done, since $\frac{dv}{du} = -\frac{v^2}{u^2}$ or 1% in this case, an uncertainty of 1 mm. in v is of the same importance in the formula as one of 10 cm. in u!

For a convex mirror the position of the virtual image can be found, using an auxiliary plane mirror.

The plane mirror M is set normal to the axis of the convex mirror and covering half of it. The object OO is placed vertically.

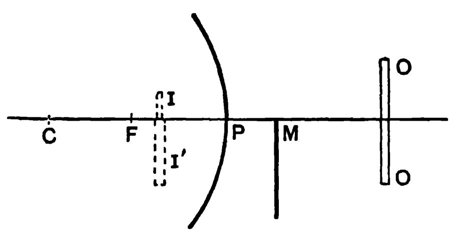


FIG. 67.—Conjugate foci for convex mirror, using auxiliary plane mirror.

I is the image of its upper part in the convex mirror and I' that of the lower part in the plane mirror. The position of M is altered until the two virtual images coincide without parallax (Fig. 67). Then $MI = MI' = MO$, so that

$$PI = MI - MP = MO - MP.$$

With the sign convention we write

$$v = +PI,$$
$$u = +PO,$$

in the formula,
$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

If it is desired to experiment with virtual images for a concave mirror, this method may be employed; but it will be impossible to make the two images I and I' of a single object coincide, so that a second pin, O', is used to produce I'.

A graphical method of combining observations and working out f without the use of reciprocal tables is of interest. Values of u are marked off along one axis and v along the other, but instead of the points being plotted in the space of the diagram, corresponding points on the axes, such as L and M, are joined. All lines

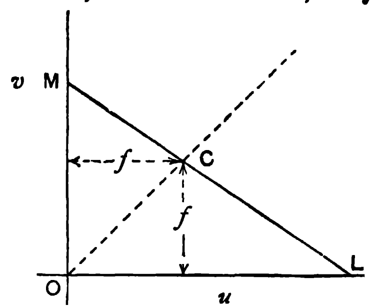


FIG. 68.—Graphical method of combining observations.

such as LM should intersect in a point C such that OC is inclined at 45° to the axis. The coordinates of C are (f, f); for the area $LOM = \frac{1}{2}uv$, area $LCO = \frac{1}{2}fu$, area $MCO = \frac{1}{2}fv$, giving $uv = fu + fv$ or $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

Method V.—A "field of view" method. A long object such as a metre scale is set up at O. Two strips of card cut the aperture of the mirror down to a width of l cm., and through this the image of actual height y of some L cm. of the object is observed. With the eye at the point marked O, OAB in the figure must be a straight line.

(*a*) *For a convex mirror* (Fig. 69*a*),

$$\frac{L}{y} = \frac{PO}{PI} \quad \text{(magnification)},$$

$$\frac{y}{l} = \frac{IO}{PO} \quad \text{(similar triangles)}.$$

So
$$\frac{L}{l} = \frac{IO}{PI} = \frac{PO + PI}{PI} = \frac{PO}{PI} + 1.$$

So
$$PI\left(\frac{L}{l} - 1\right) = PO.$$

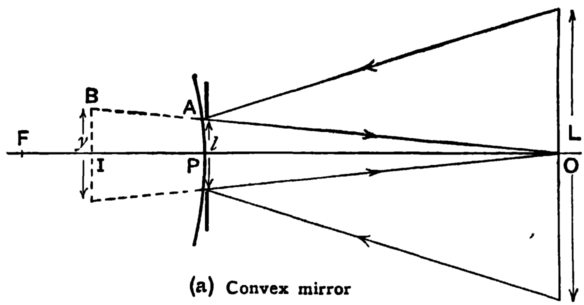
Now,
$$(+PO) = u \quad \text{(real object)},$$
$$(-PI) = v \quad \text{(virtual image)},$$

so $$v\left(1-\frac{L}{l}\right)=u \quad \text{or} \quad \frac{1}{v}=\frac{1}{u}\left(1-\frac{L}{l}\right)$$

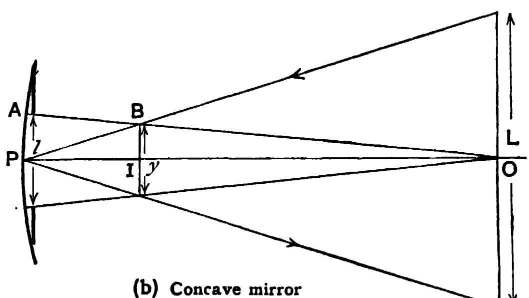
And substituting in $$\frac{1}{v}+\frac{1}{u}=\frac{2}{r},$$

$$\frac{1}{u}\left(1-\frac{L}{l}\right)+\frac{1}{u}=\frac{2}{r}.$$

Whence, $$r=\frac{2ul}{2l-L}.$$



(a) Convex mirror



(b) Concave mirror

FIG. 69.—" Field of view " method.

(*b*) *For a concave mirror.* The real inverted image is viewed from O.

As before (Fig. 69*b*),

$$\frac{L}{y}=\frac{PO}{PI}, \quad \frac{y}{l}=\frac{IO}{PO},$$

so $$\frac{L}{l}=\frac{IO}{PI}.$$

Now in this case, $IO = PO - PI$,

$$\frac{L}{l} = \frac{PO - PI}{PI} = \frac{PO}{PI} - 1,$$

and
$$\frac{PO}{PI} = \frac{L}{l} + 1.$$

As object and image are both real $u = (+PO)$ and $v = (+PI)$, so

$$\frac{u}{v} = \frac{L}{l} + 1,$$

$$\frac{1}{v} = \frac{1}{u}\left(\frac{L}{l} + 1\right);$$

and using
$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r},$$

$$\frac{1}{u}\left(\frac{L}{l} + 1\right) + \frac{1}{u} = \frac{2}{r}, \text{ whence}$$

$$r = \frac{2ul}{2l + L}.$$

QUESTIONS ON CHAPTER IV

1. Why are reflecting mirrors on motor cars usually convex? Describe in detail how you would measure the focal length of such a mirror.

Calculate the length of the image of a pole which lies along the principal axis of a convex mirror of focal length 1 foot, if one end of the pole is 4 feet from the mirror surface and the other end is 9 feet from it. (O.H.S.C.)

2. $PBCA$ is the axis of a concave spherical mirror, A being a point object, B its image, C the centre of curvature of the mirror and P the pole. Find a relation between PA, PB and PC supposing the aperture of the mirror to be small.

A concave mirror forms, on a screen, a real image of twice the linear dimensions of the object. Object and screen are then moved until the image is three times the size of the object. If the shift of the screen is 25 cm., determine the shift of the object and the focal length of the mirror. (N.U.J.M.B.H.S.C.)

3. Establish the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ for a concave mirror.

In an experiment with a concave mirror the magnification m of the image is measured for a series of values of v, and a curve is plotted between m and v. What curve would you expect to obtain, and how would you use it to deduce the focal length of the mirror?

(C.H.S.C.)

4. Show how for a concave mirror the distances of a point object and its image from the mirror surface are related to its focal length, and point out the assumptions you make.

Describe the effect of using a mirror whose aperture is large compared to its focal length. (O.H.S.C.)

5. Under what conditions does a concave mirror give an undistorted image of a small object ?

Describe and explain what you see when, in a dark room, a number of narrow beams of white light fall on a concave mirror of large aperture in directions parallel to its principal axis. (O.H.S.C.)

6. Deduce a formula connecting the distances of object and image from a spherical mirror.

What are the advantages of a concave mirror over a lens for use in an astronomical telescope?

A driving-mirror consists of a cylindrical mirror of radius 10 cm. and length (over the curved surface) of 10 cm. If the eye of the driver be assumed to be at a great distance from the mirror, find the angle of view. (O. & C.H.S.C.)

CHAPTER V

REFRACTION

Introduction.—When a beam of light travelling in air strikes the boundary of a transparent substance such as water or glass, some of the light is transmitted and some is reflected. Considering a single transmitted ray, its direction is bent towards the normal at the point of incidence except when it strikes the surface normally. This bending is called **refraction,** and the angle between the refracted ray and the normal at the point of incidence is called the **angle of refraction.**

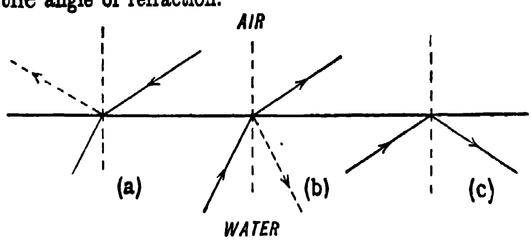


FIG. 70.—Refraction.

A beam travelling in water or glass and striking the air-bounded surface also undergoes partial reflection and transmission if the angle of incidence is less than a certain value. A single refracted ray is bent away from the normal at the point of incidence. For large angles of incidence there is no refracted ray; all the light striking the boundary at an angle exceeding a certain critical angle is reflected, the surface acting like a perfect mirror. This is called **total internal reflection.**

Fig. 70 illustrates these statements, and simple experiments to verify them can be performed using a parallel light projector and a glass-sided tank of water, to which a little fluorescein has been added to show up the track of the beam, the path in air being made visible with chalk-dust or smoke.

If rays travelling from medium A to medium B are bent towards the normal on refraction, it is said that B is *optically denser* than A. The term *dense medium* alone, used in this sense, is a generic term for substances such as glass and water which refract rays incident from air in the manner described.

The laws of refraction.

Law I.—The incident and refracted rays and the normal at the point of incidence all lie in one plane.

Law II (Snell's Law).—For two given media the ratio $\dfrac{\text{sine of angle of incidence}}{\text{sine of angle of refraction}}$ is constant, for light of a given colour.

These laws may be demonstrated with the optical disc, using a glass block of semicircular section. If a narrow parallel beam strikes the plane face AB at its centre O, whatever its direction after refraction it will always strike the curved surface normally

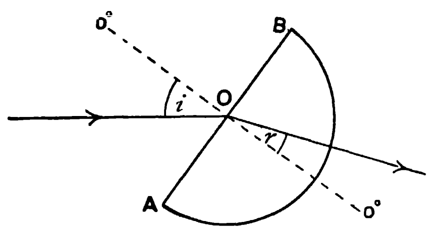


FIG. 71.—Demonstration of laws of refraction.

and be undeviated there, for O is the centre of the semicircle. We shall thus be investigating the effect of one refraction only, that at O. The block is set on the disc with AB along the 90°-90° line and O at the centre. The incident beam grazes the surface of the disc, and so does the refracted beam ; this illustrates the coplanar law. The angle of incidence i and the angle of refraction r are both read off for several different positions of the block and the results tabulated. The ratio $\frac{\sin i}{\sin r}$ is found to be constant within the limits of the experiment.

The refracted beam will be found edged with blue on the side nearer the 0°-0° line and red on the other side. The red part of the lamp's spectrum is bent less than the blue part. In the middle of the refracted beam the different colours overlap, so it is only at the edges that the colours are noticed separated. The value of $\dfrac{\sin i}{\sin r}$ is thus least for the red part of the incident light and greatest for the blue part. The difference is not sufficiently

great to be measured in this experiment with any accuracy, and is unimportant in most of the elementary experiments on refraction. In accurate experiments it will be of importance, and *light of a single colour* such as that from a sodium flame must be used. By a ray of light we must in future understand the direction in which such **monochromatic** light is travelling.

Refractive index.—The ratio $\frac{\sin i}{\sin r}$ for light passing from medium 1 to medium 2 is called the **refractive index** from medium 1 to medium 2 and is written ${}_1\mu_2$, or with other distinguishing suffixes. In the above experiment the ratio works out to about 1·5, the refractive index ${}_{\text{air}}\mu_{\text{glass}}$.

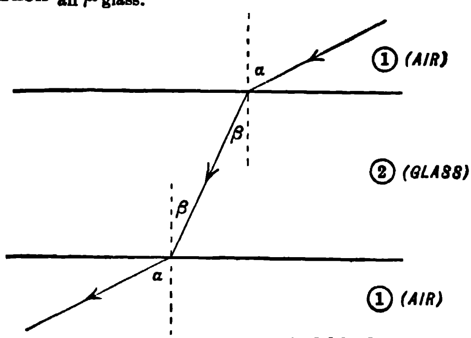


FIG. 72.—Parallel-sided block.

It is found experimentally that a parallel beam of light incident on a parallel-sided block of glass emerges after refraction at both surfaces of the block parallel to its original direction, though displaced sideways.

In Fig. 72, consider one ray from such a beam incident on a parallel-sided block of glass in air. The angle between the ray emerging into air and the normal at the point of emergence, the **angle of emergence,** must equal the angle of incidence on the first face, α. The angle of incidence at the second face equals the angle of refraction at the first, β, as these are alternate angles.

Then, $${}_1\mu_2 = \frac{\sin \alpha}{\sin \beta}, \quad {}_2\mu_1 = \frac{\sin \beta}{\sin \alpha}$$

So, $${}_2\mu_1 = \frac{1}{{}_1\mu_2}.$$

For example, $\quad {}_{\text{glass}}\mu_{\text{air}} = \dfrac{1}{1{\cdot}5} = 0{\cdot}67.$

Two or more parallel-sided slabs of different materials will clearly also give a final emergent ray parallel to the incident ray.

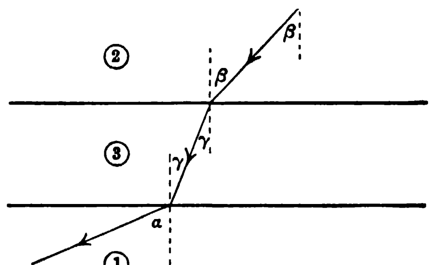


FIG. 73.—Two parallel-sided blocks.

In Fig. 73, consider a ray incident from medium 1 to medium 2, then from medium 2 to medium 3, and finally out from medium 3 to medium 1, the angles of incidence at the three boundaries being respectively α, β, γ and the angle of emergence into 1 being α.

Then, $\quad {}_1\mu_2 = \dfrac{\sin\alpha}{\sin\beta}, \quad {}_2\mu_3 = \dfrac{\sin\beta}{\sin\gamma}, \quad {}_3\mu_1 = \dfrac{\sin\gamma}{\sin\alpha}$

So, $\quad {}_1\mu_2 \cdot {}_2\mu_3 \cdot {}_3\mu_1 = \dfrac{\sin\alpha \,.\, \sin\beta \,.\, \sin\gamma}{\sin\beta \,.\, \sin\gamma \,.\, \sin\alpha} = 1.$

Or, $\quad {}_2\mu_3 = \dfrac{1}{{}_1\mu_2 \cdot {}_3\mu_1}.$

Or, as we have seen that $\quad {}_1\mu_3 = 1/{}_3\mu_1,$

$$ {}_2\mu_3 = \frac{{}_1\mu_3}{{}_1\mu_2}. $$

For example, if $\quad {}_{\text{air}}\mu_{\text{glass}} = 1{\cdot}5,$

$\quad {}_{\text{air}}\mu_{\text{water}} = 1{\cdot}3.$

Then, $\quad {}_{\text{water}}\mu_{\text{glass}} = \dfrac{1{\cdot}5}{1{\cdot}3} = 1{\cdot}15.$

Absolute refractive index.—The refractive index for a ray entering a medium from a vacuum is called the **absolute refractive index** of the medium. For air the absolute refractive index is

about 1·00028, so that for ordinary purposes the distinction between absolute refractive indices and those reckoned from air is negligible for substances like water and glass. From the equation of the preceding paragraph the absolute refractive index can be expressed in terms of that of air and the refractive index of the medium in air.

For, since $${}_2\mu_3 = \frac{{}_1\mu_3}{{}_1\mu_2},$$

$${}_{\text{air}}\mu_{\text{glass}} = \frac{{}_{\text{vac}}\mu_{\text{glass}}}{{}_{\text{vac}}\mu_{\text{air}}}$$

or $${}_{\text{vac}}\mu_{\text{glass}} = {}_{\text{air}}\mu_{\text{glass}} \times {}_{\text{vac}}\mu_{\text{air}}.$$

The equation ${}_1\mu_2 = \frac{\sin i}{\sin r}$ can be written in a more symmetrical form. For if μ be the absolute refractive index of the first medium and μ' of the second, ${}_1\mu_2 = \frac{\mu'}{\mu}$.

So, $$\frac{\mu'}{\mu} = \frac{\sin i}{\sin r}$$

or $$\mu' \sin r = \mu \sin i.$$

That is, the product of refractive index and the sine of the angle made with the normal at the point of incidence is constant for a given ray in both media.

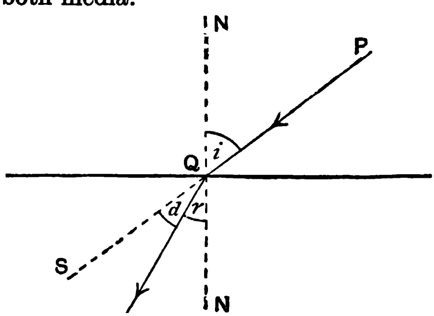


FIG. 74.—Deviation of a single ray.

The deviation of a ray by a single refraction.—The angular deviation is the angle between the directions of incident and refracted rays. In Fig. 74 the deviation $d = \widehat{SQR} = (i - r)$.

Lateral displacement of a ray by a parallel-sided block.—In Fig. 75, the ray $PQRS$ is refracted from medium 1 into medium 2 at Q, and out into medium 1 again at R. It emerges parallel to its original direction PQ, but displaced at right angles to this direction. RT is the perpendicular drawn from R to PQ produced. Let t be the thickness of the block, and the angles of incidence and refraction at Q, i and r. Then the **lateral displacement**

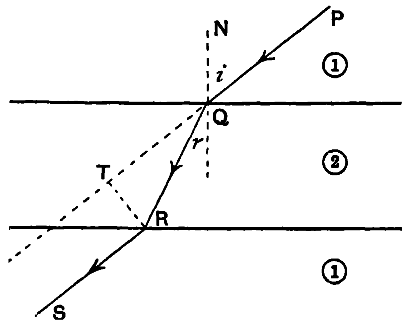


FIG. 75.—Lateral displacement by parallel-sided block.

$TR = QR \sin T\hat{Q}R = QR \sin (i - r)$. Now $QR = \dfrac{t}{\cos r}$, so

$$TR = \frac{t}{\cos r} \sin (i - r).$$

The virtual image formed by refraction at a plane surface.—P is a point object in a dense medium 1 (Fig. 76). Let PQR be a ray from P refracted at Q into medium 2. Let the angles of incidence and refraction be i and r, and the refractive indices of the media be μ and μ'.

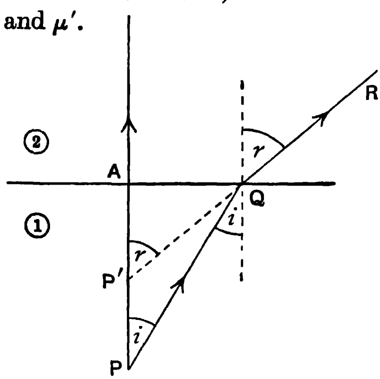


FIG. 76.—Virtual image formed by refraction.

Let AP be the normal to surface through P, and let P' be the point at which RQ produced cuts AP.

Then, $\widehat{APQ} = i$, so $\sin i = \dfrac{AQ}{PQ}$;

$$\widehat{AP'Q} = r, \text{ so } \sin r = \frac{AQ}{P'Q}.$$

Whence, $\mu \cdot \dfrac{AQ}{PQ} = \mu' \cdot \dfrac{AQ}{P'Q}$; *or, if* i *and* r *are small,*

$$\mu \cdot \frac{AQ}{AP} = \mu' \cdot \frac{AQ}{AP'}.$$

All rays from P making a small angle with the normal through P appear to diverge from a virtual image P' such that

$$\frac{AP'}{AP} = \frac{\mu'}{\mu}.$$

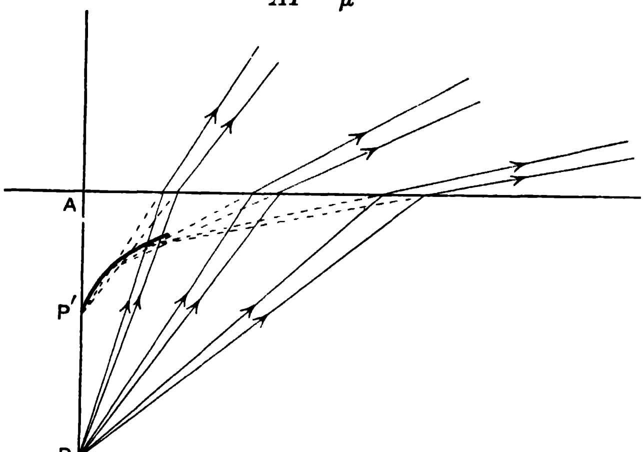


FIG. 77.—Caustic formed by refraction.

The shift in a direction normal to the surface is PP' or $AP - AP'$, which equals $AP\left(1 - \dfrac{\mu'}{\mu}\right)$.

The approximation used applies *only if the block is viewed nearly normally*, so that AP and AP' are indistinguishable from PQ and $P'Q$. An eye in the second medium looking obliquely would not see the image of P in the same position. In this case the refracted rays envelop a caustic, and the parts of this which act as virtual images of P are nearer to the surface and more distant from the normal AP for very oblique rays (Fig. 77). An

eye close to the boundary will thus see a distorted image of a large object. P, Q, R, S are four points on a line parallel to the surface. An eye at E will see the virtual images of these points

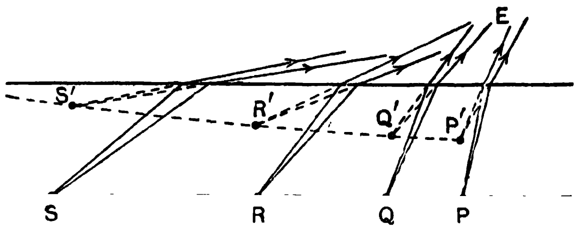


FIG. 78.—Apparent curvature of an extended object.

at P', Q', R', S' and the line thus appears to be concave towards the surface (Fig. 78).

The displacement of the virtual image by a parallel-sided block.— Consider a point O (Fig. 79) viewed normally through a parallel-sided block of glass of thickness t. Let the absolute refractive indices of air and glass be μ' and μ. Let $OPQR$ be the path of a ray refracted into the glass at P and out into the air at Q. AP is the normal to both faces through P and OB that through O. RQ produced back cuts AP in P' and OB in O'. The same argument as that of p. 74 for the virtual image formed at a single refraction holds for P and P', and $PP' = AP - AP'$; so that

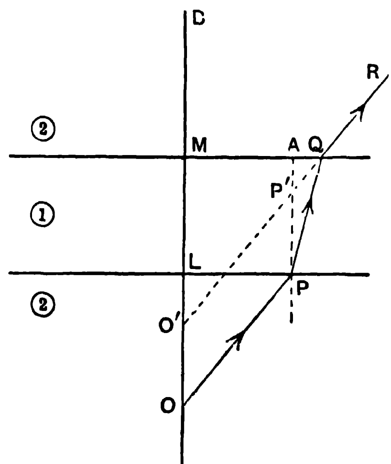


FIG. 79.—Displacement in the line of sight by a parallel-sided block.

$$OO' = PP' = AP\left(1 - \frac{\mu'}{\mu}\right) = t\left(1 - \frac{\mu'}{\mu}\right).$$

If we take $\mu' = 1$ and $\mu = 1{\cdot}5$, the displacement in the line of sight is $t(1 - \frac{2}{3})$ or $\frac{t}{3}$.

The case of the last section is simply a particular case of this, all the medium above P acting as a parallel-sided block of thickness AP.

For two or more parallel-sided blocks parallel to one another the same argument applies to each in turn, and the total displacement will be the sum of the individual displacements. For example, an object in air viewed through the bottom of a glass

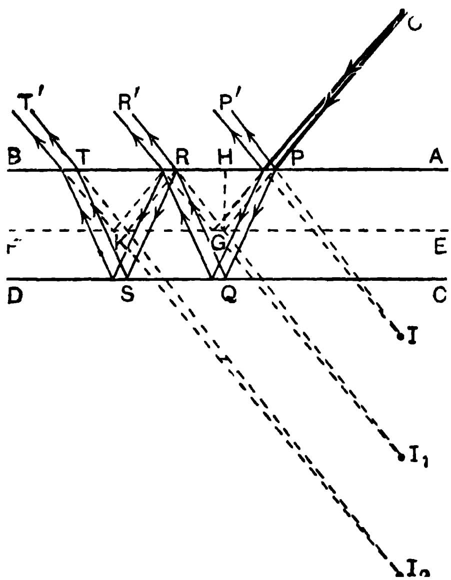


FIG. 80.—Multiple images.

tank 1 cm. thick containing 5 cm. of water will appear to be shifted up when viewed normally by $1(1-\frac{2}{3})$ cm. by the glass and $5(1-\frac{3}{4})$ cm. by the water, giving a total displacement of $(\frac{1}{3}+\frac{5}{4})$ or 1·58 cm.

Multiple images formed by reflection at a parallel-sided block.—In Fig. 80, a bright point O on the normal through A in front of one surface AB of a parallel-sided block gives an image at I by reflection at the front surface. A pencil of rays OP is partly reflected as PP' to give I, and partly transmitted along PQ. At

Q, partial transmission and reflection again takes place, and so on at R, S, T, giving a series of reflected rays RR', TT', appearing to proceed from virtual images I_1, I_2, and, of course, a similar set of transmitted rays at Q and S.

For a glass plate silvered on the back as ordinary mirrors are, no transmitted rays arise and the only important image is I_1. Producing OP and $R'R$ to meet at G, it is as if reflection took place not at Q, but at G on the plane EF, which is the virtual image of CD formed by refraction in AB. The distance QG is $t\left(1-\frac{\mu'}{\mu}\right)$ with the symbols of the last paragraph, and so the depth of G below AB is $\frac{t \cdot \mu'}{\mu}$. For a glass mirror the important reflecting surface is thus at a distance behind the front surface equal to $\frac{2}{3}$ the thickness of the glass.

The distance II_1 along the normal OA equals $2GH$ or $2t\frac{\mu'}{\mu}$, and this is the distance between the successive images along OA.

Total internal reflection.—In Fig. 81, a ray travelling from

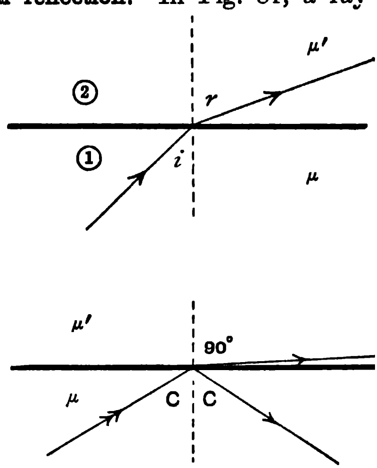


FIG. 81.—Total internal reflection.

glass (1) to air (2) is refracted away from the normal. If μ is the refractive index of glass and μ' for air,

$$\frac{\sin i}{\sin r}=\frac{\mu'}{\mu}.$$

The greatest possible value of r is 90°, when the refracted ray grazes the boundary. The greatest possible value of i for refraction to take place is given by $\frac{\sin i}{\sin 90}=\frac{\mu'}{\mu}$. This angle is called the **critical angle** C.

Hence $$\sin C=\frac{\mu'}{\mu}={}_{\text{glass}}\mu_{\text{air}}=\frac{1}{{}_{\text{air}}\mu_{\text{glass}}}.$$

For angles of incidence greater than the critical angle all the energy reaching the boundary is reflected. This is called **total internal reflection.** It can only take place if μ' is less than μ—that is, *when light is incident from the optically denser medium.* For light incident at an air face in glass, $\sin C=\frac{1}{\frac{3}{2}}=\frac{2}{3}$, so C is 42°; for a water-air face, $\sin C$ will be $\frac{3}{4}$, so C is $48\frac{1}{2}$°.

Total internal reflection at first sight introduces a rather surprising discontinuity. But we can reconcile our minds to it by considering that at all angles of incidence that part of the energy which is not refracted is reflected, so that the impossibility of refraction means that all the energy is available for reflection. Further. it leads to a new and useful way of regarding reflection as a particular case of refraction. For the relation between the angles of incidence and reflection, since they are in adjacent quadrants, is $\frac{\sin i}{\sin r}=-1$. It is just as if refraction had taken place from a medium of refractive index μ into one of refractive index $-\mu$; and this seems perfectly reasonable when we consider that the direction normal to the surface, which we have associated with a definite sign in the last chapter, has been reversed.

Atmospheric refraction.—The connection between the refractive index and density d of a gas is expressed fairly closely by the equation

$$\frac{\mu-1}{d}=\text{constant}.$$

This is known as GLADSTONE AND DALE'S LAW. At N.T.P. the refractive index of air is about 1·00029, while at 76 cm. pressure and 20° C. it will be 1·00028, the value we have already used. We can think of the atmosphere as made up of a series of parallel layers, each with refractive index a little less than the one below it. A ray of light entering the atmosphere from a star outside it will be bent downwards slightly at each refraction from one layer to the next, and will follow a curved

path as Fig. 82. An observer at E will thus see the star S at S' higher in the heavens than its actual position. The angle $\widehat{SES'}$ between the true and observed altitudes is called the atmospheric refraction. This will be zero for a star directly overhead and increase to its greatest value, about 0·5°, for a star on the horizon. This angle is greater than the angular diameter of the sun, so that when the setting sun is seen on the horizon the whole of its disc is really below it.

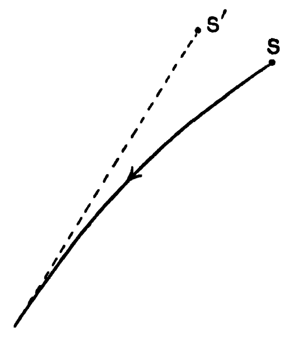


FIG. 82.—Atmospheric refraction.

The currents of hot air rising from a heated surface cause objects viewed through them to "shimmer," as the hot air has a considerably lower refractive index than the surrounding atmosphere. A method of photographing sound waves and convection currents in air, the "schlieren method" of Toepler, utilizes the change of refractive index with density. The twinkling of stars is due to small irregularities in the density of the atmosphere.

Total reflection in the atmosphere.—On a hot sunny day an observer approaching the top of a hill sees from one point on the road what appears to be a small pool at the crest, providing the

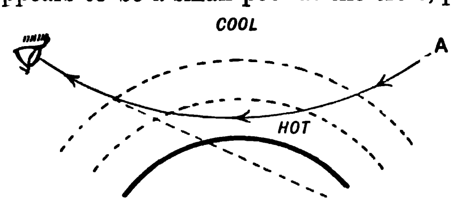


FIG. 83.—Inferior mirage.

background is the sky. The layer of air in contact with the road is warmer than that above it, μ is less, and hence there is the possibility of total reflection. The paths of rays entering the eye are as Fig. 83, which is much exaggerated. The "pool" at A' is the virtual image of the region of the sky round A.

This phenomenon on the large scale is called **mirage.** The essential conditions are a layer of warm air and light from a colder layer striking the boundary at very nearly grazing incidence, though, of course, the demarcation between the layers will not be sharp and the paths of the rays will not be sharply changed in direction. This is known as "**inferior mirage.**" As the hot air is rising the image is shimmering and unsteady. "**Superior mirage**" occurs when light is bent downwards from a layer of warm air resting on a colder layer, and is most often seen in polar regions. Fig. 84 shows how this occurs. Sometimes more than one image is seen, the images not always being in the same vertical, as the layers of equal density may not be horizontal. With superior mirage the images are steady and clear, as the atmospheric strata are stable under these conditions.

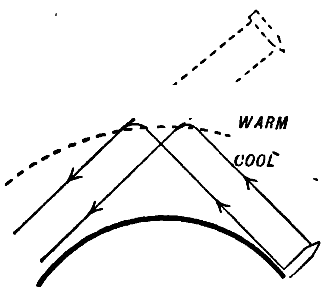


FIG. 84.—Superior mirage.

Methods of measuring the refractive index of a solid.—1. *Tracing rays through a block.* A rectangular block (Fig. 85) is placed on a sheet of drawing paper on a board and the positions XY, $X'Y'$ of two opposite sides marked on the paper. Two pins, A and B, are placed upright in the board and two more, C and D, placed so as to appear in line with their images viewed through the block. Pins and block are then removed, the lines AB and DC produced to meet XY and $X'Y'$ in E and F. Then if ABE is the path of a ray striking the block, EF is its path within it and FCD the path of the emergent ray. The normals at E and F are drawn and the angles α, β, γ, δ measured.

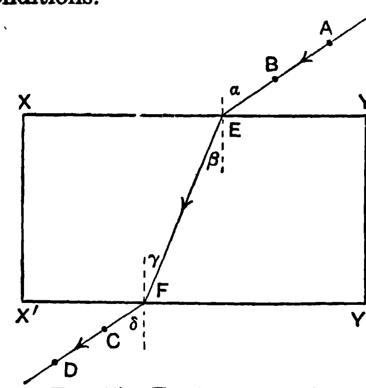


FIG. 85. –Tracing rays, using pins.

Then $$ {}_{\text{air}}\mu_{\text{glass}} = \frac{\sin \alpha}{\sin \beta}, $$

and as $$ {}_{\text{glass}}\mu_{\text{air}} = \frac{\sin \gamma}{\sin \delta}, $$

a second value for ${}_{\text{air}}\mu_{\text{glass}}$, $\dfrac{\sin \delta}{\sin \gamma}$, is obtained.

2. *Real and apparent depth, using microscope.* A microscope is a fixed focus instrument. That is, an object to be in focus must be at a constant distance from the objective. In order to bring two objects on its axis into focus in succession the microscope must then be displaced by an amount equal to their distance apart. A long-focus microscope mounted on a graduated carriage permitting motion parallel to its axis thus gives a method of locating a virtual image.

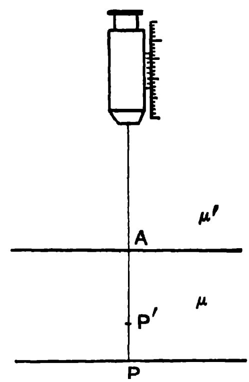


FIG. 86.—Real and apparent depth, using long-focus microscope.

The microscope is set up with its axis vertical and focussed on a mark on a piece of paper, and the reading r_1 of the scale on its carriage noted. The glass block is laid on the paper, and the microscope focussed first on the virtual image of the mark seen through the block and then on the top surface of the block made visible with a little lycopodium powder, the two readings being respectively r_2 and r_3. The depth of the mark below the surface AP is $r_3 - r_1$ and the depth of the virtual image AP', $r_3 - r_2$, so that

$$ {}_{\text{air}}\mu_{\text{glass}} = \frac{\mu}{\mu'} = \frac{AP}{AP'} = \frac{r_3 - r_1}{r_3 - r_2}. $$

This is a good method, as the long-focus microscope has a very small angular aperture, which is just the condition for the formula to apply. One precaution is important : the microscope should be provided with crosswires, and at each setting the image *focussed to coincide with the crosswires* without parallax.

Displacement of the virtual image when the object is moved.—This method is used for a liquid, which is placed in a tank with two vertical glass ends, one plane-parallel.

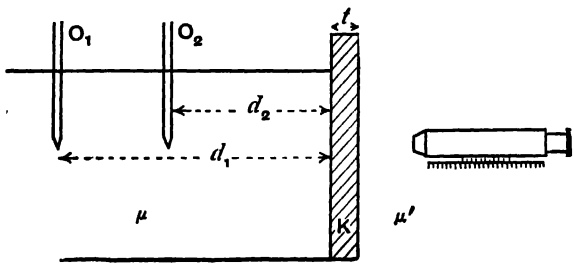


FIG. 87.—Displacement of virtual image.

A pin in the position O_1, with its tip on the axis of the microscope, which is horizontal, is at a distance d_1 from the inner face of the plane-parallel glass end. If μ, μ' and K are the refractive indices of the liquid, air and glass, and t the thickness of the glass, the distance of the virtual image from the front face of the glass is $d_1\frac{\mu'}{\mu}+t\frac{\mu'}{K}$. If the pin is shifted to O_2 at distance d_2 from the plate, the image is now $d_2\frac{\mu'}{\mu}+t\frac{\mu'}{K}$ behind the plate's front face. The shift of the image is then $(d_1-d_2)\frac{\mu'}{\mu}$ for an object shift of (d_1-d_2). So $\frac{\mu}{\mu'}=\frac{\text{distance moved by object}}{\text{distance moved by image}}$.

The concave mirror method for a liquid.—The centre of curvature C of a concave mirror is found using a pin and observing for no-parallax. Liquid of refractive index μ is poured into the mirror, and the new position C' at which the pin coincides without parallax with its real image is found. The distances AC, AC' (Fig. 88) are measured, and then

$$\frac{\mu}{\mu'}=\frac{AC}{AC'}.$$

For, consider the ray $C'D$. This is refracted at the liquid surface so as to strike the mirror normally, since it is returned along its own path; and thus DE is CD produced. So the angle of

incidence is $DC'A$, and the angle of refraction DCA. Whence,

$$\sin i = \frac{AD}{DC'}, \quad \sin r = \frac{AD}{DC}, \quad \frac{\sin i}{\sin r} = \frac{DC}{DC'};$$

and, *as the rays actually entering the eye are confined to a small pencil close to the axis of the mirror*, we can write for $\frac{DC}{DC'}$ the ratio $\frac{AC}{AC'}$.

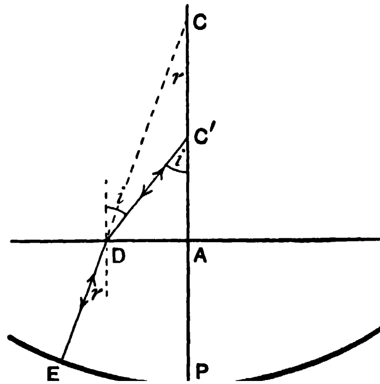


FIG. 88.—Concave mirror method.

The method is generally used with a small quantity of liquid in the mirror, and measurements made to the pole P, instead of A, the depth of the liquid being ignored. The objection to this is that with a very small quantity of liquid the upper surface may not be plane, and with a layer several millimetres deep the student is scamping a measurement that can easily be made, and in doing so wasting the accuracy of his no-parallax setting. The best method is, as shown in Bedford's *Practical Physics*, to submerge the mirror under a layer of liquid a centimetre or two in depth.

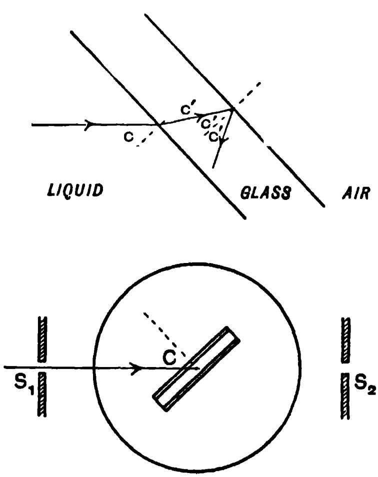


FIG. 89.—Principle of air-cell method.

The air-cell method for a liquid.—Two plane-parallel glass plates are separated by a thin strip of tinfoil at their edges, and there cemented together so as to enclose a thin film of air. This cell is placed in the liquid with the plates vertical and mounted so that it can be turned about a vertical axis and its rotation

measured. Light striking the cell at the *critical angle from liquid to air* will be totally reflected at the air film, which it will strike at the critical angle *from glass to air.*

For, let the angle of incidence from liquid to glass be C, and the angle of refraction C'. Then,

$$_{\text{liq.}}\mu_{\text{glass}} = \frac{\sin C}{\sin C'}.$$

Now if

$$\sin C = {}_{\text{liq.}}\mu_{\text{air}},$$

then as

$$_{\text{liq.}}\mu_{\text{glass}} = \frac{_{\text{liq.}}\mu_{\text{air}}}{_{\text{glass}}\mu_{\text{air}}},$$

$$\sin C' = {}_{\text{glass}}\mu_{\text{air}}.$$

The experiment is thus to measure the angle between incident light and cell at which the cell ceases to transmit any light.

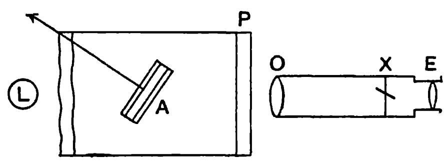


FIG. 90.—Air-cell method.

The simplest way of doing this would be to have two slits S_1, S_2, one on each side of the glass tank containing the liquid, and note the angle C through which the cell had to be turned from its initial position normal to S_1, S_2 so as to make S_1 visible from S_2. An improvement would be to have a beam of parallel light striking the cell, and view it through a telescope focussed for parallel light with its axis parallel to the beam. The ends of the tank would then have to be plane-parallel sheets of glass. A simplification of the apparatus is to dispense with the parallel-light projector and use the telescope alone ; for if it is focussed for parallel light, although light may reach its objective from all directions, it will only bring to a focus on the centre of its cross-wires parallel rays parallel to the axis. In this case only the end of the tank nearer the telescope needs to be plane-parallel. Fig. 90 shows the apparatus diagrammatically in plan. L is a source of light, A the cell, with index attached to it moving over a circular scale. The tank is closed at one end by the plane-parallel sheet of glass P. The telescope is focussed for parallel light with

the crosswires, X, horizontal and vertical and their intersection at the principal focus of the objective O. As the function of the eyepiece E is simply to view the crosswires, it may really be dispensed with and a "parallel-light detector," consisting of O and X only, used.

The cell A is turned, say, clockwise until the boundary between light and darkness is seen on the crosswires and the reading of the pointer noted; it is then turned anti-clockwise until a second such position is observed and the pointer read again. The difference between the two readings is twice the critical angle C. Taking the two readings eliminates any error due to the axis of the cell not being at the centre of the scale and avoids the necessity of setting the cell accurately normal to the telescope's axis to begin with, and of setting P normal to this axis.

The boundary will not be sharp if white light is used. For example, with water, μ is 1·329 for the extreme red end of the spectrum and 1·344 for the violet, the corresponding values of C being 49° 12′ and 48° 6′ respectively, a difference of 1° 6′. The edge will thus be of considerable width, and will shade gradually from white through orange to a deep red. For a sharp boundary monochromatic light from a sodium flame must be used. Note that monochromatic light is here necessary if an accurate setting is to be made at all. In none of the previous experiments was its use suggested, yet for water and crown glass the difference in refractive index for the extreme ends of the spectrum is about 1·5%, and for heavy flint glass 4%, limits of accuracy within which any careful experimenter should obtain results. But in setting by eye we are not concerned with the extreme ends of the spectrum, and our value of μ by these methods is an average for the yellow-green region, to which the eye is most sensitive; while the images for other colours are practically superimposed so that the presence of the rest of the spectrum is not a great nuisance.

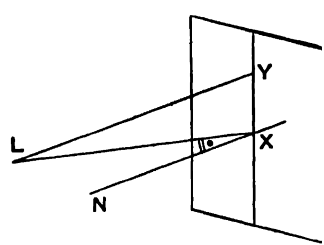


FIG. 91.

The boundary will also be curved, concave towards the bright side. The beam striking the cell is diverging, and if LX is a ray

from along the axis of the telescope incident on the cell at an angle LXN just less than the critical angle a ray LY in the same vertical plane has an angle of incidence greater than LXN and greater than the critical angle. The edges of the field of view are thus darkened before the axial part.

QUESTIONS ON CHAPTER V

1. State the laws of refraction of light.

Explain why a pool of clear water appears shallower than it really is. Find an expression for the ratio of the true to the apparent depth for normal incidence. (C.W.B.H.S.C.)

2. Explain carefully why the apparent depth of the water in a tank changes with the position of the observer.

A microscope is focussed on a scratch on the bottom of a beaker. Turpentine is poured into the beaker to a depth of 4 cm., and it is found necessary to raise the microscope through a vertical distance of 1·28 cm. to bring the scratch again into focus. Find the refractive index of the turpentine. (C.H.S.C.)

3. Describe two methods by which the index of refraction of a substance may be determined accurately, one of which is specially applicable to liquids, the other to solids. (C.H.S.C.)

4. Describe the phenomenon of total reflection at the boundary surface between two transparent media. Explain how the refractive index of a liquid may be determined by a method involving total reflection. Mention briefly any practical application of total reflection. (O. & C.H.S.C.)

5. Describe an experiment for finding the refractive index of a liquid by measuring its apparent depth.

A vessel of depth $2d$ cm. is half filled with a liquid of refractive index μ_1, and the upper half is occupied by a liquid of refractive index μ_2. Show that the apparent depth of the vessel, viewed perpendicularly, is

$$d\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right).$$ (L.H.S.C.)

6. Describe in detail how you would find accurately the refractive index of a given liquid. Give an estimate of the accuracy you would expect to obtain. (O.H.S.C.)

7. Describe and explain the phenomenon of the refraction of light, and state the laws which govern it. Explain the terms refractive index, critical angle.

Give details of *one* method of determining the refractive index of a medium, involving measurement of the critical angle.

(C.W.B.H.S.C.)

8. Find an expression for the distance through which an object appears to be displaced towards the eye when a plate of glass of thickness t and refractive index μ is interposed.

A tank contains a slab of glass 8 cm. thick and of refractive index 1·6. Above this is a depth of 4·5 cm. of a liquid of refractive index 1·5 and upon this floats 6 cm. of water ($\mu = \frac{4}{3}$). To an observer looking from above, what is the apparent position of a mark on the bottom of the tank? (O. & C.H.S.C.)

9. (*a*) Show the conditions under which total reflection occurs. Show that the phenomenon will occur in the case of light entering normally one face of an isosceles right prism of glass, but not in the case when light enters similarly a similar hollow prism full of water.

(*b*) A concave mirror of small aperture and focal length 8 cm. lies on a bench and a pin is moved vertically above it. At what point will image and object coincide if the mirror is filled with water of refractive index $\frac{4}{3}$? (N.U.J.M.B.H.S.C.)

10. Describe and explain a total reflection method of measuring the refractive index of a liquid. Discuss any effects that will be observed if white light is used in the experiment instead of monochromatic light. (N.U.J.M.B.H.S.C.)

11. A small object is placed on the principal axis of a concave spherical mirror of radius 20 cm. at a distance of 30 cm. By how much will the position and size of the image alter when a parallel-sided slab of glass, of thickness 6 cm. and refractive index 1·5, is introduced between the centre of curvature and the object? The parallel sides are perpendicular to the principal axis. Prove any formula used. (N.U.J.M.B.H.S.C.)

12. Explain the meaning of critical angle and total internal reflection. Describe fully (*a*) one natural phenomenon due to total internal reflection, (*b*) one practical application of it. Light from a luminous point on the lower face of a rectangular glass slab, 2·0 cm. thick, strikes the upper face and the totally reflected rays outline a circle of 3·2 cm. radius on the lower face. What is the refractive index of the glass? (N.U.J.M.B.H.S.C.)

13. Describe and explain the phenomenon known as mirage. Show that an object will not show this effect if ϕ, its elevation above the horizon, is greater than the value given by the equation $\cos\phi = \mu_1/\mu_2$, where μ_1 is the least and μ_2 is the greatest index of refraction of the air near the ground. (C.S.)

CHAPTER VI

PRISMS

Introduction.—Unless otherwise stated, by a prism is meant a right prism on a triangular base, which is the usual form of glass prisms. A complete and perfect prism of this kind is not necessary for the following discussion; we are interested in the refraction taking place at two of the surfaces, and provided

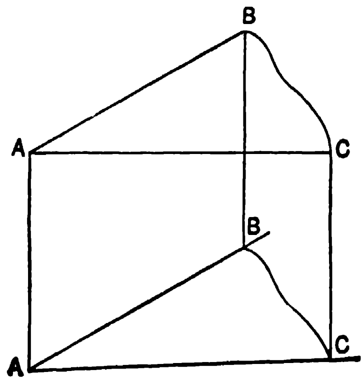


FIG. 92.—Prism.

it has two plane surfaces not parallel to one another we shall be satisfied. The line in which the two planes meet, AA (Fig. 92), is called the refracting edge of the prism, and the angle between the planes at A the refracting angle. A section of the prism by a plane perpendicular to the refracting edge, such as BAC, is called a **principal section.**

Consider a glass prism with light passing through the faces so as to be deviated away from the refracting edge. A ray striking one face in a principal section will, from the first law of refraction, continue in the same plane. In the diagram (Fig. 93),

PQRS is a ray traversing the prism in the principal section *BAC*. The ray is bent towards the normal at *Q* and away from the normal at *R*. The angle between *PQ* and *RS* is the angle of deviation, *d*. Let the angles of incidence and refraction at *Q* be *i* and *r*, the angle of incidence at *R* be *r′*, and the angle of emergence at *R*, *i′*. Let the normals *NQ*, *N′R* be produced to meet at *X*, and *PQ* and *SR* produced to meet at *Y*.

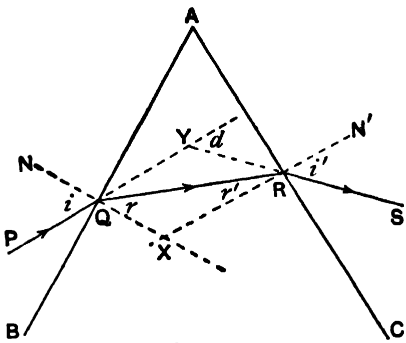


FIG. 93.—Deviation of a ray in a principal section.

Now in the triangle *YQR*,

$$Y\widehat{Q}R = i - r,$$

$$Y\widehat{R}Q = i' - r'.$$

So the exterior angle $d = (i - r) + (i' - r')$. ....(1)

In the quadrilateral *AQXR*, $A\widehat{Q}X$ and $A\widehat{R}X$ are each 90°. so that, calling the refracting angle *A*,

$$A + Q\widehat{X}R = 180°$$

and $$Q\widehat{X}R = 180° - A.$$

In the triangle *QXR*, $Q\widehat{X}R = 180° - (r + r')$.

Hence, $A = (r + r')$. ....(2)

The two expressions (1) and (2) summarize all the geometry involved in a straightforward transmissien in a principal section.

Deviation produced by a thin prism, angle of incidence small.— A thin prism is one whose refracting angle is so small that we

can, without appreciable error, take $\sin A$ and A (measured in radians) to be equal. For an angle of 5° we can verify from tables that the difference between A and $\sin A$ is about 1 part in 1000, and for an angle of 10° about 1%.

Consider a ray making a small angle of incidence i with one face of a small-angle prism, and travelling in a principal section.

Let the index of refraction of the material of the prism, from air, be μ.

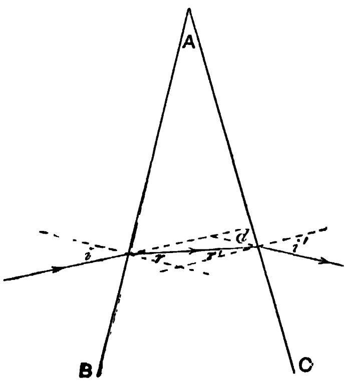


FIG. 94.—Deviation by a thin prism.

From equation (1) above,

$$d = (i - r) + (i' - r').$$

Also, $\mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin i'}{\sin r'}$.

Now as i and r are small angles, so, since A is small, will i' and r' be; and we can write $\mu = \dfrac{i}{r} = \dfrac{i'}{r'}$.

Hence, $d = \mu r - r + \mu r' - r'$,

$$d = (\mu - 1)(r + r').$$

Now, $A = (r + r')$;

$$\therefore \ d = (\mu - 1)A.$$

The i's and r's have vanished from this equation, so it must hold *for all small values of* i. **Hence for all rays making a small angle of incidence on a thin prism in a principal section the deviation is constant and is given by the above expression.** In deriving this equation we have considered the angles in radians; the result is a relation between d and A which is not, of course, dependent on any system of measuring angles. Note that if $i > \mu A$ the emergent ray will lie on the other side of the normal at the point of emergence.

A thin prism surrounded by a liquid.—If the prism were immersed in a liquid whose refractive index from air is μ', we should have for the corresponding equation of the last section,

$$i = \frac{\mu}{\mu'} r,$$

$$i' = \frac{\mu}{\mu'} r';$$

and the result obtained for the deviation in the liquid would be

$$d_1 = \left(\frac{\mu}{\mu'} - 1\right) A = \left(\frac{\mu - \mu'}{\mu'}\right) A.$$

If now the deviated rays emerge from the liquid into air through a parallel-sided plate at right angles to the original

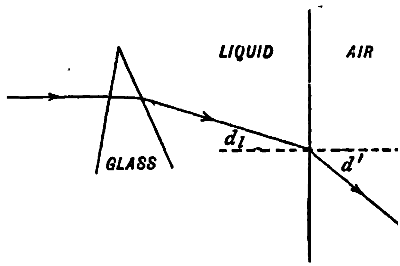


FIG. 95.—Thin prism surrounded by liquid.

direction of the light, and if d' be the deviation observed in air (Fig. 95), then $\frac{d'}{d_1} = \mu'$.

So, $$d' = \mu' d_1 = (\mu - \mu')A.$$

Combining the two equations,

$$d = (\mu - 1)A,$$
$$d' = (\mu - \mu')A,$$

we see that if μ' is known, the two measurements of d and d' should enable A and μ to be found:

$$\mu = \frac{d(\mu' - 1)}{d - d'}$$

$$A = \frac{d - d'}{(\mu' - 1)}.$$

The dispersion produced by a thin prism.—The difference between the deviations for two colours in the spectrum is called the **angular dispersion** between those two colours.

If μ_1 and μ_2 are the refractive indices for the two colours and d_1 and d_2 the deviations,

$$d_1 = (\mu_1 - 1)A,$$
$$d_2 = (\mu_2 - 1)A.$$
$$\therefore\ (d_1 - d_2) = (\mu_1 - \mu_2)A.$$

For a sample of crown glass, the refractive index for the extreme red end of the spectrum is 1·510, for the red (C) line in the hydrogen spectrum 1·513, for the yellow (D) line of the sodium flame 1·515, for the blue (F) line in the hydrogen spectrum 1·521, and for the extreme violet end 1·531.

For a 10° prism of this material the dispersion between the extreme ends of the spectrum is $(1{\cdot}531 - 1{\cdot}510) \times 10° = 0{\cdot}21°$, between the red and blue hydrogen lines

$$(1{\cdot}521 - 1{\cdot}513) \times 10° = 0{\cdot}08°,$$

between the yellow sodium line and the blue (F) hydrogen line

$$(1{\cdot}521 - 1{\cdot}515) \times 10° = 0{\cdot}06°,$$

and so on.

The angular dispersion for two colours depends not only on the difference between the refractive indices, but also on the angle of the prism. The ratio between angular dispersion and mean deviation is independent of A, and is thus a constant for the material. For if μ is the mean between μ_1 and μ_2, the mean deviation $d = (\mu - 1)A$.

So the ratio

$$\frac{\text{dispersion}}{\text{mean deviation}} = \frac{d_1 - d_2}{d} = \frac{\mu_1 - \mu_2}{\mu - 1}.$$

This ratio is called the **dispersive power** for the material **between the two colours 1 and 2.** The visually important part of the spectrum lies between the C and F hydrogen lines, and the mean of their refractive indices does not differ much from that of the D sodium line. Where values of dispersive powers are found in tables, μ_1 and μ_2 are usually the values for the F and C lines and μ is for the D line. It might be asked why we choose to define a property of a material in terms of the behaviour of so uncommon an object as a thin prism ; the reason is that in this form the constant is useful for calculations on thin lenses.

Two thin prisms combined.—Two prisms made of different materials are placed with their refracting edges parallel and pointing in opposite directions. Let their angles be A and A', their mean refractive indices μ and μ', and their refractive indices for blue and red light μ_1, μ_2 and μ_1', μ_2'.

The mean deviation produced by the combination will be

$$D=(\mu-1)A-(\mu'-1)A';$$

and the dispersion between the two colours

$$D_1-D_2=(\mu_1-\mu_2)A-(\mu_1'-\mu_2')A'.$$

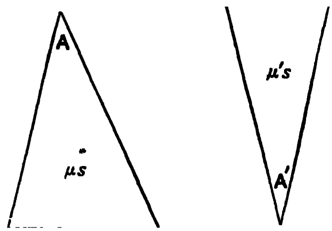


FIG. 96.—Combination of two thin prisms.

The combination will produce no deviation if $D=0$, when $\frac{A}{A'}=\frac{\mu'-1}{\mu-1}$. The dispersion given by the second equation will still be present. There will be no dispersion if $(D_1-D_2)=0$, when $\frac{A}{A'}=\frac{\mu_1'-\mu_2'}{\mu_1-\mu_2}$, though the deviation of the first equation is there. We can thus combine two thin prisms of different materials to give either dispersion without deviation or deviation without dispersion, by choosing suitable ratios for their angles.

EXAMPLE.—(i) The refractive indices of crown glass for blue and red light are 1·523 and 1·513 respectively, and for dense flint glass the corresponding figures are 1·773 and 1·743. Calculate the dispersive powers of the two materials.

For the crown glass, the average μ is $\frac{1\cdot523+1\cdot513}{2}=1\cdot518$.

So its dispersive power is $\frac{1\cdot523-1\cdot513}{1\cdot518-1}=\cdot0193$.

For the flint glass, the average μ is $\frac{1\cdot773+1\cdot743}{2}=1\cdot758$.

So dispersive power is $\frac{1\cdot773-1\cdot743}{1\cdot758-1}=0\cdot0396$.

(ii) A prism of 5° is made of the crown glass above. What must be the angle of a flint glass prism combined with it to give (*a*) no dispersion and (*b*) no deviation ?

(*a*) *No dispersion.*—The 5° crown prism gives a dispersion between the red and blue of $5(1{\cdot}523 - 1{\cdot}513) = 0{\cdot}05°$.

Let x be the angle of the flint prism arranged to give this dispersion in the opposite direction.

Then, $$x(1{\cdot}773 - 1{\cdot}743) = 0{\cdot}05.$$

So, $$x = \frac{0{\cdot}05}{0{\cdot}03} = 1{\cdot}67°.$$

The mean deviation of the 5° prism is $5 \times (1{\cdot}518 - 1) = 2{\cdot}59°$, and that of the other is $1{\cdot}67 \times (1{\cdot}758 - 1) = 1{\cdot}28°$, so that the remaining deviation is $2{\cdot}59 - 1{\cdot}28 = 1{\cdot}31°$ in the original direction.

(*b*) *No deviation.*—Let x be the angle of the flint prism which gives a mean deviation of 2·59° to balance that of the 5° prism.

Then, $$x(1{\cdot}758 - 1) = 2{\cdot}59$$

or $$x = \frac{2{\cdot}59}{{\cdot}758} = 3{\cdot}42°.$$

The dispersion produced will be

$$5(1{\cdot}523 - 1{\cdot}513) - 3{\cdot}42(1{\cdot}773 - 1{\cdot}743) = (0{\cdot}05 - 0{\cdot}102) = -0{\cdot}0502\ ;$$

that is, 0·0502° in the opposite direction to that originally produced by the crown prism.

A prism of large angle.—The usual form of prism for experimental work has a refracting angle of about 60°. Prisms of smaller angle give smaller deviation and dispersion for a given angle of incidence, and we cannot get rays transmitted through the two faces as the diagrams show if the angle exceeds about 80°, so this is a convenient size.

For a large prism the deviation is not independent of the angle of incidence. By mounting a prism section on the optical disc and using a narrow beam of light grazing the disc, or by tracking rays with pins as on p. 81, or, more accurately than either of these methods, by employing the spectrometer, we can trace the paths of incident and emergent rays in a principal section for different angles of incidence and draw graphs of angle of emergence against angle of incidence (dotted line, Fig. 97) and angle of deviation against angle of incidence (full line). The first curve is, as we should expect, symmetrical about the axes. The second shows that as i increases from the smallest angle at which it is possible to get an emergent ray, d decreases and after passing through a minimum

value increases. Taking the lowest point P on the $d-i$ curve and dropping a perpendicular on to the i axis, this cuts the $i'-i$ curve at Q. The line OQ makes an angle of 45° with the

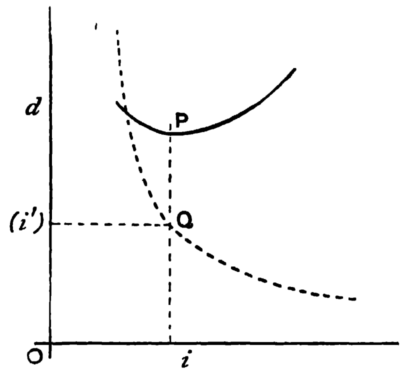


FIG. 97.—Curves showing variation with angle of incidence of the angle of deviation (full line) and angle of emergence (dotted line) for a 60° prism.

axes, so for Q, $i=i'$. *Minimum deviation thus occurs when angles of incidence and emergence are equal and the rays pass symmetrically through the prism.*

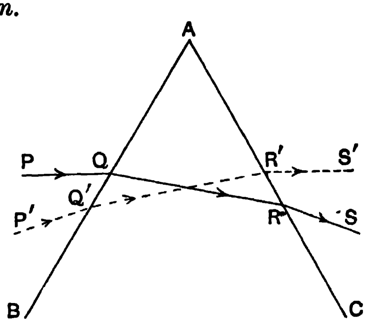


FIG. 98.

This can be proved algebraically at some length. It is easy to see, however, that if a minimum exists it must be for the symmetrical path. For if the ray $PQRS$ not passing symmetrically through undergoes minimum deviation, so does the ray $SRQP$ following the same path backwards, and so also does the

ray $P'Q'R'S'$ making an angle of incidence on AB equal to that made by SR on AC. Thus there are two positions of minimum deviation, which is contrary to observation. The symmetrical path must therefore be that of minimum deviation.

Minimum deviation formula.—For the ray $PQRS$ passing symmetrically through the prism in a principal section, $i = i'$ and $r = r'$. Let d be the angle of minimum deviation.

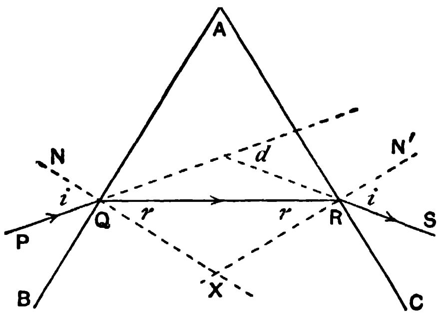


FIG. 99.—Minimum deviation.

Then,
$$d = (i - r) + (i - r) = 2(i - r).$$
$$A = r + r = 2r.$$

So,
$$i = \frac{d + A}{2},$$
$$r = \frac{A}{2}.$$

So if μ is the refractive index from air to glass of the light under consideration,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin \frac{d+A}{2}}{\sin \frac{A}{2}}$$

This is a very important result, as it provides a simple and accurate method of finding refractive indices.

The crown glass under discussion on p. 92 had refractive indices of 1·510 and 1·531 for the extreme ends of the spectrum.

For a 60° prism made of this material the angle of minimum deviation for deep red light is given by

$$1{\cdot}510 = \frac{\sin\left(\frac{d}{2}+30^\circ\right)}{\sin 30^\circ},$$

whence $$d = 38^\circ\, 2'.$$

For the extreme violet,

$$1{\cdot}531 = \frac{\sin\left(\frac{d}{2}+30^\circ\right)}{\sin 30^\circ}$$

and $$d = 39^\circ\, 56'.$$

A change of less than 1·5% for μ in the neighbourhood of $\mu = 1{\cdot}5$ thus produces a change of 1° 54′, or 114 minutes of arc, in d for a 60° prism. Measurement of d to one minute of arc will thus enable μ to be determined with high accuracy in this case. Experimental details of the method are given in Chapter XVI.

A hollow prism with sides of plane-parallel glass filled with liquid will act like a prism of liquid in air, for the glass plates will produce no deviation. The method has also been used for gases, which are enclosed at high pressures in a prism of large angle. If $\frac{\mu - 1}{\text{density}}$ is constant, and the absolute refractive index for air at 1 atmosphere pressure is 1·00028, or say 1·0003, the value for air at 11 atmospheres would be 1·0033 and its refractive index relative to the outside air 1·003. For a prism of 120° angle the angle of minimum deviation is given by $1{\cdot}003 = \frac{\sin\left(\frac{d}{2}+60\right)}{\sin 60}$, whence $d = 32'$. This rough calculation shows that a detectable deviation can be obtained with a gas, though special methods of measuring d must be used for any accuracy.

Limiting angle of a prism.—Rays striking the second face of the prism at an angle greater than the critical angle are totally reflected. It can be seen that those rays which, incident from below the normal as in all the previous diagrams, make the greatest angles of incidence on the first face strike the second most nearly normally, and so are most likely to be transmitted.

A grazing-emergence transmitted ray is possible if the prism has so large an angle that the ray incident at 90° on the first face strikes the second at the critical angle. The angles r and r' then

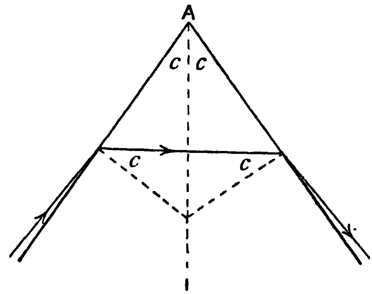


FIG. 100.—Limiting angle.

both equal C, and the angle of the prism $A = r + r' = 2C$. This value is called the **limiting angle** for prisms of the material. For greater values of A, no transmission is possible.

Critical-angle transmission. Grazing incidence.—If the surface AB is illuminated obliquely, the field viewed through AC is divided into two halves by a boundary parallel to the edge of

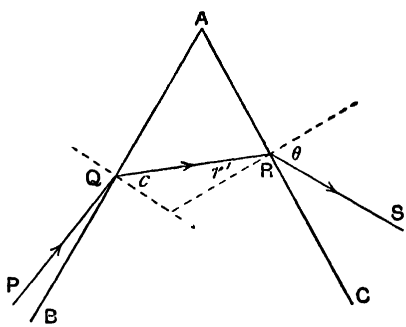


FIG. 101.—Critical-angle transmission.

the prism, though the division will not be sharp unless monochromatic light is used. The side nearer the vertex of the prism will be bright, the other completely dark. The boundary corresponds to the ray QRS, which has entered AB at the critical angle.

In the diagram, r' is the angle of incidence on AC and θ the angle of emergence.

Then, $$A = r' + C,$$

$$\frac{\sin\theta}{\sin r'} = \mu, \text{ the refractive index of the glass from air,}$$

and $$\frac{1}{\sin C} = \mu.$$

From the second of these equations,

$$\sin\theta = \mu \sin r'.$$

So, $$\sin\theta = \mu \sin(A - C)$$
$$= \mu \sin A \cos C - \mu \cos A \sin C.$$

Substituting for C from the third,

$$\sin\theta = \mu \sin A \sqrt{\frac{\mu^2 - 1}{\mu^2}} - \cos A,$$

$$\sin\theta = \sin A \sqrt{\mu^2 - 1} - \cos A\,;$$

whence, expressing μ in terms of θ,

$$\mu = \sqrt{1 + \left(\frac{\sin\theta + \cos A}{\sin A}\right)^2}.$$

This gives a second method of measuring the refractive index of the material of the prism, by determining θ and A.

For the 60° prism of crown glass, when

$$\mu = 1{\cdot}510, \quad \theta = 28°\ 39',$$

and when $$\mu = 1{\cdot}531, \quad \theta = 30°\ 2',$$

the difference is 1° 23′. This method of finding μ is thus less sensitive than the minimum deviation method (p. 97), which gives a difference of 1° 54′.

Grazing incidence with liquid on the first face.—If the face AB is covered with a layer of liquid, for which the refractive index from air to liquid is μ', less than μ, we have

$$\frac{\sin\theta}{\sin r} = \mu,$$

$$\sin C = {}_{\text{glass}}\mu_{\text{liq.}} = \frac{\mu'}{\mu},$$

$$A = r + C.$$

Now, $$\mu' = \mu \sin(A - r),$$

$$\mu' = \mu \sin A \cos r - \mu \cos A \sin r.$$

As $\sin\theta=\mu\sin r,\quad \cos r=\sqrt{1-\dfrac{\sin^2\theta}{\mu^2}},$

so
$$\mu'=\mu\sin A\sqrt{1-\frac{\sin^2\theta}{\mu^2}}-\sin\theta\cos A$$
$$=\sin A\sqrt{\mu^2-\sin^2\theta}-\sin\theta\cos A.$$

So that if the refractive index of the prism, μ, is known, the refractive index μ' of the liquid can be calculated if A and θ are determined.

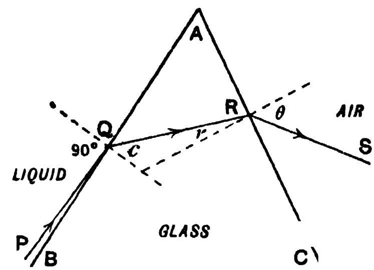


FIG. 102.—Critical-angle transmission, liquid on one face.

The greatest value of A for which this method could conceivably work is the sum of the critical angles at the two surfaces. For example, if water is on the first face, the critical angle there for a prism of refractive index 1·51, as $\sin C=\dfrac{1\cdot 33}{1\cdot 51}$, is about 62°. At the second face it is 41° 30′, so the greatest value A can have for transmission to take place is about 104°. A prism of 90° angle can thus be used with safety, when the rather complicated formula reduces to $\mu'=\sqrt{\mu^2-\sin^2\theta}$. When $A=90°$,

if $\mu=1\cdot 51$ and $\mu'=1\cdot 33$, then $\theta=45° \, 37'$;

if $\mu'=1\cdot 34$, then $\theta=44° \, 5'$.

A change of less than 1% in μ' thus produces a change in θ of 1° 32′, so the method is sufficiently sensitive to promise accuracy. It is greatly improved by making μ larger. For flint glass, for which $\mu=1\cdot 65$, when $A=90°$,

if $\mu'=1\cdot 33$, then $\theta=77° \, 36'$;

if $\mu'=1\cdot 34$, then $\theta=74° \, 19'$;

a difference of 3° 17′ for the same change. But if μ is as large as 1·67, 90° is greater than the greatest possible value for transmission.

Wollaston's method for refractive index measurements.—If a right-angled block be placed with a film of liquid underneath on a dark surface and one face, CD, be illuminated, an eye looking into the face AB will see a horizontal boundary to the bright reflection. As in previous critical angle experiments this will not be sharp with white light. It will be coloured with the violet end of the spectrum; for rays for which μ is greatest have the least C and hence the largest θ. Also, the contrast will not be between light and darkness: for the ray SS' in the diagram is the upper boundary of the region of total reflection, and partial

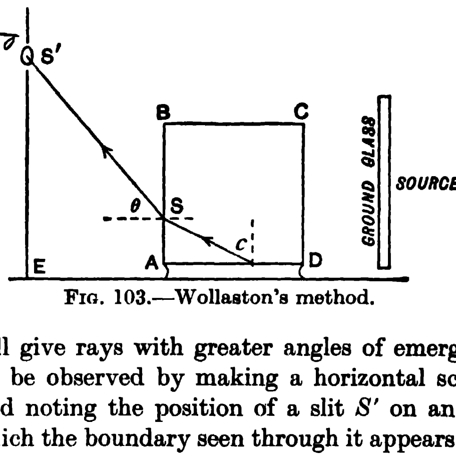


FIG. 103.—Wollaston's method.

reflection will give rays with greater angles of emergence. The angle θ may be observed by making a horizontal scratch S on the block and noting the position of a slit S' on an adjustable stand, for which the boundary seen through it appears to coincide with S. Then SS' makes an angle θ with the normal to the face AB. If the distance of the stand AE, the height ES' of the slit, and the height of the scratch AS are measured,

$$\tan\theta = \frac{ES' - AS}{AE}.$$

Another arrangement is to apply the liquid to a vertical face using black paper which the liquid will cause to adhere to the glass, to have the scratch S vertical, and to trace the line SS' with pins.

The best way of illuminating CD is to use a sodium flame behind a piece of ground glass.

The most satisfactory way of measuring θ is to use a telescope

focussed for parallel light moving over a graduated scale. This is employed in the **Pulfrich refractometer**, in which the liquid is

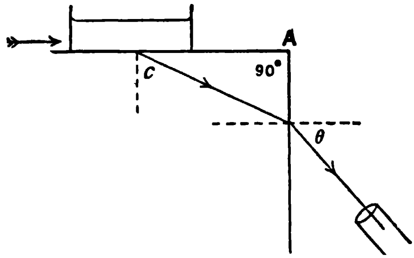


FIG. 104.—Pulfrich refractometer.

enclosed in a small cell on the upper face of the block (Fig. 104).

Special types of prism.—Glass prisms whose principal sections are right-angled 45° triangles are used for reflection. The critical angle for glass, for which $\mu = 1{\cdot}51$, is $41°\,30'$, so light incident on a glass-air face at 45° will be totally reflected.

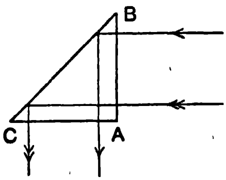


FIG. 105.

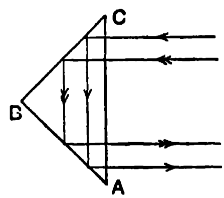


FIG. 106.

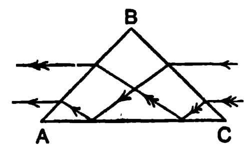


FIG. 107.

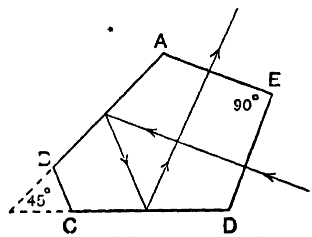
Figs. 105-107 show how a single prism can be made to give deviations of 90°, 180° and 360°. In each case lateral inversion occurs. In each case dispersion might be expected to have some effect ; but this will only give different colours a relative shift parallel to the original direction of the incident light, so that if an image is formed by a lens the effect will not be observed.

A second type of prism designed to deviate rays through a fixed angle has a pentagonal section. It depends on the principle that two reflecting surfaces inclined at an angle θ deviate all incident rays through an angle 2θ.

For example, if the faces AB and CD are inclined at 45°, and AE and ED at 90°, all rays traversing the prism as shown are deviated through 90°. The angle of incidence on both the

reflecting faces is less than the critical angle, so this is *not* a case of total internal reflection and the faces AB and CD must be silvered. A constant deviation prism to give any angle of deviation can be constructed making the angle between the faces AB and CD equal to $\frac{D}{2}$ and the angle AED equal to $(180° - D)$.

This type of prism gives no lateral inversion and no dispersion. Such prisms are used in range-finders.



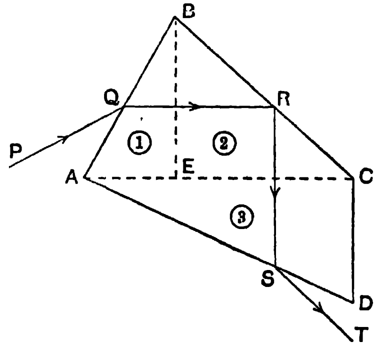


FIG. 108 and FIG. 109.—Constant deviation prisms.

Another important form of constant deviation prism is made up of a 45° prism and a 60° prism split into two, arranged so that the ray undergoing minimum deviation in the 60° prism is turned always through 90° by the system. $ABCD$ is a principal section of the prism. The angles ABE and DAC are both 30° and EBC is 45°. The prism is, of course, actually made of one piece of glass.

The ray PQ, which is refracted at AB so that it would be undergoing minimum deviation in a 60° prism (*i.e.* the angle of refraction is 30°), strikes BC at 45°, is turned through 90°, makes an angle of incidence 30° on AD, and is refracted out along ST. The angle between PQ and ST equals that between QR and RS, 90°. If the direction of the incident light is unaltered so will be the direction of ST, and as the prism is rotated about an axis perpendicular to the diagram, different parts of the spectrum are seen at minimum deviation along ST. With a plain 60° prism, if the direction of the incident light is constant we have to move not only the prism, but also the line of sight to see different colours at minimum deviation.

Advantage of using prisms for reflection.—Although total reflection occurs at the surface concerned in a 45° prism, the prism does not return all the light energy falling on it, for some will be lost by reflection on entering and leaving the glass and by absorption in it. Even so it is more efficient than a reflector of, say, speculum metal, which only returns 60%. Also, a metal surface needs careful protection against tarnishing, while the prism's reflecting surface needs only to be clean. If the metal is deposited on the back of a glass sheet the reflection from the front surface is sufficient to be a nuisance. The prism's advantages may then be summarized as brightness, clearness and permanence.

The image formed by a prism.—Tracing the paths of two rays, $PQRS$, $P'Q'R'S'$, from a bright point P, they appear after refraction in AB and AC to diverge from I_1 and I_2 respectively. I_2 is the virtual image of P formed by the prism. We found that the virtual image formed by a single refraction varied in position with the obliquity of the rays forming it, and the same will be true of I_2 formed by two refractions. Also, rays from P which do not travel in a principal section of the prism may appear to diverge from other points than I_2.

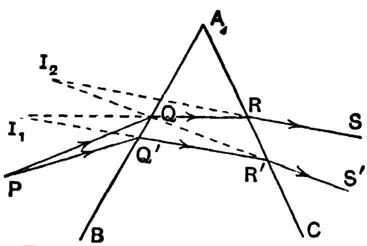


FIG. 110.—Virtual image formed by prism.

If P is a white-light source, the emergent beam will be blue-

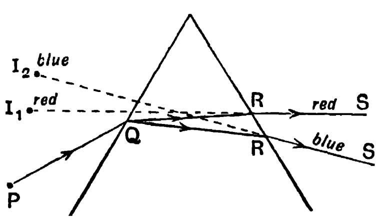


FIG. 111.—Colouring of virtual image.

violet on the side further from the vertex and red at its upper edge. But the virtual image has its blue edge nearer the vertex. Fig. 111 explains how this occurs.

QUESTIONS ON CHAPTER VI

1. Show that when a ray of light passes nearly normally through a prism of small angle α and refractive index μ, the deviation δ is given by

$$\delta = (\mu - 1)\alpha.$$

A parallel beam of light falls normally upon the first face of a prism of small angle. The portion of the beam which is refracted at the second surface is deviated through an angle of 1° 35′, and the portion which is reflected at the second surface and emerges again at the first surface makes an angle of 8° 9′ with the incident beam. Calculate the angle of the prism and the refractive index of the glass. (C.H.S.C.)

2. Define *dispersive power*.

The following table gives the refractive indices of crown and flint glass for 3 lines of the spectrum.

	C	*D*	*F*
Crown - -	1·514	1·517	1·523
Flint - -	1·644	1·650	1·664

Calculate the refracting angle of a flint glass prism which, when combined with a crown glass prism of refracting angle 5°, produces a combination that does not deviate the light corresponding to the *D* line. What separation of the rays corresponding to the *C* and *F* lines will such a compound prism produce? (L.H.S.C.)

3. Explain the meaning of *critical angle*, and describe how you would measure the critical angle for a water-air boundary.

ABCD is the plan of a glass cube. A horizontal beam of light enters the face *AB* at grazing incidence. Show that the angle θ which any rays emerging from *BC* would make with the normal to *BC* is given by $\sin \theta = \cot \alpha$, where α is the critical angle. What is the greatest value that the refractive index of glass may have if any of the light is to emerge from *BC*? (N.U.J.M.B.H.S.C.)

4. Describe a good method of measuring the refractive index of a substance such as glass and give the theory of the method.

A glass prism of angle 72° and index of refraction 1·66 is immersed in a liquid of refractive index 1·33. What is the angle of minimum deviation for a parallel beam of light passing through the prism? (L.H.S.C.)

5. What is the condition for the minimum deviation of a ray of light through a glass prism? Prove your statement.

Find an expression for the refractive index of the glass of a prism in terms of the angle of minimum deviation and the angle of the prism. (C.W.B.H.S.C.)

6. The refractive index of a prism is 1·6434 for a red line in the lithium spectrum, and 1·6852 for a violet line in the mercury spectrum. Calculate the angle of minimum deviation for each line if the angle of the prism is 60°. (N.U.J.M.B.H.S.C.)

7. Describe and explain an accurate method for measuring the refractive index of a liquid employing total internal reflection (or grazing incidence). (N.U.J.M.B.H.S.C.)

8. Prove that for a prism of small angle A the deviation of a ray of light is $(\mu - 1)A$, provided that the angle of incidence also is small. A crown glass prism of refracting angle 6° is to be achromatized for red and blue light with a flint glass prism. Using the data below and the formula above find (*a*) the angle of the flint glass prism, (*b*) the mean deviation.

	Crown glass	*Flint glass*
μ red - -	1·513	1·645
μ blue -	1·523	1·665

(N.U.J.M.B.H.S.C.)

9. How would you measure the angle of minimum deviation of a prism? (*a*) Show that the ray of light which enters the first face of a prism at grazing incidence is least likely to suffer total internal reflection at the other face. (*b*) Find the least value of the refracting angle of a prism made of glass of refractive index $\frac{7}{4}$ so that no rays incident on one of the faces containing this angle can emerge from the other. (N.U.J.M.B.H.S.C.)

10. A direct vision prism is made up of two prisms, one of flint and the other of crown glass. The flint glass has an angle of 10° and mean refractive index of 1·650. What must be the angle of the crown glass prism if its mean refractive index is 1·5137? Given that the dispersive powers of flint and crown glasses are 0·0296 and 0·0175 respectively, find the angular separation of the red and blue light on emergence from the prism. (C.S.)

CHAPTER VII

REFRACTION AT SPHERICAL SURFACES

A single refraction. Conjugate foci.—We will consider a portion of a spherical surface symmetrical about an axis which passes through the centre of the sphere C and cuts the surface at P.

O is a luminous point on the axis. A ray from O making a small angle with the axis strikes the surface at A very near to P and is refracted to pass through I on the axis. CA is the normal to the surface at A. O is supposed to be in the rarer medium.

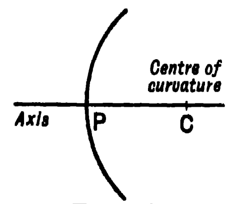


FIG. 112.

Let the angles of incidence and refraction be i_1 and i_2, and the absolute refractive indices of the first (less dense) and second (denser) media be μ_1 and μ_2 respectively.

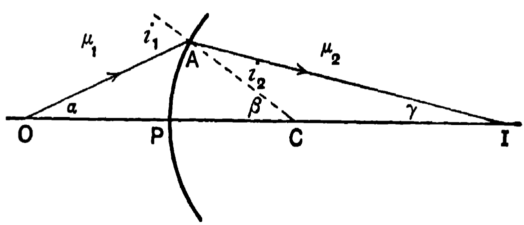


FIG. 113.

Let the small angles $\widehat{AOP}$, $\widehat{ACP}$, $\widehat{AIP}$ be α, β, γ. These angles are all so small that we can write in circular measure

$$\alpha = \frac{AP}{PO}, \quad \beta = \frac{AP}{PC} \text{ and } \gamma = \frac{AP}{PI}.$$

Now, $i_1 = \beta + \alpha$ (ext. angle to AOC and interior and opposite),

$\beta = i_2 + \gamma$ „ „ ACI „ „

So, $i_2 = \beta - \gamma$.

Now, $\mu_1 \sin i_1 = \mu_2 \sin i_2$; or, since i_1 and i_2 are both small, $\mu_1 i_1 = \mu_2 i_2$. So, $\mu_1(\beta + \alpha) = \mu_2(\beta - \gamma)$.

Now, $$\alpha = \frac{AP}{PO}, \; \beta = \frac{AP}{PC}, \; \gamma = \frac{AP}{PI}, \text{ in circular measure.}$$

Substituting for α, β and γ and dividing through by AP,

$$\mu_1\left(\frac{1}{PC} + \frac{1}{PO}\right) = \mu_2\left(\frac{1}{PC} - \frac{1}{PI}\right) \text{ or } \frac{\mu_2}{PI} + \frac{\mu_1}{PO} = \frac{\mu_2 - \mu_1}{PC}.$$

So far we have been working with a complete disregard of signs. This equation is a relation between the **numerical values** of the lengths PC, PO, PI. O and I are **conjugate foci**, for a ray IA would be refracted along AO, giving the same equation *whichever way the light travels.*

The signs to be given to the object and image distances follow the convention already given. If the object or image point is real, its distance from P is positive; for virtual points the distances are negative. In the case of Fig. 113, as both O and I are real, PO and PI are both positive.

Before giving an algebraic sign to PC, note that it is only one of the factors on which the behaviour of the surface depends; the other is the difference between the refractive indices of the dense and less dense media. The *whole expression* $\frac{\mu_2 - \mu_1}{PC}$, called the **power** of the surface, is the important physical quantity. The surface shown in the figure, which brings rays diverging from a real point O to converge to the real point I, will bring rays diverging from a distant point on the axis to a real point nearer the surface than I. This point, the **second principal focus**, discussed in the next paragraph, is real. Its distance from the surface, the **focal length**, is a **real** distance and is thus reckoned **positive**. The *power of the surface is given the same sign as the focal length.* It is, of course, the same whichever way the light goes, and is in this case **positive**. The radius of curvature is given the same sign as the power, and is in this case **positive**.

So, as PC is $+$, write $(+PC) = r$;
PO is $+$, write $(+PO) = u$ (real object),
PI is $+$, write $(+PI) = v$ (real image).

The equation in its algebraic form then becomes

$$\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r},$$

each of the symbols u, v, r, representing the *whole quantity, length with algebraic sign.*

A difficulty now arises. Remembering that O and I are conjugate foci and interchangeable, the formula must work when we put $PI=u$ and $PO=v$; but as we understand the suffixes "1" and "2" to mean "first medium" and "second medium", we must *rename* (not interchange, for they are not "conjugate media"!) the refractive indices—let us write M_1 for μ_2 and M_2 for μ_1. Then

$$\frac{M_2}{v}+\frac{M_1}{u}=\frac{M_1-M_2}{r}.$$

Now (M_1-M_2) and $(\mu_2-\mu_1)$ both stand for the same thing—the numerical difference between the refractive indices. It must be realised that *algebraic signs are exclusively restricted to distinguishing between real and virtual distances*, and are therefore meaningless when attached to a μ-difference. So $(\mu_2-\mu_1)$ always means the *numerical* difference between the two refractive indices, or $(\mu_{\text{denser}}-\mu_{\text{less dense}})$. This should cause no confusion, as it is easy enough to decide whether the surface has a positive or negative power, and after this there need be no more worrying about signs.

The sign conventions for power and radius of curvature will be:

A converging refracting surface, convex towards the less dense medium, has a positive power and positive radius of curvature.

A diverging refracting surface, concave towards the less dense medium, has a negative power and negative radius of curvature.

The diagram shows a case in which I is a real image, for the rays actually pass through it after refraction. If I were a virtual image on the same side of the surface as O, the geometrical reasoning would give us for the numerical equation

$$\mu_1\left(\frac{1}{PC}+\frac{1}{PO}\right)=\mu_2\left(\frac{1}{PC}+\frac{1}{PI}\right).$$

But as I is a virtual point, PI is now algebraically negative, so we should have

$$(+PO)=u,\ (+PC)=r,\ (-PI)=v$$

and the same algebraic equation is obtained. The object point O may also be virtual, for rays striking the surface may be converging towards some point O in the second medium, so that PO

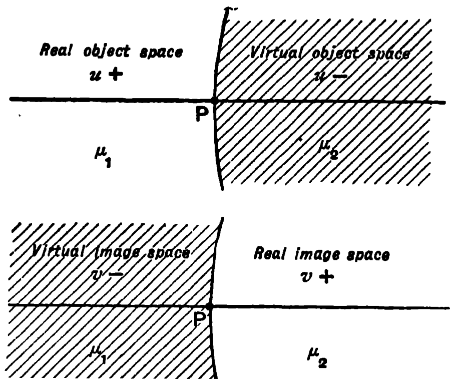


FIG. 114.—Object and image spaces for a refracting surface.

would be negative. Also, for a surface concave to the medium of lesser μ, $\frac{\mu_2 - \mu_1}{r}$ would be negative. In each case different numerical formulae are obtained geometrically which all lead to the same algebraic equation. Obviously, to give a real image, rays must converge to *beyond* the surface, so that the space in which real images are found is on the opposite side to that in which the real object is placed. Fig. 114 shows the spaces in which real and virtual objects and images occur, considering the light as travelling from medium 1 to medium 2.

Principal foci.—Suppose u is infinitely great, so that $\frac{1}{u}=0$.

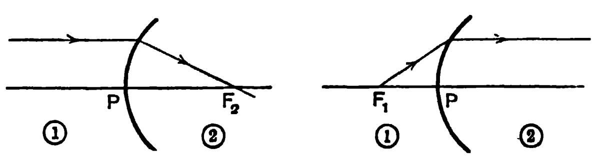


FIG. 115.—Principal foci.

The focus to which a narrow beam of *parallel* rays *parallel* to the axis (*i.e.* diverging from a point on the axis at a very great

distance) is brought after refraction is called the **second principal focus,** F_2. The distance $(+PF_2)$ is represented algebraically by f_2.

Then,
$$\frac{\mu_2}{f_2}=\frac{\mu_2-\mu_1}{r},$$

or
$$f_2=\frac{\mu_2 r}{\mu_2-\mu_1}.$$

Suppose v is infinitely great, so that $\frac{1}{v}=0$. The real object must then be placed at the **first principal focus,** F_1; the distance $(+PF_2)$ is f_1 algebraically.

Then,
$$\frac{\mu_1}{f_1}=\frac{\mu_2-\mu_1}{r},$$

or
$$f_1=\frac{\mu_1 r}{\mu_2-\mu_1};$$

f_1 and f_2 are called the first and second focal lengths of the surface.

It can be seen from the two previous equations that
$$\frac{f_1}{f_2}=\frac{\mu_1}{\mu_2}.$$

It is clear that f_2 will be positive if $\frac{\mu_2-\mu_1}{r}$ is positive; that is, for a surface convex towards the medium of less refractive index. The term $\frac{\mu_2-\mu_1}{r}$, the power of the surface, is given the symbol F.

Thus,
$$f_2=\frac{\mu_2}{F},$$
$$f_1=\frac{\mu_1}{F}.$$

Formulae in air.—If the first medium is air, then $\mu_1=1$, and μ_2 can be written simply μ since there is no need for a distinguishing suffix. The conjugate foci formula becomes
$$\frac{\mu}{v}+\frac{1}{u}=\frac{\mu-1}{r}.$$

The power F of the surface becomes $F=\frac{\mu-1}{r}$, and the focal lengths, $f_2=\frac{\mu}{F}$, $f_1=\frac{1}{F}$. *The student is advised to commit to*

memory the general formula, and not the "air formula" of this paragraph. A symmetrical formula is easier to remember accurately, and habitual use of the simplified formula will make general problems when they occur appear unduly difficult.

Reduction to reflection form.—On p. 78 we saw that a reflection could be regarded as a refraction from a medium of refractive index μ to one of refractive index $-\mu$. If we write $\mu_2 = -\mu_1$ we get for the conjugate foci formula

$$-\frac{\mu_1}{v}+\frac{\mu_1}{u}=\frac{-\mu_1-\mu_1}{r},$$

or

$$-\frac{1}{v}+\frac{1}{u}=-\frac{2}{r}.$$

This does not agree with the formula obtained directly, as it stands. But while the object spaces are the same for reflecting and refracting surfaces, the real image space for a reflection is the virtual image space for a refraction, so that when the refracting surface is acting as a mirror the signs of v and also r must be reversed, so that the mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$ is obtained.

EXAMPLE.—A glass sphere of 10 cm. radius has a small bubble 3 cm. from its centre. The bubble is viewed along a diameter of the sphere, from the side on which it lies. How far from the surface will it appear to be if the refractive index is 1·5 ?

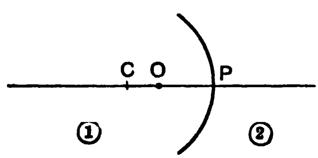


FIG. 116.—Example.

In the figure, O is the object, C the centre of the sphere, P the point on the surface nearest the eye.

Use the formula $$\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}.$$

Here,

$$\mu_1 = 1{\cdot}5,$$
$$\mu_2 = 1,$$
$$\mu_2 - \mu_1 = 0{\cdot}5 \text{ numerically.}$$

The surface is convex to the less dense medium, and its power is thus positive. So $r = +10$ cm.

The object is real, so $u = +(10-3) = +7$ cm.,

$$\frac{1}{v} + \frac{1 \cdot 5}{7} = \frac{0 \cdot 5}{+10} = +\frac{1}{20},$$

$$\frac{1}{v} = 0 \cdot 05 - 0 \cdot 214 = -0 \cdot 164,$$

$$v = -6 \cdot 1 \text{ cm.}$$

So the image is 6·1 cm. behind P and is virtual.

EXAMPLE.—With the same sphere, a beam of light strikes it, converging towards a point 40 cm. behind P. Where will the image be ?

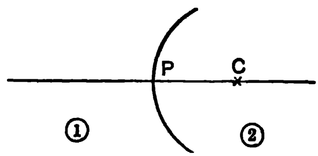


FIG. 117.—Example.

Use $$\frac{\mu_2}{v} + \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}.$$

Here, $$\mu_1 = 1,$$
$$\mu_2 = 1 \cdot 5.$$

So, $$\mu_2 - \mu_1 = 0 \cdot 5.$$

The surface is convex to the less dense medium, and its power is positive. So r must be made $+10$ cm. (*Note that the power is the important thing to fix—there is no difficulty about the sign of* r *once the sign of the power is fixed.*)

$$u = -40 \text{ cm. (virtual object)},$$

$$\frac{1 \cdot 5}{v} - \frac{1}{40} = \frac{0 \cdot 5}{+10} = +\frac{1}{20}.$$

Whence, $$v = +20 \text{ cm.}$$

So the final image will be formed on the back of the sphere and will be real.

Problems involving more than one refraction.—For several refracting surfaces with a common axis the above treatment is applied to each surface in turn, as in the next example.

EXAMPLE.—An object point is placed 60 cm. from the surface of the previous sphere along a diameter. Where will the final image formed by refraction in both surfaces lie ?

For the first face, use $\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}$.

Here,

$$\mu_1=1,$$
$$\mu_2=1\cdot5,$$
$$\mu_2-\mu_1=0\cdot5 \text{ numerically}$$

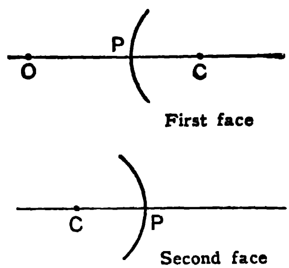


FIG. 118.—Example.

The surface is convex to the less dense medium, and so its power is positive. Hence $r=+10$ cm.

$$u=+60 \text{ cm. (real object)}.$$

$$\frac{1\cdot5}{v}+\frac{1}{+60}=\frac{0\cdot5}{10},$$

$$\frac{1\cdot5}{v}=0\cdot05-0\cdot0167=0\cdot0333,$$

$$\frac{1}{v}=\cdot022.$$

So $v=+45$, and a real image is thus formed 45 cm. beyond P.

For the second face, use

$$\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r},$$

$$\mu_2-\mu_1=0\cdot5 \text{ numerically}.$$

The power of the surface is positive, as before. So r must be 10 cm.

$$u=-(45-20)=-25 \text{ cm. (virtual object)},$$

$$\frac{1}{v}+\frac{1\cdot5}{-25}=\frac{\cdot5}{+10}=+\frac{1}{20},$$

$$\frac{1}{v}=+\frac{5}{100}+\frac{6}{100}=+\frac{11}{100},$$

and

$$v=+\frac{100}{11}+=9\cdot1 \text{ cm.}$$

The final image is real and 9·1 cm. beyond the far side of the sphere.

Note that in the course of the last three examples we have worked out the power of the sphere's surface four times. *Once* would have done, as it is always the same. So *working in terms of power can be made to save considerable labour.*

Example.—A glass reflector stud consists of a cylinder with two coaxial spherical ends. The front surface has a radius of curvature of 5 cm.; the back surface, which is silvered, a radius of 6 cm., and the thickness of the stud along the axis 6 cm. Find the position of the image of a point on the axis, 50 cm. from the first face.

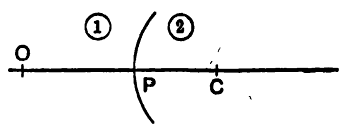


Fig. 119.—Example.

For the first refraction, use

$$\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}.$$

Here, $\mu_1=1,$

$\mu_2=1\cdot5,$

$\mu_2-\mu_1=0\cdot5$ numerically.

The surface is convex to the less dense medium, and so has a ositive power.

Hence r must be $+5$ cm.

$u=+50$ cm. (real object),

$$\frac{1\cdot5}{v}+\frac{1}{+50}=\frac{0\cdot5}{5},$$

$$v=+18\cdot75 \text{ cm.}$$

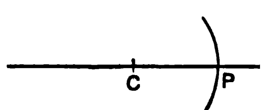


Fig. 120.—Example.

So the image is real and 18·75 cm. behind the front surface. It thus acts as a virtual object for the reflection at the rear face, being $18\cdot75-6=12\cdot75$ cm. behind this face.

For the reflection, use

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}.$$

Here, $r = +6$ cm. (concave mirror),

$u = -12{\cdot}75$ cm. (virtual object).

$$\frac{1}{v}-\frac{1}{12{\cdot}75}=\frac{2}{+6},$$

$$v = +2{\cdot}43 \text{ cm.}$$

The image is thus **2·43** cm. in front of the mirror and is real. It is thus a real object 6 − 2·43 or 3·57 cm. from the final refracting surface.

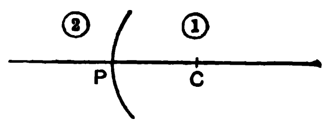


FIG. 121.—Example.

For the second refraction, use

$$\frac{\mu_2}{v}=\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}.$$

Here, $\mu_1 = 1{\cdot}5$,

$\mu_2 = 1$,

$\mu_2 - \mu_1 = 0{\cdot}5$ numerically.

The surface has a positive power, so r must be $+5$ cm.

$u = +3{\cdot}57$ cm. (real object),

$$\frac{1}{v}+\frac{1{\cdot}5}{+3{\cdot}57}=\frac{\cdot 5}{+5},$$

$$v = 3{\cdot}125 \text{ cm.}$$

So the image is virtual and 3·125 cm. behind the front surface.

Deviation produced by a sphere.—A ray of light transmitted by a sphere after two refractions undergoes a deviation of $D = 2(i-r)$. After an internal reflection (*not*, of course, total) D will be $(i-r)+(180°-2r)+(i-r)$ or $180°+2i-4r$. After two internal reflections D is $(i-r)+(180°-2r)+(180°-2r)+(i-r)$. Similarly for any number n of internal reflections the deviation is obtained by adding $n(180°-2r)$ to the refraction term $2(i-r)$.

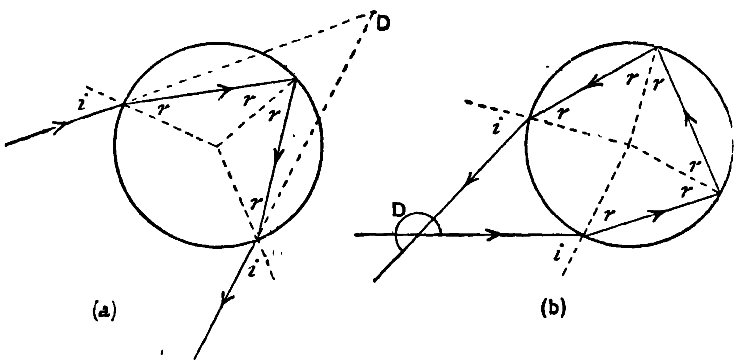
For each case a stationary value of the deviation occurs when $\frac{dD}{di}=0$.

For a single internal reflection $\frac{dD}{di}=2-4\frac{dr}{di}$,

and so, for the stationary value, $\frac{dr}{di}=\frac{1}{2}$.

Now, $$\mu \sin r = \sin i.$$

So, $$\mu \cos r \frac{dr}{di} = \cos i.$$



Single reflection. Two reflections.

FIG. 122.—Deviation by a sphere.

Whence, substituting for $\frac{dr}{di}$,

$$\frac{\mu}{2}\cos r = \cos i,$$

$$\mu^2 \cos^2 r = 4\cos^2 i,$$

$$\mu^2(1-\sin^2 r) = 4\cos^2 i,$$

$$\mu^2 - \sin^2 i = 4\cos^2 i,$$

$$\mu^2 = 3\cos^2 i + (\cos^2 i + \sin^2 i),$$

$$\frac{\mu^2-1}{3} = \cos^2 i,$$

$$\sqrt{\frac{\mu^2-1}{3}} = \cos i.$$

It can be shown, by differentiating the original expression a second time, that $\frac{d^2D}{di^2}$ is positive, so that this value of i cor-

responds to minimum deviation, and it is readily seen without this labour that it cannot be the maximum, for this occurs when $i = r = 0$ and the ray is reflected back normally undergoing a deviation of 180°.

With two internal reflections, $\frac{dr}{di}$ works out to $\frac{1}{3}$ in the stationary case, whence by a repetition of the previous argument we get for the value of i, when deviation is a minimum,

$$\cos i = \sqrt{\frac{\mu^2 - 1}{8}},$$

while for n internal reflections minimum deviation occurs when

$$\cos i = \sqrt{\frac{\mu^2 - 1}{n^2 + 2n}}.$$

The rainbow.—When the sun shines upon falling rain, an observer with his back to the sun sees one or sometimes two rainbows. These are parts of concentric circles, which have their centres on the line joining the sun to the observer's eye. The inner one, called the **primary bow**, subtends an angle of about 41° at the eye, and the outer, the **secondary bow**, an angle of about 53°. Both show the colours of an "impure" spectrum, the primary bow having its inner edge violet and outer edge red, while the orders of the colours are reversed in the secondary bow. Other bows are sometimes observed near the inner edge of the primary bow and the outer edge of the secondary bow, the nature of these **supernumerary bows** depending on the size of the raindrops. The region of the sky within the primary bow and just outside the secondary bow is always appreciably brighter than the rest of the sky, while the space between them is darker. The secondary bow is much less intense than the primary bow.

The primary and secondary bows are easily explained without regard to the size of the drops by the reasoning of the last paragraph. The value of $\frac{dD}{di}$ will be small in the neighbourhood of the minimum value of D, and thus for each colour the direction of minimum deviation will be that of greatest intensity. Substituting the values of μ for the extreme ends of the spectrum in the formula for i, and then working out the value for the angle of minimum deviation, the following results are obtained :

TABLE. (From Preston's *Light.*)

No. of internal reflections	Red light		Violet light	
	D	Acute angle between incident and emergent rays	D	Acute angle between incident and emergent rays
1	180° − 42·1°	42·1°	180° − 40·22°	40·22°
2	360° − 129·2°	51·8°	360° − 125·48°	54·52°

We conclude from this that if the primary bow is formed by light from the sun undergoing one internal reflection in the rain-drops and emerging at minimum deviation, there should be an

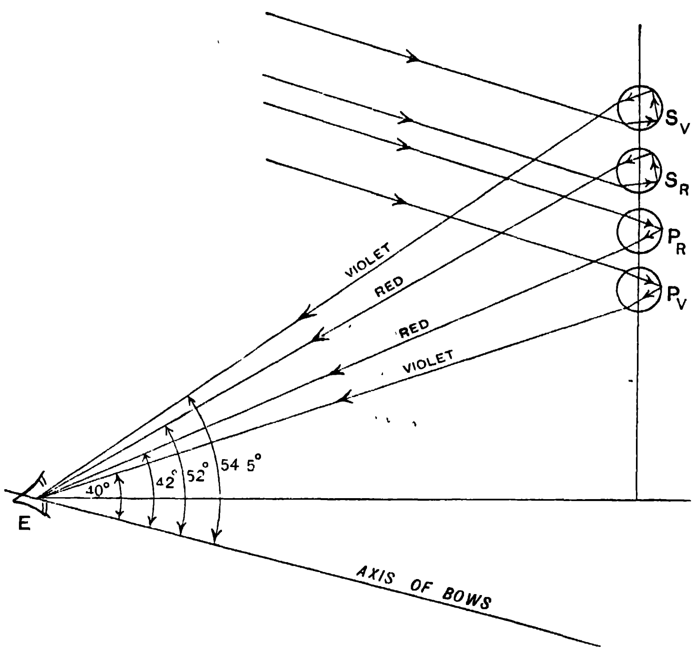


FIG. 123.—Rainbow.

inner violet edge subtending an angle of just over 40° at the eye and an outer red edge subtending an angle of about 42°, while if the secondary bow is formed by light undergoing two internal

reflections and emerging at minimum deviation, then the outer edge should be violet and subtend an angle of 54·5° while the inner edge should be red and subtend an angle of nearly 52°. Fig. 123 indicates the manner in which the eye at E sees both bows. Drops situated below the drop P_v will send light to the eye after one internal reflection, but at a deviation greater than the minimum, so that some illumination will reach the eye from within the bow. Similarly drops above S_v will give a faint illumination above the secondary bow from light which has undergone two internal reflections. In both these cases all the different colours overlap. Between the two bows no drops reflect light to the eye at E, for to do this would demand a deviation less than the minimum for either one or two reflections. The minimum deviation explanation thus fits all the observed facts.

As to the supernumerary bows, these are similar in nature to the coloured bands observed when a distant source of light is viewed through a series of narrow slits. These bands, which depend on the width of the slits and their distance apart, are discussed in the chapter on Diffraction.

A complete theory of the rainbow embracing both the true and supernumerary bows has been worked out, and according to this the angular radius of the primary bow should be a little less than the value given by the simple minimum deviation explanation and that of the secondary bow a little greater.

Aplanatic points for a spherical surface.—The equation

$$\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}$$

only holds for rays very close to the axis. A wide-angle beam will give focal lines instead of a point focus with a point object. It is possible to find two points on the axis such that all rays diverging from one appear after refraction to have come from the other.

For if O and I are points such that

$$\frac{CA}{CO}=\frac{CI}{CA},$$

then the two triangles IAC, OAC, with common angle $\widehat{ICA}$, will be similar.

Now, $\widehat{OAC} = i_1$, $\widehat{IAC} = \widehat{AOC} = i_2$.

In the triangle OAC, $$\frac{OC}{CA} = \frac{\sin OAC}{\sin AOC} = \frac{\sin i_1}{\sin i_2} = \frac{\mu_2}{\mu_1}.$$

So, $$OC = \frac{\mu_2}{\mu_1} AC,$$

$$IC = \frac{\mu_1}{\mu_2} AC.$$

This result is true whatever the values of i_1 and i_2, since we have eliminated them and made no approximations at all. So all rays diverging from O appear to come from I. O and I are called **aplanatic** points for the surface.

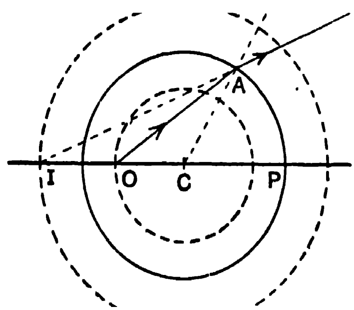


FIG. 124.—Aplanatic points for sphere.

Now if we write
$$u = (+PO) \text{ (real object)},$$
$$v = (-PI) \text{ (virtual image)},$$
$$r = (+PC),$$

we have
$$u = +r\left(1 + \frac{\mu_2}{\mu_1}\right),$$
$$v = -r\left(1 + \frac{\mu_1}{\mu_2}\right).$$

As a check it may be shown that these are possible values of u and v for axial rays by substituting in the formula for axial rays.

QUESTIONS ON CHAPTER VII

1. Prove the following relation for a ray of light refracted at a spherical surface separating two media of refractive indices 1 and μ respectively, and having radius of curvature r:

$$\frac{\mu}{v}+\frac{1}{u}=\frac{\mu-1}{r}.$$

Show that a gold-fish in a globe of water will appear nearer than it really is, at its true distance, or farther off, according as its true distance from the observer is less than, equal to, or greater than the distance of the centre from the observer.

2. Obtain a formula connecting the distances of object and image from a spherical refracting surface.

A small piece of paper is stuck on a glass sphere of 5 cm. radius and viewed through the glass from a position directly opposite. Find the position of the image. Find also the position of the image formed, by the sphere, of an object at infinity.

(O. & C.H.S.C.)

3. A glass hemisphere, 10 cm. diameter, is silvered over its curved surface. There is an air bubble in the glass 3 cm. from the plane surface along the axis. Find the positions of the images of this bubble seen by the observer looking along the axis into the flat surface of the hemisphere. ($\mu=1{\cdot}5$.) (N.U.J.M.B.H.S.C.)

4. A ray of light is incident on the surface of a transparent sphere of refractive index 1·32. After refraction it is internally reflected, and is then refracted out of the sphere again. Determine the value of the angles of incidence and deviation to the nearest degree so that the latter may be a minimum. (R.S.)

5. A large glass sphere is placed immediately behind a small hole in an opaque screen, and a small filament lamp is placed at such a distance u in front of the hole that its image falls within the sphere, and at a distance v behind the hole. (a) Sketch the course taken by the light rays in the formation of this image. (b) Derive a formula connecting the quantities u and v with the refractive index μ of the glass. (c) If $\mu=1{\cdot}5$, find the condition such that the image of the lamp is formed on that portion of the boundary surface of the sphere which is opposite to the hole. (N.U.J.M.B.H.S.C.)

6. Viewed normally through its flat surface, the greatest thickness of a plano-convex lens appears to be 2·435 cm., and through its curved surface 2·910 cm. Actually it is 3·665 cm. Find (a) the refractive index of the glass, (b) the radius of curvature of the convex surface. Do you consider this a satisfactory method of finding the radius of curvature?

(N.U.J.M.B.H.S.C.)

CHAPTER VIII

THIN LENSES

Introduction.—A portion of a transparent medium bounded by two spherical surfaces is called a **lens.** In the last chapter we saw how to calculate the position of the image formed by rays close to the axis after two refractions, but no general formula was given. If the lens is *so thin that the distance between its two surfaces can be neglected,* the treatment is simplified. Most lenses used for ordinary experiments nearly satisfy this condition.

The principal axis of the lens is the line through the centres of curvature of its faces. The tangent planes at the points P_1, P_2, in which the axis cuts these faces, are parallel, so that the middle of the lens face acts like a parallel-sided slab, and rays striking it pass through undeviated. It can be shown that for *any thin lens* there is a point C, between P_1 and P_2, through which all rays emerging parallel to their original directions must pass. This point is called the **optical centre** of the lens. For a thin lens we can regard P_1, C and P_2 as coinciding and say that the optical centre is the point in which lens and axis intersect.

A **converging lens** causes a narrow beam of parallel rays parallel to the axis and close to it to converge to a point on the far side of the lens. A **diverging lens** causes such a beam to appear to diverge from a point on the same side of the lens as the incident light. In each case the point is called the **principal focus** of the lens, and it is **real in the case of a converging lens and virtual for a diverging lens.** The distance from the optical centre to the principal focus is called the **focal length** of the lens. If the direction of the light is reversed, another principal focus on the other side of the lens is found. Where it is necessary to distinguish between the two principal foci of a lens, that on the same side as the source

of light will be called the first principal focus, and the other the second principal focus.

All distances measured to **real points** are, as usual, reckoned as **positive**, and all distances to **virtual points negative.**

The focal length of a converging lens is positive. The focal length of a diverging lens is negative.

Double-convex
Powers of both surfaces positive.

Plano-convex
Power of curved surface positive.

Convex-meniscus
Surfaces have opposite powers, but positive is greater.

FIG. 125.—Converging lenses.

Double-convex, plano-convex and **convex-meniscus** lenses, all thicker at the middle than the edges, are **converging. Double-concave, plano-concave** and **concave-meniscus** lenses, all thinner at the middle than the edges, are **diverging.**

Double-concave
Powers of both surfaces negative.

Plano-concave
Power of curved surface negative.

Concave-meniscus
Surfaces have opposite powers, but negative is greater.

FIG. 126.—Diverging lenses.

Conjugate Foci.—In Fig. 127, A and B are the two faces of a thin lens with radii of curvature r_1 and r_2. The lens is in air, and the refractive index of its glass is μ. If light is incident first on face A, the power of this face, $\frac{\mu_2 - \mu_1}{r}$, is $\frac{\mu - 1}{r}$. The *power* will be

the same, of course, whichever way the light goes through. Similarly for B, whichever way the light goes the power is $\frac{\mu-1}{r_2}$. In each case the power is positive for a surface convex to the air, and negative for a concave surface, so **the radius of curvature of a convex lens surface is positive, that of a concave lens surface negative.**

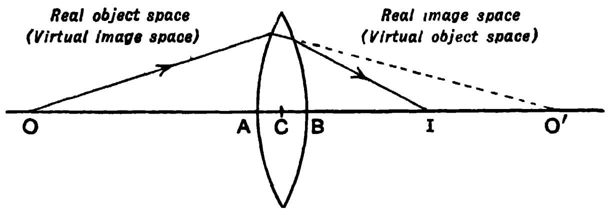


FIG. 127.

The face A forms an image O' of a point O on the principal axis ; O' acts as an object for the face B, which forms a final image at I. Let the distances of O and O' from A be u and u'.

Then in the formula $\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}=$ power of surface, we have,

since $\mu_1=1$ and $\mu_2=\mu$,

$$\frac{\mu}{u'}+\frac{1}{u}=\frac{\mu-1}{r_1}. \quad \text{..............................(1)}$$

Now if O' is in the real image space of surface A, it must be in the virtual object space for surface B. This is the case shown in the diagram. The lens is thin and we neglect the distance AB, so the distance of the virtual object O' from B is $-u'$. Also, $\mu_1=\mu$ and $\mu_2=1$ and the power of the surface B is $\frac{\mu-1}{r_2}$. The distance v of the image I from the lens is given by

$$\frac{1}{v}-\frac{\mu}{u'}=\frac{\mu-1}{r_2}. \quad \text{............................(2)}$$

Adding together both sides of equations (1) and (2),

$$\frac{1}{v}+\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right).$$

As A and B are taken to coincide with the optical centre of

the lens, u and v represent object and image distances measured from C.

Notice the symmetry of the formula ; u and v are interchangeable, and O and I are **conjugate foci.**

If u is infinite, $\frac{1}{u}=0$. The image will then be at the second principal focus and $v=f$. Thus the formula can be written

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right).$$

Putting this value for the right-hand side of the previous equation we obtain $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, the fundamental lens equation.

EXAMPLE.—A thin biconvex lens is made of glass of refractive index 1·5, and the radii of curvature of its surfaces are 20 cm. and 10 cm. What is its focal length ?

If the light is incident first on the 20 cm. face, then

$$r_1 = +20 \text{ cm. (convex to air, +power)}$$

and $$r_2 = +10 \text{ cm. (convex to air, +power).}$$

Using the formula

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right),$$

$$\frac{1}{f}=0{\cdot}5(0{\cdot}05+0{\cdot}1)=0{\cdot}075.$$

Whence, $$f=+13{\cdot}33 \text{ cm.}$$

EXAMPLE.—Find the position of the image of a point on the principal axis 20 cm. from the optical centre of the lens of the last example.

Here, $$u=+20 \text{ cm. (real object),}$$

$$f=+13{\cdot}33 \text{ cm.}$$

Using the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\frac{1}{v}+\frac{1}{+20}=\frac{1}{13{\cdot}33},$$

$$\frac{1}{v}=0{\cdot}075-0{\cdot}05=0{\cdot}025,$$

$$v=+40 \text{ cm.}$$

The image is 40 cm. from the optical centre on the far side of the lens and is real.

Power of a lens. Dioptre.—The powers of the two faces of the lens are given by

$$\mu - 1$$

$$F_B = \mu - 1$$

The power of the lens, F, is the sum of the powers of its two surfaces.

So, $$F = F_A + F_B = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{1}{f}.$$

That is, the power of a thin lens in air is the reciprocal of its focal length. The **power** of a **converging lens** will thus be **positive**, and that of a **diverging lens negative**. The unit of power, called the **dioptre**, is that of a lens of one metre focal length. For our purposes there is no advantage to be gained by working in terms of power instead of focal length, but this system saves the practising optician considerable labour with reciprocal tables.

Alternative derivation of formulae.—A section of a thin lens by a plane through the principal axis can be regarded as made up of a large number of pieces of principal sections of thin prisms,

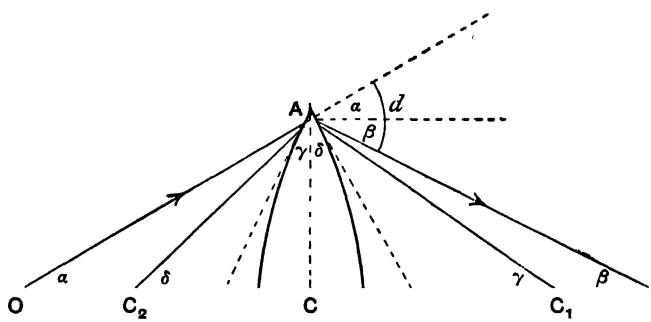


FIG. 128.—Deviation method of deriving formulae.

the parts nearest the optical centre belonging to prisms of smallest angle, the angles increasing towards the edge. Accepting the experimental fact that all rays leaving an object point O on the axis are brought to a point I on the axis provided they make a small angle with it, we can calculate the position of I from the deviation of the ray going through the edge of the lens if the aperture is small.

Let C be the optical centre of the lens, A a point on its edge, the vertex of the section, and C_1 and C_2 the centres of curvature of the two faces.

Let the ray OA make a small angle α with the axis, and after refraction along AI a small angle β with it. The deviation of this ray is $(\alpha+\beta)$. Now as the angles α and β are both small we can write for their values in radians,

$$\alpha = \frac{AC}{CO}, \quad \beta = \frac{AC}{CI}.$$

So the deviation $d = (\alpha + \beta) = \dfrac{AC}{CO} + \dfrac{AC}{CI}$.

The angle of the "prism" at A is the sum of the angles the two tangents to the traces of the surfaces make with the line AC; this is the sum of the angles the two radii AC_1 and AC_2 make with the axis, γ and δ respectively.

Now the aperture is small and so γ and δ are both small, and these in radians are

$$\gamma = \frac{AC}{CC_1}, \quad \delta = \frac{AC}{CC_2}.$$

The angle between the two faces at A is thus

$$A = \gamma + \delta = \frac{AC}{CC_1} + \frac{AC}{CC_2}.$$

Now for a thin prism of refractive index μ in air we know

$$d = (\mu - 1)A.$$

So,
$$\frac{AC}{CI} + \frac{AC}{CO} = (\mu - 1)\left(\frac{AC}{CC_1} + \frac{AC}{CC_2}\right)$$

or
$$\frac{1}{CI} + \frac{1}{CO} = (\mu - 1)\left(\frac{1}{CC_1} + \frac{1}{CC_2}\right).$$

This is a numerical equation into which no question of signs has entered. With our sign convention and the distances as in the diagram, it happens that all the algebraic signs are positive.

Hence
$$v = +CI \quad \text{(real image)},$$
$$u = +CO \quad \text{(real object)},$$
$$\left.\begin{array}{l} r_1 = +CC_1 \\ r_2 = +CC_2 \end{array}\right\} \text{(convex to air, + powers)},$$

the algebraic equation is $\dfrac{1}{v} + \dfrac{1}{u} = (\mu - 1)\left(\dfrac{1}{r_1} + \dfrac{1}{r_2}\right)$.

The deviation produced by a *thin prism* is constant for all rays making small angles of incidence on it. So the deviation of all rays through A is constant and equal to $\frac{AC}{v} + \frac{AC}{u}$ or $\frac{AC}{f}$. **The deviation of a single ray by a thin lens is thus**

$$\frac{\text{distance of point of incidence from principal axis}}{\text{focal length of lens}}.$$

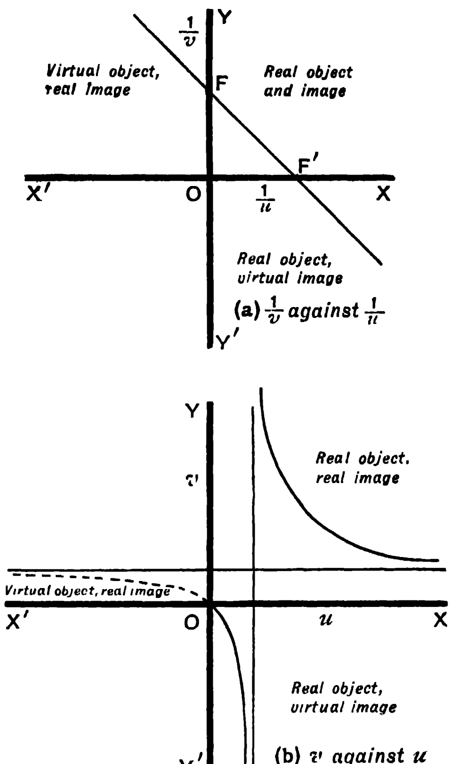


FIG. 129.—Curves for converging lens.

Examination of the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.—The formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ may be tested experimentally, using as object an illuminated pin-tip on the axis. Real images are located with a second pin

by the method of no-parallax, and virtual images with the aid of an auxiliary mirror as on p. 147.

With a converging lens the graph of $\frac{1}{v}$ against $\frac{1}{u}$ is as shown, a straight line at 45° to the axis. The distances OF, OF' should

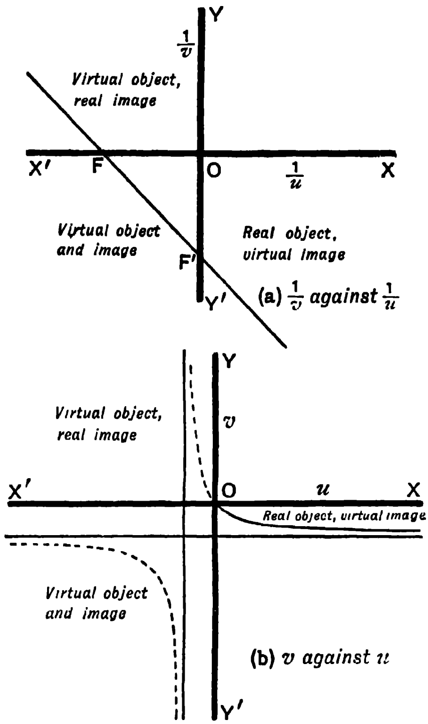


FIG. 130.—Curves for diverging lens.

each equal $\frac{1}{f}$ numerically. The graph of v against u for a converging lens is in two portions. It is a rectangular hyperbola with the lines $u = +f$, $v = +f$, as asymptotes. The lower branch could be continued as the dotted line shows if we explored negative values of u, using a beam of light converging to a virtual object point beyond the lens.

With a diverging lens the curves will be as Fig. 130.

These curves show that a converging lens gives a real image if u is numerically greater than f, while if u is less than f it gives a virtual image. This may be deduced from the formula, for $\frac{1}{v}$ is negative only if $\frac{1}{f}-\frac{1}{u}$ is negative, and as u is always positive with a real object, this means that if f is positive it must be numerically greater than u for v to be negative. For both lenses the v-u curves pass through the origin. When $u=0$, $v=0$; thus the optical centre is a self-conjugate point. Also, the slope of both curves is always such that as u increases algebraically v decreases. That is, motion of the object causes motion of the image in the same direction. This also can be seen from the formula, for $\frac{dv}{du}=-\frac{v^2}{u^2}$, which is always negative, so that as the object moves up towards the lens the image moves away. So the graphs obtained experimentally do agree with the formula. The conditions of the experiment with pin-tips and no-parallax are good, for we are dealing with a point object on the axis and narrow pencils of rays close to the axis, for which the formula holds.

Closest distance of object and image for a converging lens.—Let

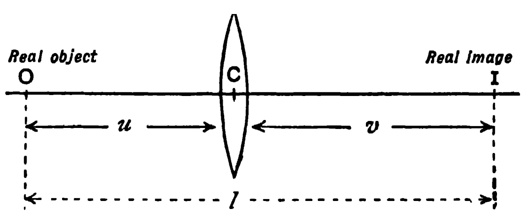


FIG. 131.—Distance between object and real image.

l denote the distance OI between object and image. Then $l=CO+CI$.

Now, $\left.\begin{array}{l}(+CO)=u\\(+CI)=v\end{array}\right\}$ (object and image both real),

So, $$l=u+v.$$

For turning values, $\frac{dl}{du}=0$.

So, $$\frac{dl}{du}=1+\frac{dv}{du}.$$

When $$\frac{dl}{du}=0,\ \frac{dv}{du}=-1.$$

But $$\frac{dv}{du}=-\frac{v^2}{u^2}\ \text{(cf. p. 55).}$$

So for a maximum or minimum value of l, $\frac{v}{u^2}=1$. The maximum values we know to be infinity where $u=\infty$ or $v=\infty$, so $v=\pm u$ gives minima.

When $v=+u$, the equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ gives $\frac{1}{u}+\frac{1}{u}=\frac{1}{f}$, or $u=2f$; hence $v=2f$ and $l=4f$.

The least distance between object and real image for a converging lens is thus four times the focal length. When $v=-u$, the only possible values for u and v are both zero, so virtual image and object coincide when both are at the optical centre of the lens

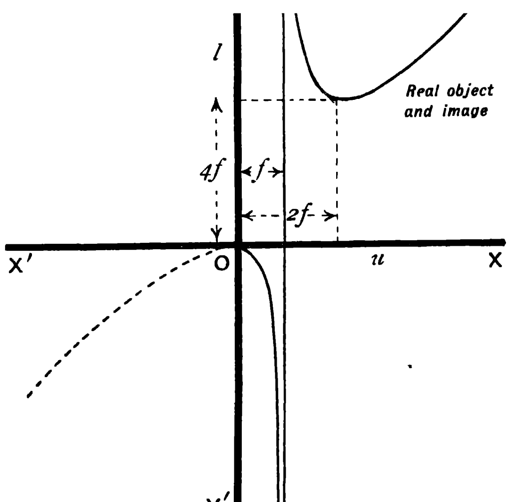


FIG. 132.—Curve showing how l varies with u.

The graph of l against u is as shown ; $l=0$, as the *least negative* value, appears as a maximum on the curve.

Newton's formula.—Let x and y be the distances of the object and image from the first and second principal foci, measured outwards.

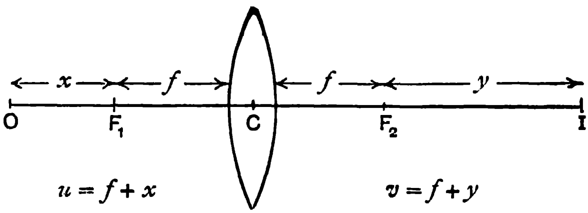


FIG. 133.—Newton's formula.

Then, $$u=f+x,$$
$$v=f+y.$$

And $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

becomes $$\frac{1}{f+y}+\frac{1}{f+x}=\frac{1}{f},$$

which simplifies to $$xy=f^2.$$

Image of a small object perpendicular to the axis.—Each point on a small object near the axis will give its image point close to

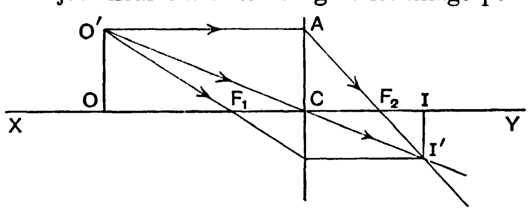


FIG. 134.—Graphical construction, first method.

the axis; and the image of a small object perpendicular to the axis will be nearly perpendicular to the axis. The position and size of the image may be found by graphical construction.

Let OO' represent to scale a small object perpendicular to the axis XY of a converging lens outside its first principal focus. Let CO and CF represent to scale the object's distance and the focal length of the lens. From the tip O' of the object two convenient rays can be traced; $O'A$ parallel to the axis is refracted through the principal focus F, and $O'C$ through the optical centre goes on undeviated. These two rays meet at I', which is

the image of O', since all other rays from O' pass through it. The perpendicular $I'I$ drawn to the principal axis represents the image of OO'. A third ray, $O'F_1$, is available ; this after refraction at the lens emerges parallel to the principal axis.

An interesting alternative method locates I instead of I'. A line is drawn through F_2 perpendicular to the axis and all rays parallel to one another, though from different points on the object, intersect at some point on this line. $O'C$ is drawn, cutting this line in X ; OD, parallel to $O'C$, will also pass through X after refraction and DX produced cuts the principal axis at I. $O'C$ intersects the line through I perpendicular to the axis in I', and II' is the image of OO'.

If the object is between F_1 and C for a converging lens, the image is found by producing back the refracted rays. It is on

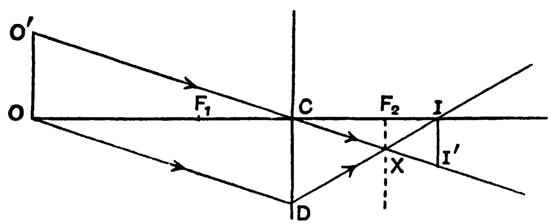


FIG. 135.—Graphical construction, second method.

the same side as the object and is virtual. For a diverging lens, the ray $O'A$ appears after refraction to have come from F_1 ; or, using the second method, the point X is somewhere on the perpendicular through F_1. The image for a real object is always virtual.

Lateral magnification, *m*.—In Fig. 134, from the similar triangles $OO'C$, $II'C$,

$$\frac{II'}{OO'}=\frac{CI}{CO}.$$

So, numerically, $\dfrac{\text{length of image}}{\text{length of object}}=\dfrac{\text{distance of image}}{\text{distance of object}}.$

Or, $$\mathrm{m}=\frac{\mathrm{II'}}{\mathrm{OO'}}=\frac{\mathrm{v}}{\mathrm{u}}.$$

If m *is positive,* u and v have the same sign and the image is **inverted ; for a real object the image is real.** (See p. 54.)

If m *is negative*, the image is **upright and for a real object will be virtual.**

It is sometimes convenient to refer to m in terms of u and f, or of v and f.

Since $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f},$$

$$\frac{1}{u}=\frac{1}{f}-\frac{1}{v}.$$

Hence, $$\mathbf{m}=\frac{\mathbf{v}}{\mathbf{u}}=\frac{\mathbf{v}}{\mathbf{f}}-1=\frac{\mathbf{v}-\mathbf{f}}{\mathbf{f}}$$

Also, $$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

$$=\frac{u-f}{fu}.$$

Hence, $$v=\frac{fu}{u-f}$$

and $$\mathbf{m}=\frac{\mathbf{v}}{\mathbf{u}}=\frac{\mathbf{f}}{\mathbf{u}-\mathbf{f}}.$$

The x and y of Newton's formula are $(u-f)$ and $(v-f)$ respectively, so that $m=\frac{y}{f}=\frac{f}{x}$.

The formulae $m=\frac{y}{f}$, $m=\frac{f}{x}$ are useful in shortening calculations of the following type:

EXAMPLE.—How far away from a thin lens of focal length $+17\cdot5$ cm. must an object be placed to give a real image magnified six times?

Here, $m=+6$ as the image is to be real,

$f=+17\cdot5$ cm.

So, using $$m=\frac{f}{x},$$

$$6=\frac{17\cdot5}{x} \text{ or } x=2\cdot91 \text{ cm.}$$

So the object must be $2\cdot91$ cm. outside the first principal focus or $20\cdot41$ cm. from the lens.

The superficial magnification of a small area normal to the axis and close to it will be m^2 or $\frac{v^2}{u^2}$.

Longitudinal magnification.

As
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f},$$
$$\frac{dv}{du}=-\frac{v^2}{u^2}$$

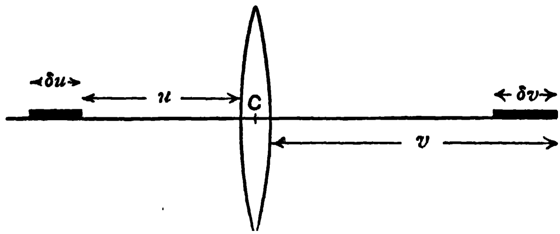


FIG. 136.—Longitudinal magnification.

A small object lying along the axis between u and $u+\delta u$ of length δu, giving an image of length δv lying between v and $v-\delta v$, thus gives a longitudinal magnification along the axis of $\frac{\delta v}{\delta u}$ or $-\frac{v^2}{u^2}$.

Two thin lenses in contact.—The same method as that used at the beginning of this chapter may be employed to find the focal length of two thin lenses in contact. Let f_1 and f_2 be the focal lengths of the lenses and f that of the combination. An object at distance u from the first gives an image at distance u', so
$$\frac{1}{u'}+\frac{1}{u}=\frac{1}{f_1}.$$
Now if the image formed by the first lens is in its real image space it must be in the virtual object space for the second lens, and *vice versa*. So that for the second lens u' as the object distance has the opposite sign to u' as the image distance for the first lens.

Let the distance of the final image from the second lens be v.

Then,
$$\frac{1}{v}-\frac{1}{u'}=\frac{1}{f_2}.$$

Adding,
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}.$$

But if f is the focal length of the combination,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

So,
$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2};$$

or, writing F, F_1, F_2 for the powers of the combination and the individual lenses,

$$F=F_1+F_2.$$

EXAMPLE.—A thin converging lens of 20 cm. focal length is placed in contact with a diverging lens of 8 cm. focal length. What is the focal length of the combination ?

Here,
$$f_1=20 \text{ cm.},$$
$$f_2=-8 \text{ cm.},$$

and
$$\frac{1}{f}=\frac{1}{20}-\frac{1}{8}=0{\cdot}05-0{\cdot}125=-0{\cdot}075.$$

So $f=-13{\cdot}33$ cm. The combination is thus diverging, and of focal length 13·33 cm.

Two thin lenses not in contact.—For a system of lenses or a single thick lens we meet with difficulty when we try to apply the simple lens formula. Conjugate foci and the first and second principal focus can be located, and from the existence of conjugate foci we may still hope to find a relation between u and v of the form $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$. But this formula does not fit experimental measurements in which u and v are taken from one face of the system, from the mid-point of the system, or from any *one point*. The problem is to find the *two points* from which the measurements must be taken.

We can locate the first and second principal foci, so let us start from them. Actual measurements of object and image distances from the foci show that the product of x and y is constant, so it appears as if the Newton formula is still satisfactory and $xy=f^2$; the focal length can be found in this way without bothering about the nature of the lens at all! Now for a thin lens we used $m=\frac{y}{f}=\frac{f}{x}$ and the *one point* from which we made all measurements, the optical centre, was such that $x=-f$, $y=-f$, and $m=-1$.

At a distance numerically equal to f measured towards the lens from the first principal focus we find a point P at which the magnification is -1. The first principal focus is at a distance f from this point; here, surely, is the reference point required on the object side of the lens. For a magnification of -1 the image will be at some other point Q, at a distance f from the second principal focus and on the lens side of it. This is the reference point required on the image side of the lens. These two points, P and Q are called the principal points of the system, and planes through P and Q perpendicular to the axis are called its **principal planes.** In the particular case of a thin lens P and Q coincided with the optical centre C, and so all measurements on both sides were made from C. Now we shall make all measurements on

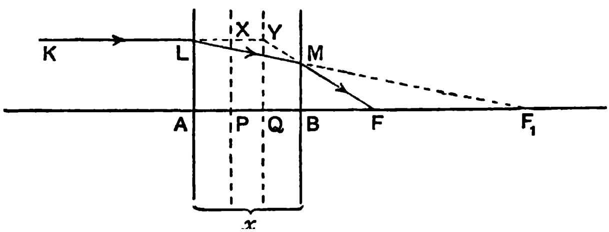


FIG. 137.—Two thin lenses not in contact.

the real-image side of the lens from Q, and on the real-object side from P, the same sign convention operating. The focal length of a lens system is thus **the distance from a principal point to the corresponding principal focus.**

Let their common principal axis cut at A and B two thin lenses, which we will also refer to as A and B, of focal lengths f_1 and f_2, and let the distance between them be x. Consider a ray KL parallel to the axis, striking the lens A at L. This is deviated so as to pass through F_1, the second principal focus of A, but striking the lens B at M it is deviated again to pass through F, the second principal focus of the combination. As F is the real image formed by the lens B of the virtual object F_1, BF is easily calculated.

Now let P and Q be the principal points on the object and image side. KL produced cuts the principal plane through P in X. Regarding PX as an object, a virtual image QY with magni-

fication -1 will be formed at the second principal plane. The ray MF will appear then to have come from Y in the principal plane through Q, where $QY = PX = AL$. QF is the focal length of the combination.

The first step is to find BF.

Since $$AF_1 = f_1,$$
$$BF_1 = (f_1 - x).$$

Using $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ for lens } B,$$
$$u = -(f_1 - x) \text{ (object virtual in figure)},$$
$$f = f_2.$$

Hence $$\frac{1}{v} - \frac{1}{(f_1 - x)} = \frac{1}{f_2},$$

and $$\frac{1}{v} = \frac{1}{f_2} + \frac{1}{f_1 - x}.$$

So, $$\frac{1}{BF} = \frac{1}{v} = \frac{f_1 + f_2 - x}{f_2(f_1 - x)}.$$

Now, $$\frac{QY}{QF} = \frac{BM}{BF} = \frac{BM}{BF_1} \cdot \frac{BF_1}{BF}.$$

But, $$\frac{BM}{BF_1} = \frac{AL}{AF_1} = \frac{QY}{AF_1}.$$

$$\therefore \quad \frac{QY}{QF} = \frac{QY}{AF_1} \cdot \frac{BF_1}{BF}.$$

Cancelling out QY and substituting for AF_1, BF_1, and BF.

$$\frac{1}{QF} = \frac{1}{f_1} \cdot (f_1 - x) \cdot \frac{f_1 + f_2 - x}{f_2(f_1 - x)}$$

or $$\frac{1}{f} = \frac{f_1 + f_2 - x}{f_1 f_2}$$

or $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}.$$

This result is our chief interest. A full discussion of thick lenses and lens theory will be found in advanced text-books.

Examples on thin lenses are grouped together at the end of Chapter IX.

CHAPTER IX

MEASUREMENTS AND PROBLEMS ON LENSES

The optical bench.—Nearly all lens measurements are best performed on an **optical bench.** Fig. 138 shows a good form. Object, lens, and a screen or pin are mounted on carriers bearing a reference mark, which slide along an axis with the marks touching a graduated scale. As the marks will not coincide with the points between which measurements are to be made, these are all

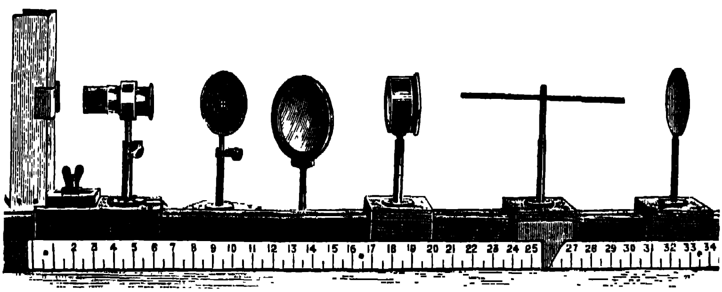


FIG. 138.—Optical bench.

effected by displacement in the following manner. A distance piece, say a knitting needle of known length, is held along the axis with one end in contact with one face of the lens. The position of the mark on the object's carrier is noted, and this carrier moved until the other end of the needle touches the object. The second position of the mark is noted, and the distance between lens face and object is (length of needle + shift of object). The principal axis of the lens must be set parallel to the axis of the bench, and the object set on the axis of the lens. Where possible it is preferable to use as object a suitably illumin-

ated pin tip on the axis and locate the image, real or virtual, by the method of no-parallax with another pin, for then an eye on the axis of the lens only receives a narrow pencil of rays close to the axis and we have all the conditions under which thin lens formulae hold.

Methods for finding the focal length of a converging lens.—(*a*) *Finding the principal focus directly.*—A real image of a distant object is formed on a screen or made to coincide without parallax with a pin tip. The distance from lens to image is f.

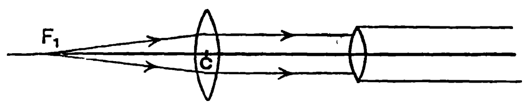


FIG. 139.—Use of telescope to determine focal length.

If a telescope focussed to receive parallel light is available, the lens may be made to give parallel light by moving an object along its axis until it is seen clearly through telescope and lens. The object is then at the first principal focus and CF_1 (Fig. 139) is the focal length.

(*b*) *Conjugate foci.*—With the object O at a distance numerically greater than f from the lens, a real inverted image I is formed. This is located with a pin P adjusted to give no-parallax. CO and CP are measured.

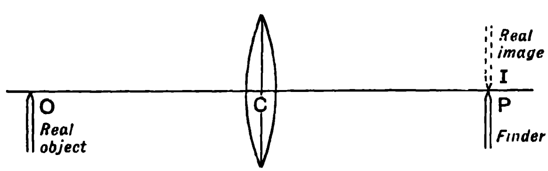


FIG. 140.—Conjugate foci for converging lens.

Then in the formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$u=(+CO),\quad v=(+CP)\ \text{(real object and image)},$$

so f is found by substitution, or by the graphical method of p. 64.

It is better to take several settings with OI in the neighbour-

hood of the minimum distance $4f$ than to take observations over a wide range, for then u and v can be determined with equal accuracy. Also, it is easier to judge no-parallax when P and I are about the same size. It may be felt that this talk of accuracy is somewhat pedantic if we are neglecting the several millimetres thickness of the lens ; but the minimum distance is also that at which neglecting the thickness makes least difference to the result. Is it worth while correcting for the thickness of the lens ? For an equiconvex lens for which $\mu = 1{\cdot}5$ the planes from which u and v should strictly be measured are $\frac{1}{3}$ the thickness of the lens behind the lens faces on either side. With such a lens of 10 cm. focal length and 5 cm. aperture the thickness will be about 6 mm. So both u and v are 2 mm. shorter than they should be; at the minimum distance u and v each equal $2f$ numerically, so that f is 1 mm. less than its true value and the error in f is some 1%. If f were 20 cm. and the aperture the same the thickness would be about 3 mm., the error in f from measurements at the minimum distance $\frac{1}{2}$ mm., and the percentage error $\frac{1}{4}$%. It is unlikely that a single setting will be made with an uncertainty of less than a millimetre, or that lenses much thicker than that of 10 cm. focal length (referred to above) will be used for simple measurements, so it is best to neglect the thickness of the lens completely.

(*c*) *Minimum distance.*—By moving lens and P alternately the position at which the distance OP is a minimum will be found. Moving the lens in either direction then causes the image to move towards the observer. OP is then four times the focal length of the lens.

A simple apparatus for facilitating the measurement of f by this method has the lens mounted at the centre of a lazy-tongs arrangement, and as this is extended object and finder at its extremities are always equidistant from the lens. The method has the advantage discussed in (*b*) and requires the measurement of only one length, and that a large one.

(*d*) *The two-position method.*—If O and P are fixed, and at a greater distance apart than four times the focal length, there will be two positions of the lens for which the real inverted image of O will coincide with P. These positions are represented by A

and B in Fig. 141. Now since O and P are at conjugate foci for the lens in the position A,

$$\left.\begin{aligned} u &= (+AO) \\ v &= (+AP) \end{aligned}\right\} \text{(real object and image).}$$

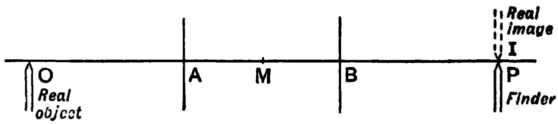


FIG. 141.—Two-position method.

Since u and v are interchangeable we should get the second position B when

$$u = (+AP) = (+BO)$$

and $$v = (+AO) = (+BP).$$

A and B are then symmetrically placed and equidistant from the mid-point M of OP. Let $OP = d$, $AB = a$.

Then $$OA = \frac{d-a}{2}, \quad AP = \frac{d+a}{2}.$$

With the lens at A,

$$u = \frac{d-a}{2}, \qquad v = \frac{d+a}{2}.$$

Using $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

$$\frac{2}{d+a} + \frac{2}{d-a} = \frac{1}{f}$$

and $$f = \frac{d^2 - a^2}{4d}.$$

Here again only one measurement between two fixed points, d, is made ; a is a displacement measurable directly.

(e) *Self-conjugate points with a lens and plane mirror.*—If a plane mirror is set behind the lens with its surface normal to the principal axis, a position of the object can be found at which the *real inverted* image I coincides with it. This is a self-conjugate point for the lens-mirror system. The diagram shows how the images of a point on the axis and of a small extended object are

formed. A point in the focal plane of the lens gives a beam of parallel rays after passing through it ; this is reflected as a parallel beam by the mirror, and so gives an image in the focal plane

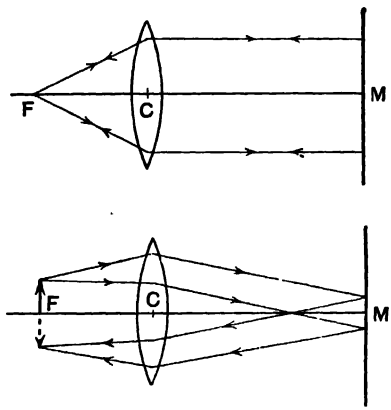


FIG. 142.—Lens and plane mirror. Object at principal focus.

of the lens after traversing it a second time. This self-conjugate point is thus the principal focus, and its distance from the lens is

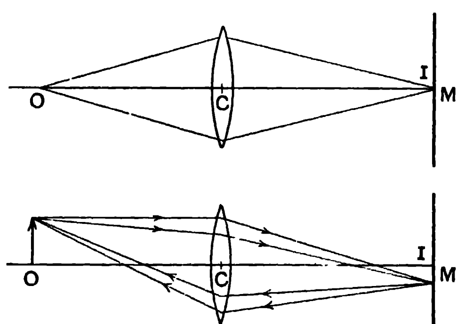


FIG. 143.—Lens and plane mirror. *Real image* formed at mirror.

measured. The distance from lens to mirror is immaterial, but there may be some difficulty in picking up the image if it is too great.

The simplest way of doing this experiment is to lay the mirror

on a horizontal bench, place the lens on top of it, and hold the object in a retort clamp.

If the mirror is at some distance greater than f behind the lens a different self-conjugate point will be found. The figures show how this occurs. The object O forms a real image I at the reflecting surface of the mirror M. The virtual image of I in M coincides with I. This acts as an object, and since I and O are conjugate foci the final image will be formed at O. This image will be *real and erect*, for inversion has taken place twice.

Writing

$$u = (+CO),$$

$$v = (+CM),$$

f may be found from the formula for conjugate foci. This experiment illustrates, more clearly than any discussion of the formula, the meaning of conjugate foci.

(f) *Newton's formula.*—The principal foci F_1 and F_2 are first located as in (a). Conjugate foci O and I are then found. The distances $F_1O = x$ and $F_2I = y$ are measured. Then $xy = f^2$. This involves *no measurements to or embracing the lens* and so will work for a thick lens or a combination of lenses.

(g) *Magnification methods.*—The object may be two fine parallel pencil lines, at a distance K apart, drawn on ground glass illuminated from behind.

The real image may be received on a screen. Object and screen are set up at a distance greater numerically than $4f$ apart, and the lengths l_1 and l_2 of the images formed when the lens is in the two positions A and B of Fig. 141 are measured. The lateral magnifications m_1 and m_2 are $\frac{l_1}{K}$ and $\frac{l_2}{K}$ respectively.

If $\qquad v_1 = (+AI)$ and $v_2 = (+BI)$

then (p. 135) $\qquad m_1 = \frac{v_1}{f} - 1, \quad m_2 = \frac{v_2}{f} - 1.$

So, $$m_1 - m_2 = \frac{v_1 - v_2}{f}$$

and $$f = \frac{v_1 - v_2}{m_1 - m_2}.$$

Or, writing a for the displacement of the lens, $(v_1 - v_2)$,

$$f = \frac{a}{m_1 - m_2}.$$

This involves *no measurements to or embracing the lens* and will apply for all lenses.

In Abbe's method the lens is kept fixed and the magnifications for two positions of object and screen determined. The formula for f in terms of the two magnifications and the displacement of either object or screen can be worked out on lines similar to the above.

(*h*) *Lenses of very long focal length.*—None of the foregoing methods could be used with any convenience in an ordinary laboratory for a lens of very long focal length. An auxiliary converging lens of short focal length f_1 can be placed in contact with the long-focus lens of long focal length f_2 and the focal length f of the combination found.

Then $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$, from which f_2 is calculated.

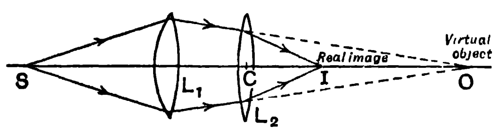


FIG. 144.—Long-focus converging lens.

Alternatively the short-focus lens L_1 can be used to form a virtual object at O, of which the long-focus lens L_2 forms a real image at I (Fig. 144).

Then for the lens L_2,

$$u = (-CO) \text{ (virtual object)},$$
$$v = (+CI),$$

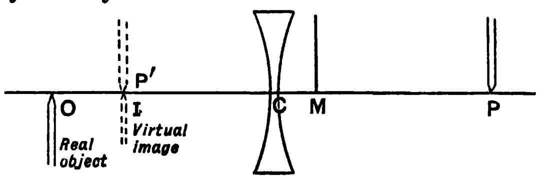
and these distances are measured and substituted in the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

Methods for finding the focal length of a diverging lens.—(*a*) *Conjugate foci.*—The virtual image I of O can be located using a pin P and a plane mirror M covering half the aperture of the lens. Measure CO, CM, PM.

Then, $$CI = (P'M - CM)$$
$$= (PM - CM)$$

and $$u = (+CO) \text{ (real object)},$$
$$v = (-CI) \text{ (virtual image)},$$

for substitution in the formula. If O is a very distant object, CI gives f directly.



ɪ. 145.—Conjugate foci for diverging lens, using auxiliary plane mirror.

(*b*) *With a converging lens in contact.*—If the diverging lens is placed in contact with a converging lens of shorter focal length the two will form a converging combination. If f_1 is the focal length of the converging lens, f that of the combination, and f_2 that of the diverging lens then, since $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$,

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}.$$

(*c*) *With a converging lens of any focal length not in contact.*—The converging lens L_1 (Fig. 146) gives a real image of O at the point O'.

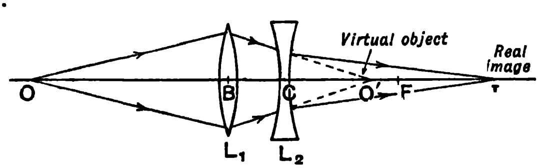


FIG. 146.—Converging lens *not in contact* with diverging lens.

If CO' is less than the focal length of L_2 a real image I will be formed, O' serving as a virtual object.

Here, $$u = (-CO') \text{ (virtual object)},$$
$$v = (+CI) \text{ (real image)},$$

for substitution in the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

With a plane mirror M (Fig. 147) set normal to the common axis of L_1 and L_2 for any position of the two lenses, a position of the object O can be found such that the real inverted image formed by rays which have traversed the two lenses twice and undergone reflection at the mirror coincides with I. In this case the rays reaching M must be parallel, so that O' is at the principal focus of L_2.

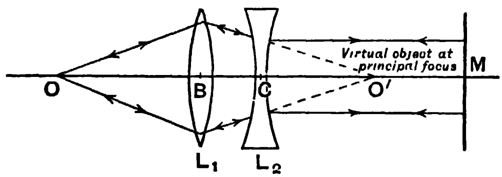


FIG. 147.

So $(-CO')$ is the value of u when $\frac{1}{v}=0$; hence $f=(-CO')$.

(*d*) *Using a concave mirror.*—The lens is placed in front of a concave mirror M, so that their principal axes coincide (Fig. 148). The object O is moved until a real inverted image coincident with it is formed.

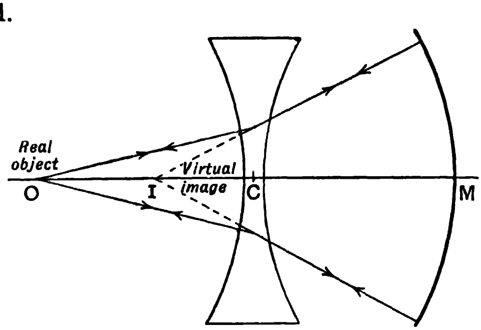


FIG. 148.

The virtual image I formed by the lens L is thus at the centre of curvature of the mirror.

Then, $\quad CI=(MI-MC)$

and $\quad u=(+CO)$ (real object),

$\quad v=(-CI)$ (virtual image),

in the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

To find the radii of curvature of the surfaces of a convex lens by Boys's method.—The object should be well illuminated and the internal reflection at the surface intensified by floating the lens on mercury with this face downwards.

A pin tip is moved along the axis of the lens until it coincides with its real image formed by reflection in the under surface B (Fig. 149). Rays are then striking B normally, as if they came

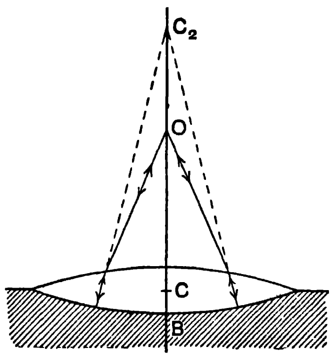


FIG. 149.—Boys's method.

from C_2, its centre of curvature. For the refraction at the first surface A of the lens, O and C_2 are therefore conjugate points. But the rays striking B are incident normally, so that *were they transmitted instead of being reflected* they would go through B without deviation at all. Hence if we were using the lens in the ordinary way, *in this one position of the object* the second face B would contribute nothing to its performance. So O and C_2 are conjugate foci *for the lens as a whole.*

Let f be the focal length of the lens, r_2 the radius of curvature of B; then,

$$u=(+CO) \quad \text{(real object)},$$
$$v=r_2=(-CC_2) \quad \text{(virtual image)}.$$

So, using $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f},$$

$$\frac{1}{r_2}+\frac{1}{u}=\frac{1}{f}, \quad \frac{1}{r_2}=\frac{1}{f}-\frac{1}{u}.$$

Or, $$r_2=\frac{fu}{u-f}.$$

The value of r_2 given by this equation will be negative in sign, out this must necessarily happen as C_2 is a virtual image point and no account need be taken of it.

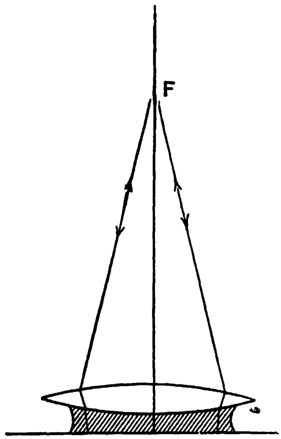


FIG. 150.—Refractive index of a small quantity of liquid.

To find the refractive index of a small quantity of liquid.—The focal length of a convex lens f_1 and the radius of curvature of one of its faces, r, are found. The lens is placed on a large drop of liquid on a horizontal plane mirror, thus forming a converging combination with a plano-concave lens of liquid. The focal length f of the combination is found by locating the self-conjugate point F as shown in Fig. 150. The focal length f_2 of the liquid lens is thus given by

$$\frac{1}{f_2}=\frac{1}{f}-\frac{1}{f_1}.$$

Now for the liquid lens,

$$\frac{1}{f_2}=(\mu \quad 1)\left(\frac{1}{r}+\frac{1}{\infty}\right)=\mu-1$$

So,
$$\frac{\mu-1}{r}=\frac{1}{f}-\frac{1}{f_1}.$$

Or,
$$\mu=1+r\left(\frac{1}{f}-\frac{1}{f_1}\right.$$

Here both r and the expression in the bracket are algebraically negative.

Focal length of a thin lens not bounded on both sides by air.—

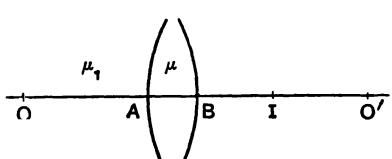


FIG. 151.—Thin lens not surrounded by air.

Let the absolute refractive index of the lens material be μ,

and those for the media bounding faces A and B (Fig. 151), be μ_1 and μ_2.

The powers of the faces are then $\frac{\mu-\mu_1}{r_1}, \frac{\mu-\mu_2}{r_2}$.

Reworking the calculation of p. 125 we have for the first refraction,

$$\frac{\mu}{u'}+\frac{\mu_1}{u}=\frac{\mu-\mu_1}{r_1};$$

and for the second,

$$\frac{\mu_2}{v}-\frac{\mu}{u'}=\frac{\mu-\mu_2}{r_2}.$$

Whence,
$$\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu-\mu_1}{r_1}+\frac{\mu-\mu_2}{r_2}.$$

The right-hand side is, as before, the sum of the powers of the two surfaces and we will denote this by F, so that

$$\frac{\mu_2}{v}+\frac{\mu_1}{u}=F.$$

Now if
$$v=\infty, \quad \frac{\mu_1}{u}=F.$$

The distance of the first principal focus from the optical centre, f_1, is thus $\frac{\mu_1}{F}$.

If $u=\infty$, $\frac{\mu_2}{v}=F$, so that the distance of the second principal focus from the optical centre, f_2, is $\frac{\mu_2}{F}$.

So the first and second focal lengths will not be the same and will be in the ratio $\frac{f_1}{f_2}=\frac{\mu_1}{\mu_2}$.

If the initial and final media are the same, but not air, we can put $\mu_2=\mu_1$ in the expression first obtained.

Then,
$$\frac{1}{v}+\frac{1}{u}=\frac{\mu-\mu_1}{\mu_1}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

Or,
$$\frac{1}{v}+\frac{1}{u}=\left(\frac{\mu}{\mu_1}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right).$$

So that instead of using the refractive index from air to glass, μ, we should use the refractive index from the surrounding medium to glass, $\frac{\mu}{\mu_1}$, in the usual expression for the focal length.

Flare spots.—When a really intense beam of parallel light falls on a convex lens, bright spots on the axis can be detected between the lens surface and the chief bright spot, the principal focus F. These are formed by light which has been internally reflected, and Fig. 152 shows the formation of F_1, the brightest. They are called flare spots. By treating in turn the refraction at A,

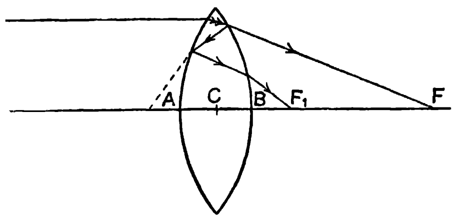


FIG. 152.—Flare spots.

the two internal reflections at B and A, and finally the refraction at B, using the image calculated at each step as the object for the next and eliminating the radii of curvature of the surfaces, using $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$, it is seen that the distance CF_1 is $\frac{\mu-1}{3\mu-1}f$. The actual working out of this is left as an exercise for the student. For glass for which $\mu=1{\cdot}5$, $CF_1=\frac{{\cdot}5}{3{\cdot}5}f=\frac{1}{7}f$.

This gives a method of finding the refractive index of the material of a lens without measuring the radii of its surfaces, and also suggests a method of finding the focal length of a long-focus lens.

Other spots should be formed on the same side of the lens as F_1 at distances

$$\frac{\mu-1}{5\mu-1}f,\quad \frac{\mu-1}{7\mu-1}f,$$

and so on, but these will generally be too small and too faint to be seen. There will also be a corresponding set of spots on the other side of the lens at distances

$$\frac{\mu-1}{2\mu-1}f,\quad \frac{\mu-1}{4\mu-1}f,\quad \frac{\mu-1}{6\mu-1}f,\ \ldots,$$

and of these the first and second, at distances of $\frac{f}{4}$ and $\frac{f}{10}$ for a lens for which $\mu=1{\cdot}5$, can be seen fairly easily. Meier recom-

mends a lens of at least 3 inches diameter and focal length less than a foot, with a 100 watt lamp, and screens to prevent direct light from the lamp entering the eye.

QUESTIONS ON CHAPTER IX

1. Define the focal length of (*a*) a diverging lens, (*b*) a converging lens. Explain how the focal length of a diverging lens may be conveniently measured if a converging lens is available having a numerically shorter focal length.

A small object is placed at zero on the scale of an optical bench. A converging lens for which $f=10$ cm. and a diverging lens for which $f=8$ cm. are set up at 35 cm. and 45 cm. respectively, the object being on the common axis of the lenses. At what point on the scale must a screen be placed to receive the real image formed by the combination ? Calculate the magnification. (C.W.B.H.S.C.)

2. Show that the focal length of a thin lens can be calculated from a knowledge of the refractive index and the radii of curvature of the two faces.

What will be the focal length in water of a biconvex lens whose radii of curvature are 10 cm. and 15 cm. ? (O. & C.H.S.C.)

3. (*a*) Find an expression for the focal length of a combination of two thin lenses in contact.

(*b*) A symmetrical convex glass lens, the radii of curvature of which are 3 cm., is situated just below the surface of a tank of water which is 40 cm. deep. An illuminated scratch on the bottom of the tank is viewed vertically downwards through the lens and the water. Where is the image, and where should the eye of the observer be placed in order to see it ? The refractive indices of glass and water may be taken as $\frac{3}{2}$ and $\frac{4}{3}$ respectively. (O. & C.H.S.C.)

4. Describe an optical method of finding the radius of curvature of a surface of a thin convex lens.

An object is placed on the axis of a thin plano-convex lens, and is adjusted so that it coincides with its own image formed by light which has been refracted into the lens at the first surface, internally reflected at the second surface, and refracted out again at the first surface. It is found that the distance of the object from the lens is 20·5 cm. when the convex surface faces the object, and 7·9 cm. when the plane surface faces the object. Calculate (*a*) the focal length of the lens, (*b*) the radius of curvature of the surface, (*c*) the refractive index of the glass. (C.H.S.C.)

5. A convex lens mounted on an optical bench forms an image of an object on a screen, the magnification being 2·5. The object and screen are kept fixed and the lens is moved through a distance of

10 cm., when a sharp image is again formed on the screen, the magnification being now 0·4. What is the focal length of the lens ? (C.H.S.C.)

6. A convex lens is set up in front of a mirror at a distance of 2 in. from its surface. A pin is placed on the axis of the lens on the far side from the mirror and is adjusted until it gives no parallax with its own image. With a plane mirror the distance of the pin from the lens is 6 in. ; with a convex mirror it is 8 in., and with a concave mirror 4 in. Explain, with the aid of diagrams, how the final image is formed in each case, and calculate the focal length of the lens and the radii of curvature of the two spherical mirrors. (O.H.S.C.)

7. Deduce a formula for the focal length of a thin lens, given the radii of curvature of its surfaces and the refractive index of the glass.

An object 8 cm. long is placed upright on the axis of a convex lens and 15 cm. from the lens. If the image is formed 60 cm. from the lens on the side remote from the object, determine the focal length of the lens and the size of the image. (C.W.B.H.S.C.)

8. Describe a method you would use to find the focal length of a concave lens.

A beam of light is converging towards a point 1 foot beyond a lens, and after refraction it converges to a point 20 inches beyond the lens.

Calculate the focal length of the lens and say whether it is convex or concave. (O.H.S.C.)

9. What factors determine the focal length of a thin lens ?

A plano-concave water lens is formed between a plane glass plate and one surface of a thin double-convex glass lens of focal length 15·8 cm., and the focal length of the convergent combination is found to be 21·6 cm. When the glass lens is turned over, so that the other surface is in contact with the water, the focal length of the combination is 21·3 cm. Find the refractive index of the glass. (C.H.S.C.)

10. A convex lens of focal length 30 cm. is placed 20 cm. away from a concave lens of focal length 5 cm. An object is placed 6 metres distant from the convex lens (which is the nearer to it) and on the common axis of the system. Determine the position, magnification, and nature of the image formed. (O. & C.H.S.C.)

11. A thin plano-convex lens forms a window in the vertical face of a tank, with its plane face inside the tank. An object 1 inch in length is placed vertically, outside the tank, with its lower end on the axis of the lens and at a distance of 3 inches from it. The focal length of the lens is 2 inches. Determine, by a graphical construction or otherwise, the position and size of the image (*a*) when the tank is empty, (*b*) when it is filled with water of refractive index $\frac{4}{3}$. (C.H.S.C.)

12. Show that, in general, there are two coaxial positions of a convergent lens which will give, on a fixed screen, a sharp image of a fixed object.

If the distance between object and screen is 96 cm. and the ratio of the lengths of the two images 4·84, what is the focal length of the lens ? (N.U.J.M.B.H.S.C.)

13. Describe two methods for the determination of the focal length of a concave lens.

A thin equiconvex lens is placed on a horizontal plane mirror and a pin held 20 cm. vertically above the lens coincides in position with its own image. The space between the under surface of the lens and the mirror is filled with water (refractive index 1·33) and then, to coincide with its image as before, the pin has to be raised until its distance from the lens is 27·5 cm. Find the radius of curvature of the surfaces of the lens. (N.U.J.M.B.H.S.C.)

14. A concave lens is placed at a distance of 25 cm. in front of a concave mirror of focal length 20 cm., and it is found that a pin placed 68·6 cm. in front of the lens coincides with its own inverted image formed by the lens and mirror. Find the focal length of the lens, and draw a diagram to show how the image is formed.

(C.H.S.C.)

15. Derive a formula expressing the relation between the position of the object and that of the image formed by a concave mirror of radius r. Trace the changes in the position and character of the image when the object is moved from infinity up to the mirror.

An object is placed in front of a concave lens at a distance equal to its focal length. A concave mirror of focal length 25 cm. is placed 40 cm. behind the lens, and a real image is observed to be coincident with the object. Find the focal length of the lens. (O. & C.H.S.C.)

16. Draw a diagram to illustrate the principle of the convex driving mirror on a motor car.

A convex lens of focal length 24 cm. is placed 12 cm. in front of a convex mirror. It is found that when a pin is placed 36 cm. in front of the lens it coincides with its own inverted image formed by the lens and mirror. Find the focal length of the mirror. (C.H.S.C.)

17. Give an account of a method of measuring the focal length of a concave lens, preferably without the use of an auxiliary convex lens. A small luminous object and a screen are placed on an optical bench and a convex lens is placed between them so as to throw a sharp image of the object on the screen ; the linear magnification of the image is found to be 2·5. The lens is now moved 30 cm. nearer the screen and a sharp image again formed. Calculate the focal length of the lens. (N.U.J.M.B.H.S.C.)

18. Describe how you would find the focal length of a concave lens by a method involving the use of an auxiliary convex lens *not*

placed in contact with the concave lens. A thin concave lens has a focal length of 30 cm. Find the position of the image which it forms of an object 10 cm. away (*a*) if the object is real, (*b*) if the object is virtual. In each case draw a pencil of rays showing how an eye, suitably placed, may see a non-axial point on the image.

(N.U.J.M.B.H.S.C.)

19. Using a convex lens, an image of a small source of light is focussed on a screen 50 cm. from the lens. A man holds one of his spectacle lenses between the convex lens and the screen and 5 cm. away from the lens. The screen is now moved 15 cm. nearer the lens in order to refocus the image. Is the man long- or short-sighted? What is the focal length of his spectacle lens?

A biconvex lens has faces of equal radii, and its focal length is 15 cm. A small object is moved along the axis of the lens until it coincides with its own image formed by refraction at the first face of the lens and internal reflection at the second face. This happens at a distance of 8 cm. from the lens. Find the refractive index of the glass. (N.U.J.M.B.H.S.C.)

CHAPTER X

DEFECTS OF LENSES

Spherical aberration.—When a wide parallel beam of light falls on a lens in a direction parallel to its principal axis it will not be brought to one point. Rays close to the axis pass through F, the principal focus, after refraction, while rays through the edge of the lens converge to a point P on the axis nearer the lens than F. The distance between P and F is called the **longitudinal spherical aberration.** Clearly there will be no sharp image point

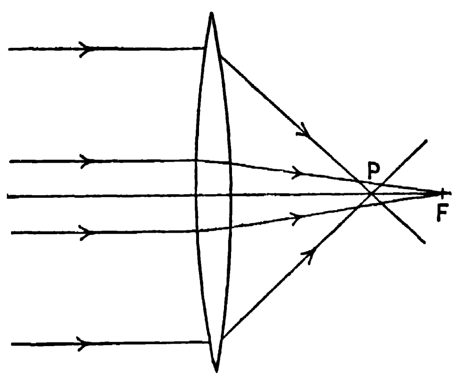


FIG. 153.—Spherical aberration.

but somewhere between P and F the section of the refracted beam by a plane perpendicular to the axis will have its least area —the **circle of least confusion.**

A stop reducing the aperture of the lens to a small central portion will improve definition. It may (p. 193) reduce the brightness of the image, and if sufficiently small will also affect appreciably the detail observable for another reason (p. 293). The defect can also be reduced by suitable design of the lens

surfaces. Before Newton had shown that the chief cause of imperfect definition was chromatic aberration, with spherical aberration playing a smaller part, mathematicians had laboured at the calculation of surfaces which would minimize spherical aberration and Descartes showed that the best surface was given by a "surface of the fourth degree." Such a surface would give perfect correction for only one position of the object, and would in any case be difficult to make. The problem is to arrange ordinary spherical surfaces so that spherical aberration is cut down as much as possible.

In discussing refraction at a single spherical surface, we deduced a formula for the image of a point on the principal axis formed by rays close to the axis and used a limiting approximation $\mu_1 i_1 = \mu_2 i_2$ instead of $\mu_1 \sin i_1 = \mu_2 \sin i_2$ (p. 107). Now the sine of an angle can be expressed as a series, $\sin \theta = \theta - \frac{\theta^3}{6} + \frac{\theta^5}{120} \ldots$ and so on; if θ is very small only the first term is important, while for values of θ greater than about $\frac{1}{10}$ radian the second term begins to be appreciable. A better approximation to the formula $\mu_1 \sin i_1 = \mu_2 \sin i_2$ for larger angles of incidence would be

$$\mu_1\left(i_1 - \frac{i_1^3}{6}\right) = \mu_2\left(i_2 - \frac{i_2^3}{6}\right)$$

or

$$\mu_1 i_1\left(1 - \frac{i_1^2}{6}\right) = \mu_2 i_2\left(1 - \frac{i_2^2}{6}\right).$$

There is no necessity to pursue this analysis further. The point is that for a single surface the inadequacy of the approximate formula increases rapidly as i_1 and i_2 increase. The angle of incidence on any single surface should therefore be kept small. For a lens, the deviation must be *shared as equally as possible by both surfaces*, and this is aided by having the more convex surface facing the incident light, *if this is parallel.*

The spherical aberration of a plano-convex lens with *parallel light* incident first on the convex surface is small, and this is nearly the most favourable form for a *telescope objective.* Note that if the light strikes the plane surface first, all the deviation (with a parallel beam) occurs at the second surface, so that this would be among the worst of arrangements.

The best possible form for a converging lens used for parallel light depends on the refractive index of its material, and can be

calculated. A lens so designed is called a "crossed lens." If $\mu = 1{\cdot}50$, $\frac{r_1}{r_2}$ should be $\frac{1}{6}$; if $\mu = 1{\cdot}67$, the plano-convex is best; while if μ is greater than this, the best form is convex-meniscus.

The best form to deal with rays diverging from a point *at a definite distance from the lens* (*e.g.* a microscope objective) can also be computed.

There is an advantage to be gained by using a glass of high refractive index. For the power of each surface is given by $\frac{\mu_2 - \mu_1}{r}$, and if $\mu_2 - \mu_1$ is large r can be made larger for a given power. The effect of this, as the student can verify for himself, is that both i_1 and i_2 will be smaller for a given deviation.

Fig. 153 suggests another method of attack. It looks as if it might be possible to use a diverging lens between C and P so as to bring the images of P and F nearer coincidence. This is a solution, but in practice it would be more effective to increase the radii of both the surfaces of the convex lens if a longer focal length is required.

Two converging lenses can be combined to reduce spherical aberration. The deviation is then distributed over *four surfaces* instead of two, and the best arrangement will be when each lens shares equally in this.

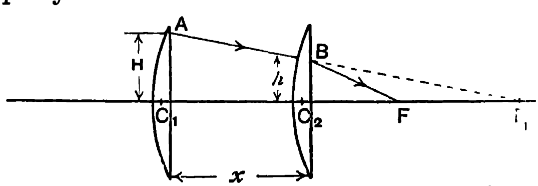


FIG. 154.—Condition for least spherical aberration.

The deviation of a single ray by a thin lens is (p. 129)

$$\frac{\text{distance of point of incidence from principal axis}}{\text{focal length of lens}}.$$

With two thin lenses of focal lengths f_1 and f_2, separated by a distance x (Fig. 154), a ray striking the first lens parallel to the axis at distance H undergoes a deviation $\frac{H}{f_1}$. This ray strikes

the second lens at distance h from the axis and the second deviation is $\frac{h}{f_2}$.

For the two deviations to be equal, $\frac{H}{f_1}=\frac{h}{f_2}$.

Now from the similar triangles AC_1F_1, BC_2F_1,

$$\frac{H}{f_1}=\frac{h}{f_1-x}.$$

Comparing these two equations, $f_2=f_1-x$ or $x=f_1-f_2$. So the two lenses should be arranged with light **incident on the lens of longer focal length** and the distance between them should be **the difference between their focal lengths.**

Astigmatism.—A narrow pencil of rays striking the lens obliquely is not brought to a point, but to **two focal lines.** The nearest approach to a point is a patch somewhere between the two lines, the circle of least confusion. Any rays passing through

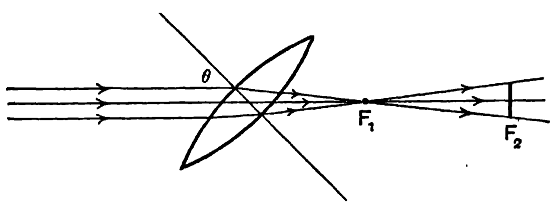


FIG. 155.—Astigmatism.

the edge of a lens are striking at least one surface obliquely, so that spherical aberration must always be accompanied by **astigmatism.** Astigmatism alone may be studied by using a narrow beam of light incident close to the centre of the lens face and rotating the lens about, say, a vertical axis, perpendicular to the paper.

It will be found that instead of coming to a single point F on the direction of the beam produced, the light passing through the lens gives two sharp lines, F_1 and F_2, vertical and horizontal. The vertical one is as shown, nearer the lens than the horizontal one. If θ be the angle between the principal axis of the lens and the direction of the incident beam the **astigmatic difference,** F_1F_2, is nearly proportional to θ^2 for small values of θ.

As in the case of reflection, already discussed (p. 58), the effective curvature of each surface is now, from the point of view of the incident light, different in horizontal and vertical planes. The same effect can be produced with a beam incident along the principal axis of a **toric lens** whose surfaces are made with different horizontal and vertical principal curvatures, or by the use of two equal plano-cylindrical lenses which give a single principal focus when the axes of the cylinders are at right angles, but otherwise give the two focal lines.

With an extended object, vertical lines will appear sharply focussed at F_1 and horizontal ones at F_2; and between F_1 and F_2 will be the nearest approach to a sharply focussed image of the whole.

Effect of a stop.—A stop reducing the aperture of the lens will, as already mentioned, reduce spherical aberration and that part of the astigmatic defect that goes with it. The stop is best placed close to the surface of a thin lens, and at some point between the members of a lens combination.

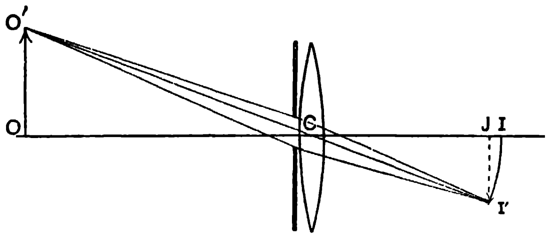


FIG. 156.—Stop placed close to lens.

With a stop the image is sharp, but it will also be **curved**. For (Fig. 156) O' is further from C than is O, so that CI' and CJ will be less than CI. The camera obscura, a seaside peepshow of unsophisticated days, avoided this defect by receiving the image on a concave white bowl.

If the stop is not placed in its best position, another defect becomes pronounced. The image is still sharp, but is now **distorted** as well as curved. Consider a converging lens forming a real image, and let u stand for the distance of a point on the object (measured along the rays) from the lens. With the stop on the *object side* of the lens, rays from *axial points* on the object

go through the *centre* of the lens, and those from the *edge* of the object through the *more distant* edge of the lens. So for the extremities of the object, u will be numerically greater than if rays through C were used, and hence the magnification (which can be written $m = \frac{f}{u-f}$) of the edges of the image will be less than that

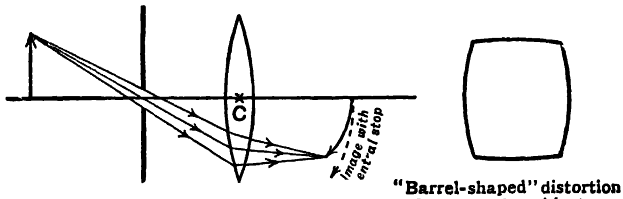


FIG. 157.—Stop on object side of lens, giving "barrel-shaped" distortion of real image.

at its centre. The image will thus appear *pulled in at the corners*, and the image of a rectangular object will appear "barrel-shaped."

If the stop is on the *image side* of the lens, then only those rays from the *extremities* of the object for which u is *numerically less than for central rays* are used. Hence, on the preceding argument, the edges of the image will be more magnified than its central part and it will appear *pulled out at the corners*, so that

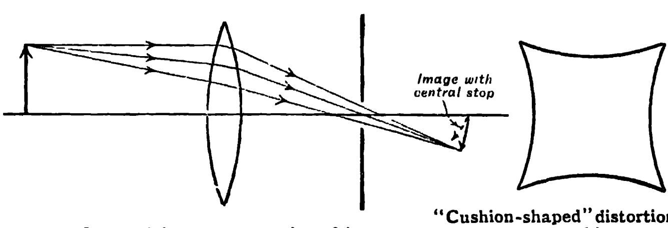


FIG. 158.—Stop on image side of lens, giving "cushion-shaped" distortion.

a rectangular object looks "cushion-shaped." A stop between the two members of a combination will give barrel-shaped distortion for the first member and cushion-shaped distortion for the second, the two defects tending to cancel one another.

In the last paragraph we were considering curvature and distortion of the *clear image* produced *with a stop.* Spherical aberration, making the effective focal length for marginal rays less than for axial rays, of course gives curvature and distortion as well as the general blurring when a wide aperture is used ; the distortion for a converging lens is barrel-shaped for a real image and cushion-shaped for a virtual image.

Chromatic aberration.—The refractive index of glass is greater for blue light than for red, and since $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$, the focal length of a lens will be less for the blue end of the spectrum than for the red end. Each portion of the spectrum will give an image of a white object in a different position, and unless these are superimposed, as in the magnifying glass for example, blurring will result.

Newton was the first to investigate this, with a " nice and troublesome experiment." He cast a spectrum on a printed book and formed real images of the print on a screen, using a lens at the minimum distance. The distance from object to screen was about twelve feet, and the difference in this distance for the parts of the object illuminated by the extreme ends of the spectrum was $5\frac{1}{3}$ inches, so that the difference between the focal lengths of the lens for these extreme colours was $\frac{1}{27}$ of the mean focal length. He showed that the circle of least confusion between the red and violet foci had a diameter more than 5000 times that due to spherical aberration in the lens he used, though the eye is so insensitive to the ends of the spectrum that the full extent of this would not be appreciated in use. In another experiment, with a card painted red and blue, he found the minimum distance for blue light to be $1\frac{1}{2}$ inches less than for red, so that the difference in focal lengths for the ends of the visually important parts of the spectrum was about 1% and the effective circle of least confusion still more than 1000 times that due to spherical aberration. *So spherical aberration is almost negligible in comparison with this enormous effect.*

If an ordinary lens is used to give a real image of, say, a small motor headlamp bulb placed on its axis, and a screen moved outwards from the lens, the patch of light will appear orange-red on the outside until the blurred image is reached, and beyond this

point the outside will be blue. Closer investigation will show that the extreme edges are a deep red and violet. This simple experiment shows the effect well, and also demonstrates that the eye takes little notice of the extreme ends of the spectrum unless they are looked for with some attention. A lens can be made to give approximately monochromatic light by an extension of this experiment, originally due to Rubens. The centre of the lens face is covered to prevent direct light through it reaching the axis, and a screen with a small hole placed with the hole at the point on the axis at which the particular colour it is required to isolate is brought to a focus.

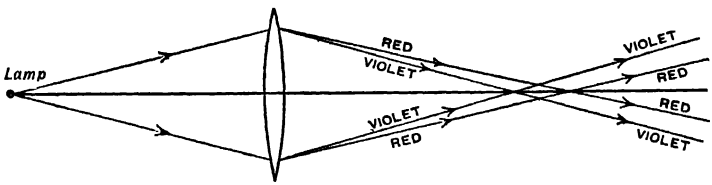


FIG. 159.—Demonstration of chromatic aberration.

The design of an achromatic lens.—In dealing with thin prisms, it was shown how to combine prisms of crown and flint glass of different dispersive powers to produce deviation without dispersion. Similarly a converging lens, achromatic for two colours, can be constructed by combining a converging lens of crown glass with a diverging lens of flint glass.

The calculation is rather long, and is best taken in separate steps: first, establishing a general formula for the change in focal length caused by a difference in refractive index, and so for the change of focal length in terms of dispersive power; then working out the focal lengths of the two components which will give a combination of the correct focal length, and which has this focal length the same for the two colours we are considering; and finally, some discussion of the best way of fitting these two together to make a working lens.

First step.—We know

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right).$$

Here $\frac{1}{f}$ and not f is expressed explicitly as a function of μ, so we shall work in terms of $\frac{1}{f}$.

Let a small change in μ to $(\mu+\delta\mu)$ produce a small change, $\delta\left(\frac{1}{f}\right)$, in $\frac{1}{f}$.

Then, $$\delta\left(\frac{1}{f}\right)=\delta\mu\left(\frac{1}{r_1}+\frac{1}{r_2}\right).$$

Now, $$\left(\frac{1}{r_1}+\frac{1}{r_2}\right)=\frac{1}{f(\mu-1)};$$

so, $$\delta\left(\frac{1}{f}\right)=\frac{1}{f}\cdot\frac{\delta\mu}{\mu-1}.$$

In this equation $\frac{\delta\mu}{\mu-1}$ is the dispersive power ν of the glass between μ and $(\mu+\delta\mu)$, and f is the focal length for refractive index μ. It is clear that we should get just the same expression (with a negative sign) if we considered the change in $\left(\frac{1}{f}\right)$ between μ and $(\mu-\delta\mu)$. Take f and μ as *mean values* over the range to be covered; taking $\delta\mu$ as the change in μ between the visually important ends of the spectrum, red and blue, and writing $\delta\mu=(\mu_{\text{blue}}-\mu_{\text{red}})$;

then, $$\delta\left(\frac{1}{f}\right)=\frac{1}{f}\cdot\frac{\mu_{\text{blue}}-\mu_{\text{red}}}{\mu-1}.$$

But $\frac{\mu_{\text{blue}}-\mu_{\text{red}}}{\mu-1}$ is the dispersive power ν of the glass between these two colours (p. 92).

Hence, $$\delta\left(\frac{1}{f}\right)=\frac{\nu}{f}.$$

Second step.—Let F be the focal length of the combination, f_1 be the focal length of the crown glass lens of dispersive power ν_1, f_2 be the focal length of the flint glass lens of dispersive power ν_2.

Then $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$, if the two are in contact.

If F is to be the same for red and blue light, $\delta\left(\frac{1}{F}\right)=0$.

So, $$\delta\left(\frac{1}{f_1}\right)+\delta\left(\frac{1}{f_2}\right)=0.$$

But, $$\delta\left(\frac{1}{f_1}\right)=\frac{\nu_1}{f_1} \quad \text{and} \quad \delta\left(\frac{1}{f_2}\right)=\frac{\nu_2}{f_2}$$

Hence, $$\frac{\nu_1}{f_1}+\frac{\nu_2}{f_2}=0.$$

From the two equations, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$ and $\frac{\nu_1}{f_1}+\frac{\nu_2}{f_2}=0$, the focal lengths of the components are calculated.

Third step.—The task is not yet finished. We have found f_1 and f_2, but the focal lengths fix only the sum of the powers of the two surfaces of each lens, and the ratio of the radii of curvature for each can be chosen to the best advantage.

As the two lenses are to be used in contact, the two faces in contact should have the same radius of curvature; they may then be cemented together with Canada balsam to reduce reflection losses. Also, spherical aberration will be lessened if the combination is plano-convex. So we should start with the diverging lens and make one face plane ; calculate the radius of curvature of its concave face; and then make one face of the converging lens fit this, calculating the curvature required for its other face.

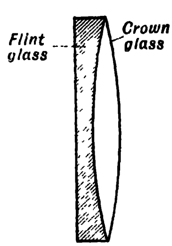


FIG. 160.—Achromatic doublet.

Such a lens is called an **achromatic doublet.** It is not, of course, *completely* without chromatic aberration as its name might suggest. The two ends of the spectrum are superposed, but there is a remaining colour defect called the "**secondary spectrum.**" A combination of three lenses made of different glasses can be made achromatic for three colours on the same principles as the doublet.

Newton did not suspect that different media had different dispersive powers, concluded that no improvement could be made in refracting telescopes, and turned his attention to reflectors. The first achromatic doublet was made by Dollond in 1758.

EXAMPLE.—The refractive indices of crown glass for red and blue light are 1·517 and 1·523 respectively, and the corresponding

values for dense flint glass are 1·650 and 1·664. Design a plano-convex achromatic doublet of 50 cm. focal length made of these two glasses.

Let f_1 be the focal length of the crown glass lens and f_2 that of the flint glass lens.

Then,
$$\frac{1}{f_1}+\frac{1}{f_2}=+\frac{1}{50}.$$

For the crown glass, $(\mu_{\text{blue}}-\mu_{\text{red}})=1\cdot523-1\cdot517=0\cdot006$ and $\mu=(1\cdot523+1\cdot517)/2=1\cdot520$, so the dispersive power is
$$0\cdot006/0\cdot520=\cdot0115.$$

For flint glass, $(\mu_{\text{blue}}-\mu_{\text{red}})=1\cdot664-1\cdot650=0\cdot014$ and
$$\mu=(1\cdot664+1\cdot650)/2=1\cdot657,$$
so the dispersive power is $0\cdot014/0\cdot657=0\cdot0213$.

$$\therefore \qquad \frac{0\cdot0115}{f_1}+\frac{0\cdot0213}{f_2}=0 \quad \text{and} \quad \frac{1}{f_1}=-\frac{0\cdot0213}{0\cdot0115}\cdot\frac{1}{f_2}.$$

Substituting for $\frac{1}{f_1}$ in the first equation, $\frac{-98}{115}\cdot\frac{1}{f_2}=+\frac{1}{50}$; then $f_2=-42\cdot6$ cm., whence $\frac{1}{f_1}-\frac{1}{42\cdot6}=\frac{1}{50}$, and $f_1=23\cdot0$ cm.

The combination thus consists of a crown glass converging lens of 23·0 cm. focal length, and a flint glass diverging lens of 42·6 cm. focal length.

Now for the diverging lens, if r is the radius of curvature of the concave face, using $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$, $-\frac{1}{42\cdot6}=\frac{0\cdot657}{r}$, and $r=-42\cdot6\times0\cdot657=-27\cdot99$ cm., the negative sign indicating concave to the air.

One face of the converging lens must have a radius of curvature of 27·99 cm. The radius of curvature of the other is found, using $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$, when $\frac{1}{23\cdot0}=0\cdot520\left(\frac{1}{r_1}+\frac{1}{27\cdot99}\right)$ and $r_1=20\cdot89$ cm. The radius of curvature of the common surface is thus 27·99 cm., and that of the front face of the converging lens 20·89 cm.

Achromatic combination of two thin lenses.—With two thin lenses of the *same material* not in contact, it is possible to make a system which would, *if no other defects were present*, be achromatic for all colours.

Let f_1 and f_2 be their focal lengths, x the distance apart, F the focal length of the combination, and ν the dispersive power of each between any two colours.

Then (p. 139), $$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1f_2}.$$

So, $$\delta\left(\frac{1}{F}\right)=\delta\left(\frac{1}{f_1}\right)+\delta\left(\frac{1}{f_2}\right)-\delta\left(\frac{x}{f_1f_2}\right)$$
$$=\delta\left(\frac{1}{f_1}\right)+\delta\left(\frac{1}{f_2}\right)-\frac{x}{f_1}\delta\left(\frac{1}{f_2}\right)-\frac{x}{f_2}\delta\left(\frac{1}{f_1}\right),$$
where the last two terms result from the ordinary algebraic approach to differentiation.

Now if the focal length is the same for both these colours,
$$\left(\frac{1}{F}\right)=0.$$

So, $$\delta\left(\frac{1}{f_1}\right).\left(1-\frac{x}{f_2}\right)+\delta\left(\frac{1}{f_2}\right).\left(1-\frac{x}{f_1}\right)=0.$$

Now, $$\delta\left(\frac{1}{f_1}\right)=\frac{\nu}{f_1},\quad \delta\left(\frac{1}{f_2}\right)=\frac{\nu}{f_2}.$$

So, $$\frac{\nu}{f_1}\left(1-\frac{x}{f_2}\right)+\frac{\nu}{f_2}\left(1-\frac{x}{f_1}\right)=0,$$
$$\frac{1}{f_1}-\frac{x}{f_1f_2}+\frac{1}{f_2}-\frac{x}{f_1f_2}=0,$$
$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{2x}{f_1f_2},$$
$$x=\frac{f_1+f_2}{2}.$$

As ν has been eliminated this result holds *for all colours.* The combination then has the *same focal length* for all colours. As it is a *combination* and not a thin lens, this does *not* mean that the principal foci for all colours coincide. It means that rays of all colours initially parallel to the axis are deviated through the same angle, so that *each coloured image subtends the same angle at the focus for that colour* and an eye looking at a virtual image through the combination will see all the coloured images as if superposed. The achromatism of the combination is then similar to that of an ordinary magnifying glass (p. 197). As the combination can also (p. 159) be made to minimize spherical aberration, which is different for different colours, the edge of the field has less of the colouring due to this than is observed when a single lens forms a virtual image.

QUESTIONS ON CHAPTER X

1. Find the condition that must be satisfied in order that the focal lengths of a combination of two thin lenses in contact may be the same for light of two different colours.

The focal lengths of a thin convex lens are 100·0 cm. and 96·8 cm. for red and blue rays respectively. Find the dispersive power of the glass as accurately as these data permit.

How would you measure the focal length for red light in this case? (N.U.J.M.B.H.S.C.)

2. Explain the dispersion produced by a simple lens, and show how the defect may be corrected.

Why is such correction unnecessary in the case of a simple convex lens used as a magnifying glass held close to the eye? (O. & C.H.S.C.)

3. A thin biconvex lens is placed with its principal axis first along a beam of parallel red light and then along a beam of parallel blue light. If the refractive indices of the lens for red and for blue light are respectively 1·514 and 1·524, and if the radii of curvature of its faces are 30 cm. and 20 cm., calculate the separation of the foci for red and blue light. What relation does the result bear to the dispersive power of the lens for the two kinds of light? (N.U.J.M.B.H.S.C.)

4. What is dispersive power? Show that a combination of thin lenses is achromatic if the sum of the products for the two lenses of dispersive powers and reciprocals of focal lengths is zero. (R.S.)

5. What is meant by chromatic aberration? When a simple convex lens is used as a magnifying glass, the image appears almost free from colour at the edges, but when such a lens is used as the object glass of a telescope the colour effect is more marked. Draw diagrams illustrating the formation of images by the red and violet components of the light, and point out why one image appears to overlap the other to a greater extent in the case of the telescope.

An achromatic lens (converging) is to be formed of two components, one having twice the dispersive power of the other. The combination has to have a focal length of 30 cm. What are the focal lengths of the components? (O.H.S.C.)

6. Explain how it is possible to construct achromatic lenses. Why did Newton consider it impossible? An achromatic objective of 100 cm. focal length is to be made, using two lenses of the glasses shown below. Find the focal length of each lens, stating whether it is convergent or divergent.

	Glass A	*Glass B*
μ red - -	1·5155	1·641
μ blue - -	1·5245	1·659

(N.U.J.M.B.H.S.C.)

7. Show how it is possible to construct an achromatic combination from two thin lenses of the same material. (C.S.)

CHAPTER XI

THE CAMERA AND PROJECTION LANTERN

The pinhole camera.—A real image may be formed on a screen by both lens and mirror (pp. 47, 125). In both cases rays striking the aperture from a point on the object were made to pass through a single point. Something very similar to a real image is given by a small pinhole, P (Fig. 161). A narrow pencil of rays from a point A on the object passes through and strikes a screen placed behind P at a, giving a bright patch there; other points, B and C, give corresponding patches, and so on, and an inverted

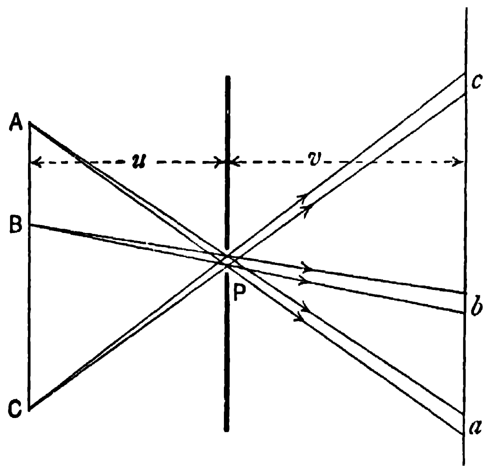


FIG. 161.—Pinhole camera.

picture of the object will thus be formed. It cannot *strictly* be called an image, for the rays do not give definite points, and so it can be formed *in any position*. Furthermore, a real image does not need a screen for its presentation and can be seen (as in no-parallax experiments) as an "aerial image" by the eye; the pinhole camera's picture *must* be received on a screen.

By similar triangles, it is easily seen that if d is the diameter of the hole, D that of the patch formed by rays from a single point on the screen, u and v the object and screen distances from the hole,

$$D = d \cdot \frac{u+v}{u};$$

whence, if u is large compared with v, quite a large pinhole will give a tolerably sharp image. Thus the spaces between the leaves of a tree give fairly sharp circular pictures of the sun on the ground. Even a keyhole will give recognizably clear pictures of buildings more than thirty yards or so away in a darkened room.

The eye can just resolve two points 0·1 mm. apart at a distance somewhat greater than 25 cm. A pinhole $\frac{1}{10}$ mm. in diameter ought thus to give a picture which is indistinguishable to the naked eye from that formed by the best of lenses if the object is so distant that $\frac{u+v}{u}$ can be taken as 1. This would be true if the propagation of light were perfectly rectilinear, but owing to diffraction (p. 281) so small a hole would give blurred images if the screen were more distant than about 1·5 cm. from it. With a hole 1 mm. in diameter the blurring due to diffraction would not be evident until the screen was about 15 cm. from the hole, and this is roughly equivalent to the smallest convenient pinhole.

A large pinhole may be regarded as made up of numerous small ones whose pictures overlap, and the illumination at any point of the picture is the sum of all the illuminations of the overlapping patches. The number of these will be proportional to the area, a, of the hole, so that the illumination of the screen and hence the brightness of the picture is proportional to $\frac{a}{(u+v)^2}$.

The linear magnification of the picture is $m = \frac{v}{u}$. This will be the same for all parts of the picture if the pinhole is on a line normal to the planes of object and picture, so that there will be no curvature or distortion, though the edges will be less sharp and less bright than the centre. The *size of the pinhole* has no effect on the size of the picture. The *shape of the pinhole* also has no effect on the picture, but it shapes the boundary of the field.

The camera.—A real image is formed on the sensitive plate, P, held in a light-tight chamber whose interior is blackened. The converging lens, L, is mounted on a slide, so that its distance from P can be altered. When the object is very distant, P will be at the principal focus of the lens, while for nearer objects P must be further away, so that the chamber is made, for the most part, in the form of an extensible **bellows.** An adjustable **stop,** usually in the form of an iris diaphragm, is placed in the best position

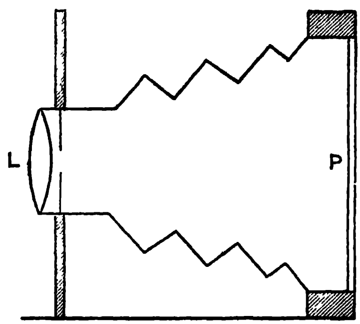


FIG. 162.—Camera.

for the particular lens and the **shutter** is usually close to this, though in some forms the plate is exposed in successive strips by a shutter in the form of a **roller blind** with a slit in it, working just in front of P. The roller-blind shutter causes distortion of a fast-moving object's picture.

From the point of view of the plate we can regard the lens aperture as a source of light, which it collects from the object and then passes on. As a collector of light from the object its illumination will depend on the intensity of the object. As a source, its brightness is proportional to its illumination, and its intensity is brightness × area. The illumination of the plate at its centre, when a distant object is focussed, is, by the law of inverse squares, proportional to $1/f^2$. So that the illumination for a given object is proportional to $\dfrac{\text{area of aperture}}{f^2}$; or, writing d for the diameter of the aperture, illumination is proportional to $\dfrac{d^2}{f^2}$.

The quantity d/f is called the **relative aperture** of the lens. It is clear that different lenses with the same d/f will give the same

illumination for a given object; hence this is the important factor to be used in compiling exposure tables. Good lenses corrected for spherical aberration can be used at large relative aperture without noticeable loss of definition, and hence will give bright images requiring short exposures. Assuming that the exposure required is inversely proportional to the brightness of the image on the plate, it will be proportional to $1/(d/f)^2$.

With a wide relative aperture, points not accurately in focus will give comparatively large circles on the plate (see Fig. 164), the range of object distances giving reasonably clear images will be small, and there is said to be little **depth of focus.** Stopping down the lens aperture then *increases* the depth of focus, besides *improving definition* of the accurately focussed image.

Telephoto lens.—To give a large magnification of a distant object f must be large. For example, to give an image 2 cm. high of a cricketer rather less than 2 metres high, needs a linear magnification of about $\frac{1}{100}$. If the camera is in the pavilion, u may be 100 metres.

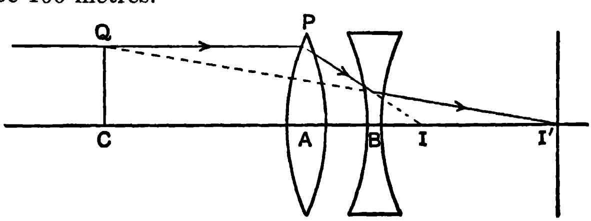


FIG. 163.—Telephoto lens.

So as
$$m = \frac{f}{u-f},$$
$$\frac{1}{100} = \frac{f}{100-f}.$$
Whence
$$f = \frac{100}{101} \text{ metres.}$$

A camera with such a lens mounted in the ordinary way would be unwieldy. But it is possible to combine two lenses in such a way that the second principal plane of the system, from which v is measured, is well in front of the first of the pair; both can then be placed in an ordinary camera.

A convex lens, A, of short focal length gives an image of a distant object at I (Fig. 163). A diverging lens, B, is placed

between A and I so that BI is less than its numerical focal length, and gives a final real image, I', on the plate. Rays reaching I' appear as if they had been deviated by a single lens, Q, placed at C. The equivalent focal length of the combination is then CI'.

Increasing the distance BI will increase BI'. The effective focal length and magnification of the combination is thereby increased, but so also is the distance from lens to plate which the system is designed to reduce.

Photographs produced in this way show very little perspective and look very " flat." Perspective in a photograph is caused by differences in magnification between nearly equally well focussed images of objects at different distances. A metre or two difference in u makes little difference if u is as large as 100 metres. To get a 2 cm. image of our cricketer with a lens of 10 cm. focal length u would have to be 10 metres, and a metre or two would make a great deal of difference to the magnification.

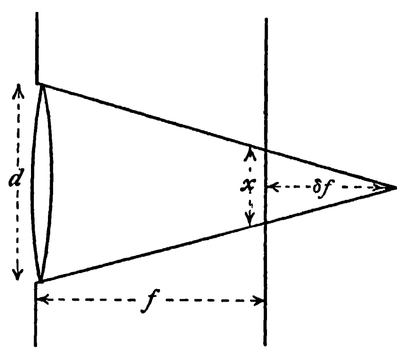


FIG. 164.—Depth of focus.

Fixed focus cameras.—Cheap box cameras need no focussing at all. The sensitive film is at a distance f from the lens, and objects at all distances from infinity to perhaps two metres or less are sufficiently well focussed to give a satisfying picture. The pencil of rays from a point on the object converges to the film, and gives there a circular patch which becomes smaller as the distance of the object is increased. We reduced the size of similar patches in the pinhole camera by reducing the aperture,

and the same method is used here. *Very great depth of focus* is secured by using the lens at *very small relative aperture.*

Let d be the diameter of the lens aperture and f the focal length. A sharp image of a really distant object is formed at f.

A nearer object at distance u gives an image at distance $f+\delta f$, which gives a patch of diameter x on the film at f. If x is less than $\frac{1}{10}$ mm. no noticeable blurring is produced.

Now by similar triangles, $\frac{x}{\delta f}=\frac{d}{f+\delta f}$,

$$x=\frac{d\,.\,\delta f}{f} \quad \text{approximately.}$$

Now,
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

Substituting for v,
$$\frac{1}{f+\delta f}+\frac{1}{u}\quad\frac{1}{f},$$

$$\frac{1}{f+\delta f}-\frac{1}{f}=-\frac{1}{u}$$

$$-\frac{\delta f}{f^2}=-\frac{1}{} \quad \text{approximately.}$$

So,
$$\frac{\delta f}{f}=\frac{f}{u}$$

Substituting the value of $\frac{\delta f}{f}$ in the expression for x,

$$x=\frac{d\,.\,f}{u}.$$

Now if x is to be 0·01 cm. and if $f=10$ cm. and $d=f/10=1$ cm., $u=100$ cm., so that all objects at greater distances than a metre will be adequately clear. It is easy to see why enlargements of photographs taken with a box camera are frequently unsatisfactory when viewed from a distance of about 25 cm. The best overall definition would be obtained with the plate set midway between the focal plane of the lens and the position for focussing the nearest object sharply.

Projection lantern.—This gives a real enlarged image of a transparent object on a screen. The problem of projecting the image is simple enough ; the design of the instrument is directed

to securing uniform and intense illumination as efficiently as possible, and ensuring that no image of the source of light can intrude on the picture.

The slide, O, is placed just outside the first principal focus, F_L, of the lens, L, which is an objective corrected for both chromatic and spherical aberration. A real, inverted and enlarged image is formed on the screen at I, some distance away. In order to make as much light as possible go through the slide, in such a direction that the objective can use it, the source, S, is chosen as

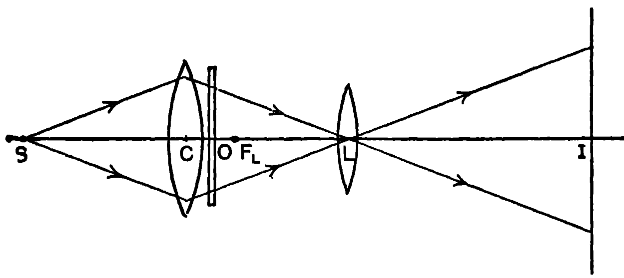


FIG. 165.—Principle of projection lantern.

small as possible, and a **condensing lens**, C, of wide aperture and short focal length, makes the rays converge through O to L. The best arrangement is with S and the optical centre of the lens L (or its first principal plane) as conjugate foci for C. Then all the light goes through the central part of L, and also no image of S can possibly be formed by L. It can be seen that the focal length of the condenser must be about half that of the objective.

It is perhaps hard to see from Fig. 165 how the image of O comes to be formed, but this is not intended as a construction diagram for the image. All rays passing through an element of O are made to strike a corresponding element of I in the usual manner; but here the elements of O do *not* emit light in all directions, but only in the directions prescribed by the condenser.

The magnification is $\frac{v}{f} - 1$. Here $\frac{v}{f}$ must be large, so that a short focal length is chosen for the objective.

The **episcope** is a lantern adapted for the projection of opaque objects. Intense illumination is required, as the object absorbs

light and scatters most of that reflected in useless directions. The source is usually a 1000-watt lamp. The objective has a much wider aperture than is necessary for an ordinary lantern. The **epidiascope** is a combined episcope and lantern. Fig. 166 shows the principle of the Ross epidiascope. The lamp L is mounted in a square chamber faced with two condensers C_1, C_2, and two concave reflectors A_1, A_2. This chamber is set as in Fig. 166 (*b*) for episcopic projection, and can be turned through 45° as in Fig. 166 (*c*) for ordinary lantern work, when only C_2 and A_1 are used. An opaque object S (Fig. 166 (*a*)), held firmly beneath a sheet of plate glass, is illuminated by light reflected down from C_1 and C_2 by the tilted curved mirrors R_1 and R_2. The reflection of S in the surface-silvered mirror M is the object

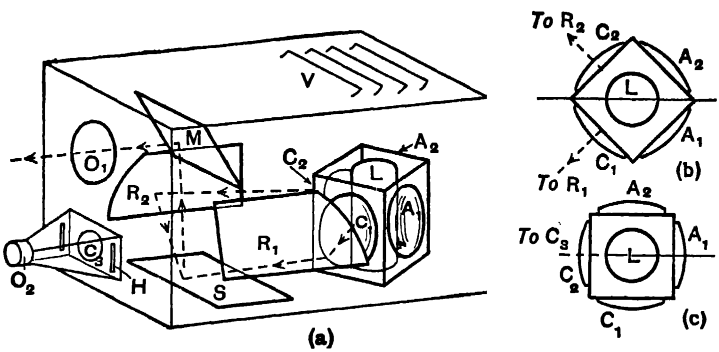


FIG. 166.—Principle of " Ross " epidiascope.

of which the objective O_1 projects a real image. A separate optical system with condenser C_3 and objective O_2 is used to project slides inserted at H. The lamp chamber forms a " chimney " which, continued to the ventilator V, cools the lamp by convection. Other types are sometimes provided with a motor-driven fan for cooling.

Lenses producing real images.—A lens designed to produce a real image on a screen must be corrected for chromatic and spherical aberration. The more perfect the correction of the lens itself, the less is the need for a stop, so that a good lens can be used at high relative aperture and will give bright images.

The correction of these two aberrations would cause the lens to give good sharp images of points close to its principal axis, but this is not sufficient. A photograph must be sharp and clear all over, and special attention is paid in designing lenses to the importance of obtaining a sharp image of points well away from the axis without distortion. Astigmatism and curvature and distortion have to be corrected for, and these defects are relatively much more important in a camera lens than, say, in a telescope objective or eyepiece. A simple type, the **rapid rectilinear** lens, consists of two achromatic lenses, with a stop in the space between them to reduce distortion, as explained on p. 162.

QUESTIONS ON CHAPTER XI

1. Explain the principles which govern the size and appearance of the image and the necessary exposure of the negative in a pinhole camera, and point out the effect of the size and shape of the pinhole.

If such a camera is 6 inches long, what is the greatest possible diameter of the hole if two objects 1 foot apart and 60 yards from the pinhole give image spots which just do not overlap ? (O.H.S.C.)

2. Describe the optical arrangement of a photographic camera, indicating in general terms how and why the lens differs from a simple convergent lens.

A camera has a convex lens of 7 in. focal length mounted 3 in. in front of a concave lens of 6 in. focal length. Compare the size of the image of a distant object formed by this combination with the size of the image which would be obtained if the convex lens alone was used. (N.U.J.M.B.H.S.C.)

3. Give an account of the optical system of a simple type of photographic camera.

What are the essential requirements of a good photographic lens ? (O. & C.H.S.C.)

4. Describe the optical system of a projection lantern.

A lantern is required for the projection of slides 3 in. square on to a screen 7 ft. square. The distance between front of lantern and screen is to be 20 ft. What focal length of projection lens (to the nearest inch) would you consider most suitable ? (C.W.B.H.S.C.)

5. A fixed pinhole camera is used on a calm day to take several snapshots on the same plate of an airship in flight. Explain how to determine from the photographs (*a*) the height of the ship, (*b*) its speed. Assume the ship to be nearly vertically above and of known length. (N.U.J.M.B.H.S.C.)

6. A box camera has a rack focussing arrangement and three points on the sliding portion of the box are marked "infinity," "10 feet," and "6 feet" respectively, giving the positions of the rack when the camera is focussed upon objects at these distances. The first two points are $\frac{1}{2}$ inch apart. Find the focal length of the lens and the distance of the third point from the first. (R.S.)

7. Draw a careful diagram showing the construction of, and the path of a beam of light through, a photographic camera. To the eye of a photographer the moon's diameter subtends an angle of half a degree. What will be the size of the moon's image when photographed, using a camera of which the lens has a focal length of 11·4 cm.? (N.U.J.M.B.H.S.C.)

CHAPTER XII

THE EYE

General structure of the eye.—The eye is a miniature camera. It forms a real inverted image on a sensitive screen, can regulate its aperture, and can focus objects at different distances.

The eyeball is held in its socket by six muscles, which also serve to rotate it. The outer coating, S, is called the **sclerotic,** and consists of nearly opaque fibrous tissue for the most part. At its front it merges into the transparent meniscus, C, called the **cornea,** whose front surface may be regarded as a sphere of about 7 mm. radius of curvature; its refractive index is about 1·36. The inside of the sclerotic is covered with a dark pigmented membrane, the **choroid,** Ch, which serves the same purpose as the blackened interior of a camera and prevents the reflection of scattered light. At the front of the eye the sclerotic merges into the **iris,** I, the coloured part of the eye; this is a diaphragm, whose function is similar to that of the stop in a camera. The central orifice is called the **pupil.** L is the **crystalline lens,** connected to the walls of the eye by the **suspensory ligament,** SL. Between the crystalline lens and the cornea is the **aqueous humour,** a watery liquid; between the lens and the interior surface of the eye a transparent jelly called the **vitreous humour.** The straight line OX through the centres of cornea and crystalline lens is called the **optic axis** of the eye. The line FV represents the **visual axis** of the

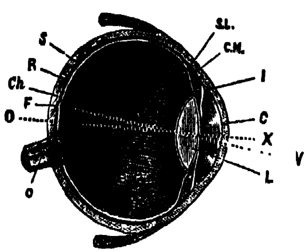


FIG. 167 (*a*).—Structure of the eye.

eye, the direction in which the eye looks to see things clearly; this is not the same as OX.

The inner surface of the eye is coated with a nearly transparent membrane rich in nerve-fibres and blood-vessels—the **retina**, R. This is the part of the eye which is sensitive to light. Two types of nerve-fibre terminations can be distinguished, **rods** and **cones**.

The most sensitive spot, the **fovea centralis**, F, is where the visual axis of the system intersects the retina. It is surrounded by a

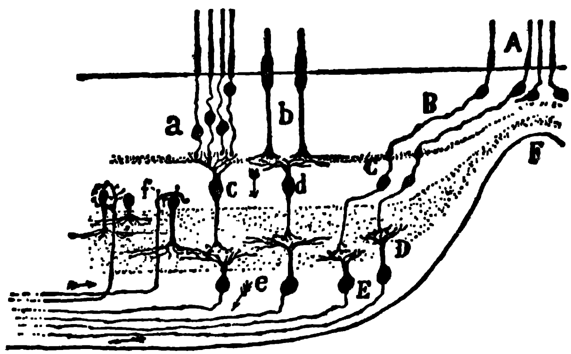


FIG. 167 (*b*).—Structure of the retina, showing rods and cones.
From "Vision and Colour Vision," by R. A. Houston (Longmans, Green & Co., Ltd.)

region called the yellow spot, and the image of an object viewed directly is focussed sharply there—that is why the eye is moved to look straight at objects. The yellow spot is made up almost

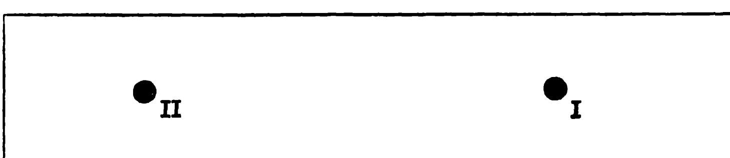


FIG. 168.—Blind spot.

entirely of cones; it is the most sensitive part for colour, and the perception of detail is greatest there. The maximum sensitiveness to light (as distinct from the useful sensitiveness to colour difference and detail) is not on the yellow spot, but on the part of the retina round it. The smallest quantity of energy perceptible by the normal eye is about 2×10^{-10} ergs.

The region at which the optic nerve enters the eye is not sensitive to light and forms the **blind spot**, *B*. Its existence may be demonstrated by looking at the spot *I* with the left eye and moving the page away; for a certain distance of the page *II* disappears, for its image has fallen on *B*. As the retinal image is inverted the blind spot must be to the right of the visual axis of the eye, nearer to the nose than the yellow spot.

That the eye actually does give an inverted image on the retina is shown by the following experiment. An object close to the surface of a lens illuminated by a pencil of light will give no image of any kind on a screen behind the lens, but simply a shadow the same way up as the object. If the eye looks at a bright source of light through a small hole in a card held at arm's length and a pin held close to the eye is moved up, the shadow is seen to move down. There is no doubt that the actual motion of the shadow on the retina was upwards, and the mind has interpreted this as down. So the image on the retina is inverted, and from this the mind constructs a visual picture which fits in with that of the other senses.

The lens system. Accommodation.—Cornea, aqueous humour, and crystalline lens together can be considered as a single lens combination, with air on one side and vitreous humour of refractive index 1·336 on the other. The focal length of the combination when the normal eye is looking at a distant object is 15·5 mm. in air, and hence $15{\cdot}5 \times 1{\cdot}336$ or 20·7 mm. in vitreous humour; its power is rather more than $+60D$, where D stands for dioptres.

The closest distance at which objects can be seen clearly is called the **near point**; its distance from the eye depends chiefly on a person's age but 25 cm. is reckoned as the normal distance for calculations, as in Chapter XIII. A normal or **emmetropic** eye has a range of vision from the near point to infinity, the normal **far point.**

The process by which the eye adapts itself for objects at different distances is called **accommodation.** By means of muscles, called the **ciliary muscles,** the focal length of the crystalline lens is altered. When the eye is at rest the power of the crystalline lens is about $+20D$. The action of the muscles increases its power by about $+4D$, chiefly by increasing the curvature of its back

surface. Distant objects are thus seen when the eye is at rest; near objects require the effort of accommodation. The lens itself can be made to move forward slightly to aid the action of accommodation for near objects.

Defects of vision and their correction.—The chief defects which can be corrected by the aid of lenses placed close to the eye are **presbyopia** (sometimes called *far sight*), **hypermetropia** (called *long sight* and frequently confused with far sight), **myopia** or *short sight*, and **astigmatism.** The optician's test for a defective optical system as in hypermetropia and myopia, as distinct from an ageing normal one as in presbyopia, is the position of the *far point.*

Far sight.—A person whose near point is further from the eye than the normal 25 cm. is said to be far sighted if his far point is normal. This is the condition met with in advancing age, for the power of accommodation declines like other muscular abilities and the lens cannot be made sufficiently converging to bring rays diverging from a point as near as 25 cm. to a focus on the retina.

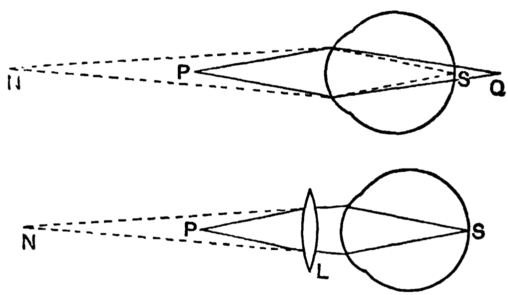


FIG. 169.—Far sight and its correction.

An object at P, 25 cm. away, will give an image at Q behind the retina, while one at N, the actual near point, just gives its image at S on the retina. If the eye is to see an object at P, then rays from P must be made to diverge less, as if they came from N. The correction will then be a *converging lens* placed in front of the eye, so that N is its virtual image of P. The lens L will not be in contact with the cornea, of course. But as it is close to it, and as P will be 25 cm. away and N even more, we can without grave error consider that distances measured from lens and cornea do not differ.

If d is the numerical value of the distance of N, then we can substitute in the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ for the thin lens C,

$$v=(-d) \qquad \text{(virtual image at near point),}$$
$$u=(+25 \text{ cm.}) \qquad \text{(real object).}$$

Whence, $$-\frac{1}{d}+\frac{1}{25}=\frac{1}{f} \quad \text{or} \quad f=\frac{25d}{d-25}.$$

Example.—A far-sighted man has his near point at 100 cm. and his far point is normal. What glasses must he use to make his near point normal, and what will be his far point when wearing them ?

Suppose the lens is close to the eye. An object placed 25 cm. away must give a virtual image 100 cm. away from it. Let f be the focal length of the lens. Then, using

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f},$$
$$u=(+25 \text{ cm.}) \qquad \text{(real object),}$$
$$v=(-100 \text{ cm.}) \qquad \text{(virtual image).}$$

Whence, $$-\frac{1}{100}+\frac{1}{25}=\frac{1}{f} \quad \text{or} \quad f=+33\tfrac{1}{3} \text{ cm.}$$

(If we wished to take into account the fact that the lens may be, say, 2 cm. from the eye, we should have

$$u=(+23 \text{ cm.}) \qquad \text{(real object),}$$
$$v=(-98 \text{ cm.}) \qquad \text{(virtual image),}$$

whence $-\frac{1}{98}+\frac{1}{23}=\frac{1}{f}$ or $f=+30{\cdot}0$ cm., with which the approximate result agrees sufficiently accurately for this purpose.)

With this lens, for the virtual image to appear at infinity, u has the value given by $\frac{1}{\infty}+\frac{1}{u}=\frac{1}{33\frac{1}{3}}$, so the far point will be $33\frac{1}{3}$ cm. from the lens.

Note the simplicity of this calculation. It is a straightforward problem on conjugate foci for the correcting lens. It is not only tedious but actually *wrong* to endeavour to treat L and the eye as two thin lenses in contact, for the eye is certainly not a thin lens and the two are separated by so great a distance that L is at or beyond the first principal focus of the eye. The equation $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2}$ shows that a lens L_1 placed at the principal focus

of another, L_2 ($x = f_2$), gives a combination whose focal length is the same as that of L_2, so that if the lens is at the eye's first principal focus, about 15 mm. from the cornea, there is *no change in focal length at all.* The distance of the image from the eye's second principal plane is thus unaltered, but (compare, for example, what has been said about the telephoto lens) this principal plane has been shifted towards the incident light by a distance equal to SQ, so that the image is moved forward by this amount. In this position there can be said to be *no gain in magnification due to L.*

Moving the lens further from the eye increases the magnification. That is why people whose far sight has outgrown their glasses perch them on the end of the nose, in an effort to make magnification compensate for indistinctness.

True long-sightedness or *hypermetropia* really differs from far-sightedness. In this case the eye cannot see distant objects without accommodation—*i.e.* when it is at rest only rays

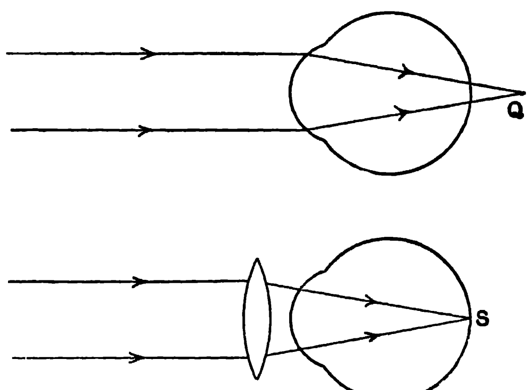


FIG. 170.—True long sight. The *unaccommodated* eye enabled to see distant objects with the aid of a converging lens.

incident on the eye so that they are converging to a point behind it can be focussed on the retina, so the far point is virtual. It may even happen that the eye cannot deal with diverging rays at all, and the near point is then also virtual and behind the retina. A *converging lens* corrects this. In the previous example, if the near point were 100 cm. behind the lens, we should have $v = +100$ and f would have to be $+20$ cm.

Short sight or *myopia.*—If the eyeball is longer than it should be, parallel light will be brought to a focus in front of the retina. The far point will be nearer than infinity. If accommodation is normal the near point will be much closer than 25 cm., but this is no disadvantage.

If a distant object is to be seen, its virtual image must be brought to the far point, F. Clearly a *diverging lens* with F at its principal focus must be used close to the eye.

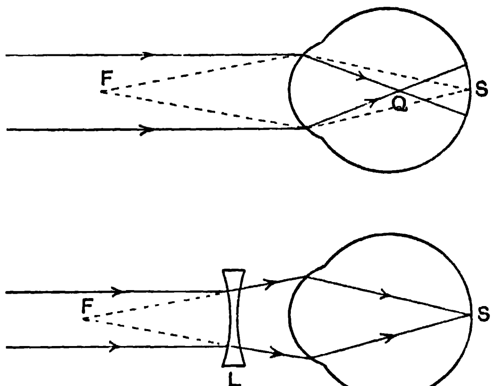


FIG. 171.—Short sight and its correction.

EXAMPLE.—The far point and near point of a short-sighted man are 100 cm. and 18 cm. from his eye. What lens will be needed to enable distant objects to be seen clearly, and what will be his near point with this lens ?

If the lens is close to the eye, consider all distances to be measured from its optical centre. A distant object must give a virtual image 100 cm. away. Let f be its focal length.

Use $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

Here $u=(+\infty)$,

$v=(-100 \text{ cm.})$ (virtual image).

So $\frac{1}{f}=-\frac{1}{100}$ and $f=-100$ cm. and a diverging lens of 100 cm. focal length must be used.

Now let an object at distance u give an image at the near point of the eye.

Here $v = (-\ 18 \text{ cm.})$ (virtual image),
$f = (-\ 100 \text{ cm.})$.

So $\frac{1}{u} = +\frac{1}{18} - \frac{1}{100}$ and $u = +21{\cdot}9$ cm., giving the new near point.

As before, if the lens is about 15 mm. from the cornea, no magnification will be produced by it.

Astigmatism.—The most important refracting surface is the front of the cornea. This is rarely spherical, and if the curvatures in, say, horizontal and vertical planes are widely different there will be no sharp image, each point giving two focal lines. The eye tries to focus both at the same time, with the result that muscular strain is caused. Astigmatism may be corrected using **cylindrical** or **toric lenses**, fixed in their frames so that the principal planes of their astigmatism coincide with those of the eye, which need not, of course, necessarily be horizontal and vertical. A toric lens is one whose surfaces are parts not of a sphere, but of a *tore* or anchor-ring—*i.e.* a cylinder whose axis has been bent into a circle, giving two principal radii of curvature, those of the cylinder and the circle.

Chromatic and spherical aberration. Curvature.—The eye's spherical aberration is over-compensated, and the focal length of the lens system is greatest for rays going nearest to its edge. Chromatic aberration can be demonstrated easily ; if two bright coloured patches, one blue and one red, be thrown on a screen most people will find it impossible to focus both sharply at once. Curvature of the real image on the curved retina is no disadvantage. Its existence can be shown by an experiment due to Helmholtz, in which a distorted diagram of a chessboard appears undistorted when held close to the eye.

Visual angle. Visual acuity.—The size of the image on the retina determines the apparent size of the object. This depends only on the **visual angle** subtended at the eye by the extreme parts of the object. The two objects AB and $A'B'$ will each produce equal retinal images ab, and so appear equally large. *In dealing with the magnification produced by an instrument we shall thus consider the angles object and image subtend at the eye, and not their linear dimensions.*

Two points on a photograph $\frac{1}{10}$ mm. apart could not be reproduced as distinct by the coarse-screen half-tone process used in daily papers, for both would appear on the same spot. Similarly the eye cannot distinguish as separate two points so close together that their retinal images fall on the same cone on the yellow spot. An eye is said to have a **visual acuity of unity** if two points whose angular separation is 1′ of arc can just be distinguished. The **vernier acuity**, or power of distinguishing when two straight lines are non-collinear, is much better than this, the minimum angular separation necessary for non-coincidence to be detected being about 12″.

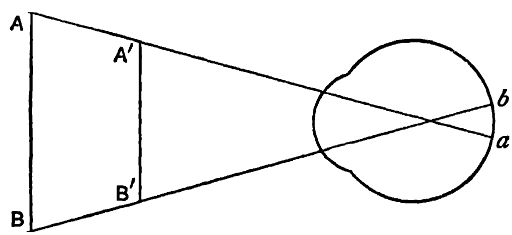


FIG. 172.—Visual angle.

Colour vision.—It is thought that the sensitiveness to colour resides in the cones, as nearly all the eye's power of detecting colour difference is in the yellow spot. This is why a small field of view is important in the flicker photometer.

In bright light the eye is most sensitive to the yellow-green part of the spectrum; for very feeble illumination it is most sensitive to the violet end.

Binocular vision.—When both eyes are looking at a point P their visual axes converge and meet in P. From the muscular effort this convergence requires experience enables us to estimate the distance of P. Two slightly different views of a solid object at P are seen, and fused by the mind to give what we experience as solidity in relief. Distances of objects can be gauged roughly with one eye alone if we already know the true size of the objects, for the greater their distance the less will be their apparent size. How rough this judgment is may be tested by a simple action such as dipping a pen in an ink-well when one eye is shut. It can even be definitely misleading, as in the case of the setting sun. The axes of both eyes are parallel when we look at a distant object, so we must rely on apparent size for the distance

of a really distant object. As most of our distance judging is done horizontally, we tend to underestimate vertical distances. Although the sun, of course, subtends the same angle at the eye wherever it is, we imagine it is much more distant when near the horizon than when overhead and so persuade ourselves that it must be larger. Parallax and accommodation also play a part in distance judging with one eye.

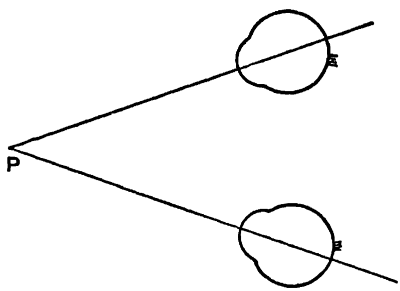


FIG. 173.—Binocular vision.

Two photographs of a solid object taken from slightly different angles will, if placed so that one eye looks at each at a suitable distance, give the effect of solidity. The arrangement for viewing them is called a **stereoscope.** The two pictures are placed side by side and viewed through "binoculars," consisting of two equal converging lenses. Two prisms, their vertices pointing inwards, superpose the two virtual images, so that each eye views its picture in the direction in which the camera pointed to take it.

QUESTIONS ON CHAPTER XII

1. Draw a diagram to show the optical structure of the eye, and explain the meaning of the terms *blind spot, accommodation, least distance of distinct vision.*

An eye, whose least distance of distinct vision is 25 cm., views the virtual magnified image of an object formed by a convex lens of focal length 5 cm. If the lens is held close to the eye, what is the greatest angular magnification of the image that can be obtained ? (C.H.S.C.)

2. Describe the eye as an optical instrument, and explain how the defects of short and long sight can be remedied by the use of lenses.

What lens would enable a short-sighted person to see distant objects distinctly, if with the unaided eye he cannot see distinctly objects which are more than 6 in. from his eye ? (C.W.B.H.S.C.)

3. Draw diagrams to illustrate long sight and short sight. Draw also diagrams showing the correction of these defects by suitable lenses. A person can focus objects only when they lie between 50 cm. and 300 cm. from his eyes. What spectacles should he use (*a*) to increase his maximum distance of distinct vision to infinity, (*b*) to reduce his least distance of distinct vision to 25 cm.? Find his range of distinct vision using each pair. (N.U.J.M.B.H.S.C.)

4. Draw a diagram showing a section of the human eye, labelling those parts of special interest in the study of its optical behaviour. An eye from which the lens has been removed may be considered to be filled with a homogeneous refracting medium of $\mu = \frac{4}{3}$. If the distance from the cornea to the retina of such an eye is 2·2 cm., and the radius of curvature of the cornea is 8 mm., find the focal length of a spectacle lens which, placed close to the eye, will enable it to focus a distant object on the retina. (N.U.J.M.B.H.S.C.)

5. Explain how the normal eye can see distinctly objects at different distances. Give diagrams showing the paths of rays from a distant object in an eye which is both short-sighted and astigmatic. A short-sighted man can only see distinctly objects between 15 cm. and 90 cm. from his eyes. What lenses will he need for viewing distant objects, and what will be his shortest distance of distinct vision when wearing these lenses? (N.U.J.M.B.H.S.C.)

6. What is astigmatism? A person suffering from astigmatism notices that a set of vertical straight lines appears clearer than a set of horizontal straight lines. Explain the reason for this, and show how the disability may be corrected with the aid of cylindrical glass spectacles. (R.S.)

7. Give a short description of the eye, and indicate some of its common defects. What spectacles would be required by a person whose near point is 5 feet in order that he might see clearly at a distance of 20 inches? (R.S.)

CHAPTER XIII

OPTICAL INSTRUMENTS

General considerations. *Power.*—The compound microscope and telescope consist essentially of two thin lenses with the same principal axis. Such a combination has a focal length which can be calculated from the formula on p. 139, and if this is expressed in metres its reciprocal will be the **power** of the system in **dioptres.**

We can express the power of a thin lens in a way which involves *no measurement to or embracing its optical centre.* For if AC and BC are two rays through the optical centre C from the extremities

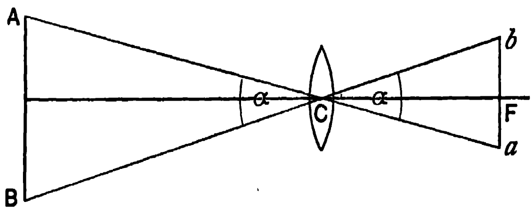


FIG. 174.—Power.

of a very distant object subtending a small angle α at C, the extremities ab of its image subtend the same angle α at C. Now the distance of ab from C equals CF, the focal length. As α is small we have, in radians,

$$\frac{ab}{CF}=\alpha$$

and $$\frac{1}{CF}=\frac{\alpha}{ab}.$$

So if ab and CF are measured in metres the power of the lens is $\frac{\alpha}{ab}$ dioptres.

Here we have a way of expressing the power which involves no measurements along the axis, and thus can be applied to *any*

system; and the power of any optical instrument can be defined in terms more general than the reciprocal of a length, which cannot in all cases be measured conveniently. The following definitions are those given in the Physical Society's *Report on the Teaching of Geometrical Optics*:

(*a*) A reflecting or refracting instrument is said to be of zero power if the image it forms of an infinitely distant object is at an infinite distance from the instrument.

(*b*) Axially symmetrical instruments not of zero power are either of positive or negative power. An instrument is said to be of positive power if it produces an inverted image of an infinitely distant object, and of negative power if it produces an upright image of an infinitely distant object.

(*c*) The numerical value of the power of the instrument (measured in dioptres) is the small angle (in radians) subtended by an infinitely distant object divided by the length (measured in metres) of its image. The measurements are assumed to be made in air.

These definitions are of importance in more advanced work. But since we shall have occasion to mention the power of different instruments as they are described, it is as well to use the proper definitions.

Magnification.—The optical instruments we shall consider are designed to increase the apparent size of objects—*i.e.* the angle they subtend at the eye. We are thus solely concerned with *angular magnification*, which is the ratio

$$\frac{\text{angle subtended at eye by image}}{\text{angle subtended at eye by object}}.$$

In defining this for any particular instrument *in normal adjustment*, object and image are usually considered to be at the same distance from the eye. The term " magnifying power " is also used for this ratio. We shall avoid it in the text as " power " has already been used in another sense.

Brightness.—The illumination of the image formed on the retina determines the **apparent brightness** of an object. We shall prove:

(1) That the apparent brightness is independent of the distance of the object from the eye, and (2) that the brightness of an aerial image can never exceed that of the object, provided in each case that the object is not exceedingly small.

(1) For the eye, let the area of the object be A sq. cm., u cm. its distance from the eye, B its actual brightness in lumens/cm.^{2} and S sq. cm. the area of the pupil (Fig. 175). Let the area of the image be a, and v its distance from the pupil. Then the intensity of the object regarded as a source is BA lumens.

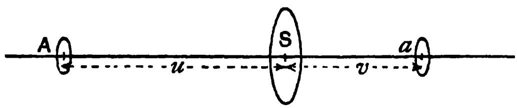


FIG. 175.

The solid angle subtended by the pupil at each point on the object is S/u^2. So the flux through the pupil is BAS/u^2 lumens.

This flux falls on an area a sq. cm., and so the illumination of the image is

$$BAS/u^2 \div a = BS/u^2 \cdot \frac{A}{a}.$$

But the superficial magnification $a/A = \frac{v^2}{u^2}$, so $\frac{A}{a} = u^2/v^2$.

Hence the illumination is

$$\frac{BS}{u^2} \cdot \frac{u^2}{v^2} \quad \text{or} \quad \frac{BS}{v^2} \text{ lumens/cm.}^2.$$

For the eye, v is necessarily constant, so the illumination of the image on the retina, and hence the apparent brightness of the object, depends only on the actual brightness of the object and the area of the pupil.

If the object is so small that its image is seen as a point, the effective area of the retina stimulated, whatever the size of the image, is a single cone. In this case A/a is constant, and the illumination increases as u is decreased.

Before proceeding to the second calculation, let us be clear as to what we have found. We have determined the *illumination of a surface*. A similar calculation holds for the camera or any case in which a *real image is formed on a screen*. *If the surface were emitting according to the cosine law*, its brightness would be proportional to the illumination. *If, however, it were constrained to emit all its light within a given solid angle*, its intensity and hence its brightness would depend not only on the *total luminous flux* it emitted, but on the *solid angle* as well.

(2) For the **aerial image** formed by a lens, use the same notation as before. The lens of area S forms an image at I of the object at O (Fig. 176). The image is viewed by the cone of rays diverging from it, and all the flux it emits is concentrated within the cone.

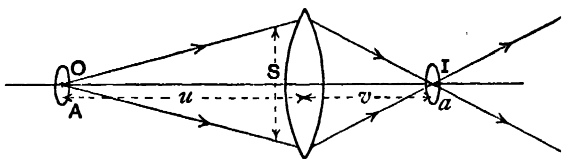


FIG. 176.

As before, the flux through the lens from the object is BAS/u^2 lumens. This is the flux through the image if there is no loss of light in passing through the lens. *It fills a cone of solid angle* S/v^2 *in converging towards the image and also in diverging from it.*

So the intensity of the image is $BAS/u^2 \times v^2/S$ and the brightness of the image is $BAS/u^2 \times v^2/S \times 1/a$; and as $a/A = v^2/u^2$ everything cancels but B. So if there is no loss of light the image appears as bright as the object.

In this case the aperture of the lens makes no difference to the brightness of the image. If the aperture is halved the light received by the image is halved, but *so is the solid angle it has to fill.* To sum up, the aperture does not affect the brightness of the aerial image; it does affect the illumination of an image received on a screen.

Field of view.—Through an optical instrument, as through a window, a definite **field of view** is seen. This may be expressed as the angle subtended by the extreme portions of the field at the instrument.

Focussing.—Optical instruments can be focussed to form an image at a definite distance from the eye of objects at different distances. They can also be adjusted to make this distance from the eye anything convenient for the observer.

Tracing rays through the instrument.—Since each lens uses as object the image formed by the preceding one, the final image can be found by graphical construction, each image being located

by the method given on p. 133. This is not sufficient for a full discussion of the performance of the instrument. In order to see what the field of view and brightness (for the pupil may not be filled) will be, two or more rays must be traced right through the system. This can, of course, only be done after the first constructions; the method is similar to that of p. 39 for two inclined plane mirrors.

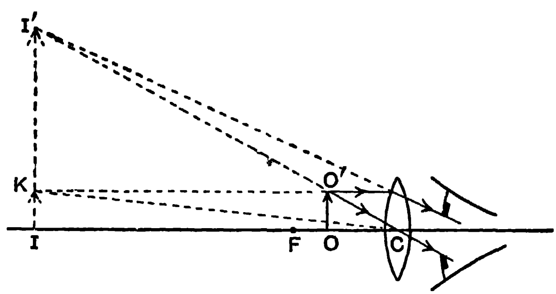


FIG. 177.—Simple Microscope.

The simple microscope, or magnifying glass.—The angle subtended at the eye by an object can be increased by bringing it up to the near point. If it is brought nearer the angle is still further increased but clearness is lost. If the near point is brought nearer to the eye we shall be able to move the object up, and we have already seen how to bring the near point closer for a far-sighted man using a single convex lens. The same type of lens can be used to act as a microscope. The lens is held close to the eye and the object OO' moved until a virtual image II' is formed at the near point. It was stated on p. 185 that a single lens close to the eye produced no increase in magnification. The same is true here, for the angle subtended at C by II' is the same as that subtended by OO'. But this is merely with regard to the lens. As far as the eye is concerned, if the object had been placed at the near point, with O at I and O' at K, the greatest angle it could have subtended at C would have been $\widehat{KCI}$; this has been increased to $\widehat{I'CI}$. So we can define the effective magnification as

$$m=\frac{\text{angle subtended at eye by image at near point}}{\text{angle subtended at eye by object if it were put at near point}}.$$

Hence,
$$m=\frac{\widehat{I'CI}}{\widehat{KCI}}$$
$$=\frac{II'}{IC}\div\frac{IK}{IC} \quad \text{(if both angles are small)}$$
$$=\frac{II'}{IK}.$$

This happens to be the same as the linear magnification, II'/OO'. Now for the linear magnification, $m=\frac{v}{f}-1$, where f is the focal length of the lens and $v=(-25\text{ cm.})$, for the normal eye. So $m=-\left(1+\frac{25}{f}\right)$, the negative sign indicating a virtual image. The smaller f is, the greater the magnification. If $f=+10$ cm., $m=3{\cdot}5$. If $f=+5$ cm., $m=6$, and so on.

To view objects at different distances, eye and lens would both be moved until OC was such as to give an image at the near point.

The image may be formed anywhere between the near point and infinity. In this case O is somewhere nearer F and the angle $O'CO$, and hence $I'CI$ will be smaller. When O is at F, the eye is receiving parallel rays and the angular magnification is now

$$m=\frac{\text{angle subtended at eye by image at infinity}}{\text{angle subtended at eye by object at near point}},$$

which can be seen to be $\frac{25}{f}$, numerically.

If the eye is not close to the lens the near-point magnification is reduced, but the infinity magnification is unchanged.

Example.—What is the greatest magnification obtainable with a lens of 10 cm. focal length held 5 cm. from the eye ?

Magnification is greatest when the image is at the near point ; it will then be 20 cm. from the lens. For the lens, using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, $v=(-20)$, $f=(+10)$, whence $u=+6\frac{2}{3}$ cm., and the linear magnification the lens gives is -3. The virtual image at the near point is three times as big as the object, and the angle it subtends at the

eye three times that which the object would give if placed there. The maximum angular magnification is thus 3.

The centre of the field shows no chromatic aberration, while colouring is evident at the edges. This is because II' always subtends the same angle at C as OO'; the focal length f makes no difference to this, and hence it is unaffected by any change of f with colour. The images of all the different colours appear superimposed. The colouring at the edges is due to differences in the spherical aberration for different colours. It is interesting to compare the appearance of printed type viewed through a magnifying glass used properly with that when the lens is held at some distance from the eye and forms a real inverted image, which is observed. There is no doubt about the presence of chromatic aberration in the second case, and this simple experiment will emphasize the point that a single lens only acts achromatically when giving a virtual image. Another point worth noting is that the images for *all colours* are combined, whereas an achromatic doublet giving a real image superposes only *two* colours.

The compound microscope.—The magnification of a single thin lens has its maximum when the image is at the near point, $-\left(1+\frac{25}{f}\right)$. It is impracticable to make single lenses of focal lengths considerably shorter than 1 cm.; a plano-convex lens for which $f=1$ cm. would have a radius of curvature of 5 mm., and a diameter of 1 cm. if made as a hemisphere, while its usable aperture would be very much smaller than this. So for high magnifications some other method must be found. If we can cause the object to be magnified before the eye views it through the lens the difficulty is overcome; this is the principle of the compound microscope.

The **objective**, a lens of really short focal length, forms a real image of an object placed just outside its first principal focus. This is viewed by the **eyepiece**, a lens of 2-5 cm. focal length. Both lenses in an actual instrument will be combinations designed to overcome the defects listed in Chapter X. The distance between objective and eyepiece is usually fixed, the standard length of tube being about 16 cm.

Fig. 178 shows the constructions for the final image.

This should be self-explanatory. The objective forms the real image, I_1, of O. This has been used as object to construct the final image, I_2, at the near point. Geometrically this is correct, but it gives an unfair impression, because no rays going through

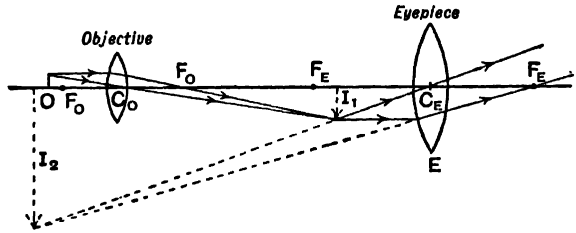


FIG. 178.—Compound microscope.

the tip of I_1 can possibly go through the optical centre of the eyepiece; and of the pencil leaving the tip, only part would strike the eyepiece at all, and this would go through its edge and miss an eye placed near the eyepiece.

Now that I_1 and I_2 have been found, put these in a fresh diagram and follow the paths of three rays from a point P on the object; one through C_0 and the others through the objective's edges.

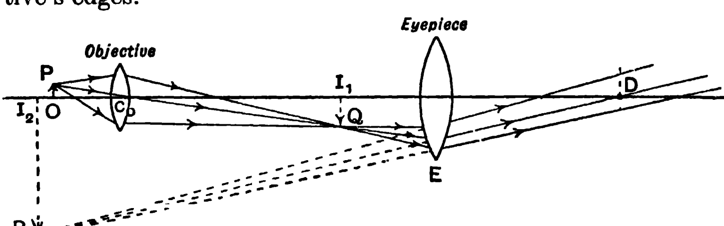


FIG. 179.—Rays traced through compound microscope.

First find Q, the image of P at I_1, and R, the image of Q at I_2. The ray PC_0 passes through Q and, after refraction at E, actually passes through D the image of C_0 formed by E, while appearing to come from R, which is thus found. The paths of the extreme rays can now be put in, as they also pass through Q and appear to come from R.

We can make the following deductions from the diagram :

(1) D is the image of C_0 formed by the eyepiece. So the bright patch of light which would be received on a screen at D is the real image of the objective. If this is as large as, or larger than, the pupil the brightness of the image will have its maximum value. All rays from any point on the object which go through C_0 and strike the eyepiece pass through D. D is thus the best position for the eye, and is where the **eye-ring**, a stop with a hole of aperture S, should be placed.

(2) If we assume that points on the object which send rays through the optical centre of the objective to the eye are visible and others are not, the rays limiting the field pass from C_0 through the edge of the eyepiece. If d is the diameter of the eyepiece and l the distance between the lenses, this angle is d/l radians.

Thus the *aperture of the eyepiece determines the field of view, while that of the objective, if this is small, determines the brightness of the image* for a given pair of lenses. In practice things are not quite as simple as this, for reducing the aperture of the objective reduces the diameter of that part of the eyepiece actually used, and hence also cuts down the field of view.

Focussing.—It is usual to have a pair of fine threads intersecting at right angles at I_1 in the instrument and first adjust the eyepiece so that the image of these **crosswires** is seen at the near point or wherever the observer chooses—the near point gives greatest magnification, infinity least strain. The first image is brought to coincide with the crosswires without parallax. The distance of I_1 from the objective is thus fixed, so *objects must always be at the same distance from the objective to be in focus* ; so to focus objects at different distances the microscope *as a whole* is moved.

Magnification.—The objective's magnification is $m_0 = \dfrac{v}{u}$, where u and v denote the distances from the objective of O and I_1. This can also be written $m_0 = \left(\dfrac{v}{f_0} - 1\right)$, f_0 being the objective's focal length.

The angular magnification of the eyepiece, $m_e = -\left(1 + \dfrac{25}{f_e}\right)$ for near-point vision or $-\dfrac{25}{f_e}$ for infinity vision for a normal eye, *where* f_e *is the focal length of the eyepiece.*

The final magnification for near-point vision is

$$m_n = m_0 \times m_e = -\left(\frac{v}{f_0} - 1\right)\left(1 + \frac{25}{f_e}\right),$$

the negative sign indicating a virtual image.

As f_0 is short compared with v, we can write $\frac{v}{f_0}$ approximately for the first bracket, so that

$$m_n = -\frac{v}{f_0}\left(1 + \frac{25}{f_e}\right).$$

If the final image is at infinity, this becomes

$$m_\infty = -\frac{v}{f_0} \cdot \frac{25}{f_e}.$$

If the eye is at the eye-ring, which must be at least as far from the eyepiece as its second principal focus, the near-point magnification approximates to that for infinity; for instead of 25 cm. for the distance of the near point from the *lens* we must write nearly $(25 - f_e)$ numerically, so that m_e is nearly $\frac{25}{f_e}$ and m_n becomes approximately $-\frac{v}{f_0} \cdot \frac{25}{f_e}$.

The magnification will always be negative, indicating that the final image is virtual. The power of the instrument will be negative, since an infinitely distant object will give an erect image.

There are three ways of increasing m; reducing f_0, reducing f_e, and increasing v. Expensive microscopes have interchangeable eyepieces and objectives of various focal lengths. Some types of inexpensive school microscope have tubes of adjustable length, which can be extended to increase v and so m.

For a model compound microscope made in the laboratory from two thin lenses on the optical bench, the magnification may be measured by finding f_0, f_e, and v separately. A second method is to use as object a millimetre scale and view this with one eye through the system, while the other eye looks at a similar scale held 25 cm. away. If K divisions of the directly viewed scale cover one of the image, the angular near-point magnification, m_n, is K. Details of these experiments will be found in the practical text-books.

Examples are best worked from first principles, as shown.

EXAMPLE.—A compound microscope has as objective and eyepiece thin lenses of focal lengths 1 cm. and 5 cm. respectively. An object is placed 11 mm. from the objective and the final image is 25 cm. from the eye. What is the magnification produced and the separation of the lenses?

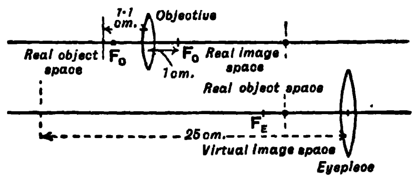


FIG. 180.—Problem.

For the objective, using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$u=(+1\cdot1 \text{ cm.}) \quad \text{(real object)},$$
$$f=(+1 \text{ cm.}).$$

So $$\frac{1}{v}+\frac{1}{1\cdot1}=1,$$

whence $$v=+11 \text{ cm.}$$

and $$m_0=\frac{v}{u}=+10.$$

For the eyepiece, the final image is 25 cm. in front of the lens, so, using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$v=(-25 \text{ cm.}) \quad \text{(virtual image)},$$
$$f=(+5 \text{ cm.}),$$
$$-\frac{1}{25}+\frac{1}{u}=\frac{1}{5}.$$

So, $$u=+\tfrac{25}{6}=+4\tfrac{1}{6} \text{ cm.}$$

and $$\frac{v}{u}=\frac{-25}{4\frac{1}{6}}=-6,$$

which equals the eyepiece's angular magnification. The total magnification, m_n, is thus -10×6 or -60, and the lenses are $(11+4\frac{1}{6})$ cm. or $15\frac{1}{6}$ cm. apart.

EXAMPLE.—A convex lens of $\frac{1}{2}$ in. focal length is 9 in. from another convex lens of 2 in. focal length. Find how far the object is from the $\frac{1}{2}$ in. lens if the final image is 10 in. from the eye when

the combination is used as a compound microscope. Find also the magnification.

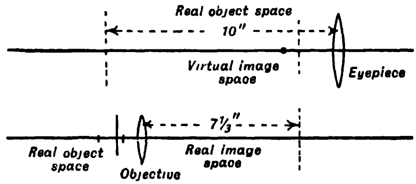


FIG. 181.—Problem.

For the eyepiece, the 2 in. lens, use $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

Here, $$v=(-10\text{ in.})\quad\text{(virtual image)},$$
$$f=(+2\text{ in.}).$$

So, $$-\frac{1}{10}+\frac{1}{u}=\frac{1}{2},$$
$$u=+1\tfrac{2}{3}\text{ in.}$$

The real image which acts as an object for the eyepiece is thus $1\frac{2}{3}$ in. in front of it, and so must be $7\frac{1}{3}$ in. behind the objective.

For the objective, the $\frac{1}{2}$ in. lens, use $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

$$v=(+\tfrac{22}{3}\text{ in.})\quad\text{(real image)},$$
$$f=(+\tfrac{1}{2}\text{ in.}),$$

and $$\frac{3}{22}+\frac{1}{u}=2,$$
$$u=\frac{+22}{41}\text{ in.}$$

So the object is $\frac{22}{41}$ in. in front of the objective.

The magnification given by the objective is

$$m_0=\tfrac{22}{3}\times\tfrac{41}{22}=+\tfrac{41}{3},$$

and by the eyepiece,

$$m_e=-10\times\tfrac{3}{5}=-6,$$

and this is also the near-point angular magnification. The total magnification, m_n, is thus $m_0\times m_e=-82$.

The illumination of the object.—Microscope objects are usually transparent sections mounted, often in Canada balsam, on a glass slide under a thin glass cover-slip. They are viewed by transmitted light and are illuminated from beneath by means of a con-

denser, like a lantern slide. There is one important difference in the function of the condenser in the two cases. In the lantern, we wish for *illumination of a screen,* and focus an image of the source on the objective to get as much light as possible through the lens aperture. In the microscope, we require a bright *aerial*

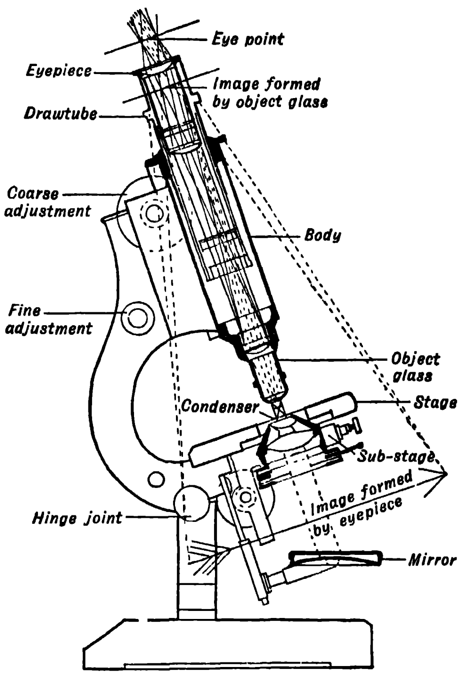


FIG. 182.—Beck microscope, showing illumination of object and details of construction.
(*Courtesy of Messrs. R. J. Beck, Ltd.*)

image, and the condenser is focussed on the object, each point of which is to emit as much light as possible. The behaviour of the condenser is shown in Fig. 182. This type of illumination is called **bright ground** illumination.

Should the microscope be required for counting small opaque objects, they are best rendered visible, like dust in a sunbeam, by the light they scatter. The centre of the condenser is covered

with a stop, so that the object field is illuminated but no light from the condenser can enter the objective directly ; particles in the field are then seen as bright specks on a dark background. Very small particles can thus be made visible. This is called **dark ground** illumination. The *ultramicroscope* of Siedentopf and Zsigmondy uses a very intense beam at right angles to the microscope's axis ; with this the Brownian movement of smoke particles can be watched.

Microscope objectives.—In an actual microscope the objective is a system corrected for chromatic and spherical aberration. This correction can never be perfect, and will be best for rays whose paths lie closest to the axis.

The object is frequently immersed in oil, such as cedar oil of refractive index 1·5, and the first surface of the objective, a plane face, dips into the oil. If AB is the objective's face, μ the refractive index of the liquid, and α the inclination to the axis of the extreme rays from the point object O, if the object had been placed in air it would have had to be placed at P, from which the greatest angle to the axis at which a ray can enter the objective is given by α', where

$$\mu \sin \alpha = 1 \times \sin \alpha'.$$

The aim is to reduce the actual angle α and so reduce aberrations. As P, the position of the object in air is fixed, so is α' ; hence $\mu \sin \alpha$ will be a constant for the objective, called its **numerical aperture.** The greatest reduction in α would be made by having μ as large as possible ; but it is best to have the refractive index of the oil the same as that of the first lens of the objective. The object is then effectively inside a sphere of glass, and can be placed at one of the aplanatic points of the sphere.

The chief use of **oil immersion objectives** is in microscopes using high magnification, for only in this case will the object need to be so close to the objective as to make α' very large.

Advantages of the arrangement are :

(*a*) Increased *brightness* of the object, as light losses by reflection, particularly the large loss by internal reflection at the upper face of the slide, are reduced.

(*b*) The *resolving power*, or detail observable, is increased. Diffraction (p. 294) takes place at the edge of any obstacle, and

though that at the rim of the objective is small it is sufficient to cause blurring at high magnifications. The least distance that can be separated by a microscope in monochromatic light is given as $\frac{\lambda}{2\mu \sin \alpha}$, where λ is the wavelength of the light and $\mu \sin \alpha$ the numerical aperture of the objective.

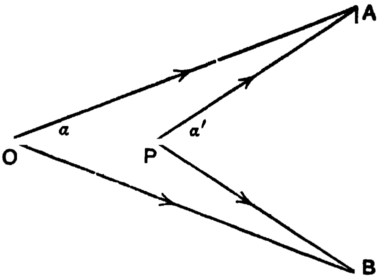


FIG. 183.—Reason for oil immersion.

Eyepieces.—The eyepiece usually consists of two lenses, a **field lens** and an **eye lens**—the former nearer the objective. The combination is designed to be as free as possible from chromatic and spherical aberration, and besides improving the image in this way it *increases the field of view.*

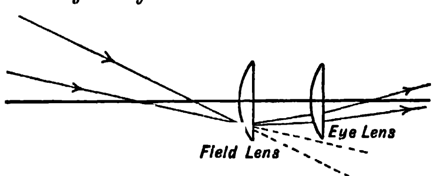


FIG. 184.—Kellner's eyepiece.

The simplest type of eyepiece, **Kellner's**, has the field lens at the principal focus of the eye lens. If the microscope is focussed to give a final image at infinity the field lens will have no effect on focussing or magnification, since for it both u and v are zero. Spherical aberration is lessened as the deviation is spread over four surfaces instead of two. If both lenses have the same focal length the condition for achromatism, $x = \frac{f_1+f_2}{2}$, is satisfied. Whatever the focal length of the field lens it will

deviate through the eye-lens rays, which would otherwise have missed its edge. With the short focal lengths used all rays reaching the field lens are deviated through the eye lens, whose aperture can be reduced without sacrificing field, as rays which would never have reached it unaided now pass through it near its centre.

Huygens' eyepiece was designed to give equal deviation by both lenses. The condition for this is that they should be separated by a distance $x = f_1 - f_2$, where f_1 is the focal length of the field lens and f_2 that of the eye lens. Huygens chose a field lens whose focal length was three times that of the eye lens and separated them by a distance of twice the latter's focal length. This combination also gives $x = \frac{f_1 + f_2}{2}$ and is achromatic. Both lenses are plano-convex, with the convex side facing the incident light. The diameter of the eye lens is usually less than that of the field lens.

In Huygens' eyepiece the real image, I_1, is formed between the lenses ; this is clear, for it must be nearer to the eye lens than its principal focus if the final image is to be virtual. So the eyepiece *cannot be used to focus on crosswires outside it* like a single lens ; the crosswires must be fixed between the two. This is a disadvantage for some purposes.

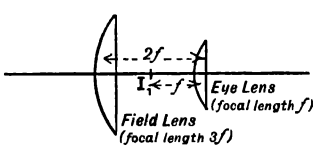


FIG. 185.—Huygens' eyepiece.

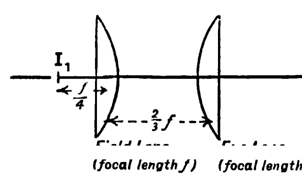


FIG. 186.—Ramsden's eyepiece.

Ramsden's eyepiece consists of two equal plano-convex lenses separated by a distance of $\frac{2}{3}$ their common focal length f, with their convex faces inwards. The real image is formed at a distance of $f/4$ in front of the field lens, and so external crosswires can be viewed. It is not as well compensated as Huygens', but the spherical aberration is not considerable and the chromatic aberration can be reduced by making each lens an achromatic doublet—though its chromatic aberration is in any case small.

The astronomical telescope.—The **objective** is a lens of long focal length and the **eyepiece** has a short focal length. If the distance apart of the two lenses is the sum of their focal lengths, an image of a very distant object will be formed at the principal focus of the objective and seen at infinity through the eyepiece. The telescope is then receiving and delivering parallel rays, and is said to be in **normal adjustment**. Its power, as defined on p. 192, is zero.

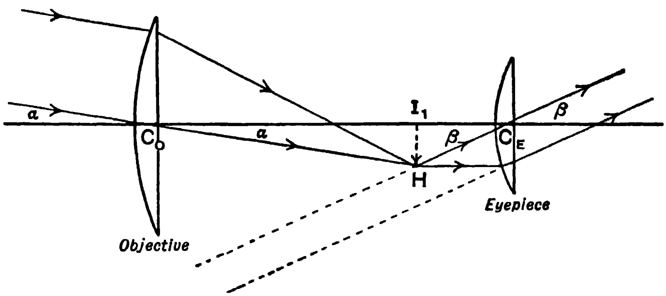


FIG. 187.—Astronomical telescope.

Fig. 187 shows the construction for the real image at I_1. The two rays shown are two out of the beam of parallel rays from the tip of an erect arrow-object at a very great distance. The ordinary construction cannot be done ; but the second method is useful here, for we know the ray through C_0 is undeviated and that the image is in the focal plane of the objective, so where this ray cuts the focal plane at H is the real image of the tip. The construction for the parallel rays emerging from the eyepiece is done in the ordinary way.

The astronomical telescope is frequently used with the image of a distant object formed at the **near point.** The eyepiece is then nearer the objective than is the case for normal adjustment.

To focus on near objects, with the final image at infinity, the eyepiece must be moved away from the objective by the same distance as that moved through by I_1.

The relative positions of objective and eyepiece for near-point vision of the image of a near object can be worked out from first principles in any particular case.

In Fig. 188 the paths of three rays, one through the centre of the objective and the others through its edges, have been traced. As with the microscope, we can see from this ray diagram :

(1) The point D on the axis is the image of C_0 formed by the eyepiece, and that the disc of light formed by the rays here is

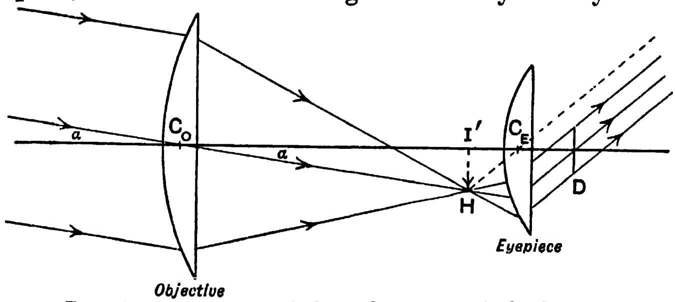


FIG. 188.—Rays traced through astronomical telescope.

the real image of the objective formed by E. This will be the best place for an eye on the axis, and the eye-ring should be placed here.

(2) If the angle α made with the axis by the rays from a point on the object is less than $\dfrac{d}{2(f_0+f_e)}$, where d is the diameter of the eyepiece, then the ray through C_0 from that point will reach the eyepiece and the point will be in the field of view.

(3) The brightness of the image will depend on the aperture of the objective if the diameter of the disc at D is less than that of the pupil.

Magnification.—In normal adjustment the magnification is

$$m=\frac{\text{angle subtended at eye by image at infinity}}{\text{angle subtended at eye by object at infinity}}.$$

From Fig. 187, considering the rays C_0H and HC_E, this ratio is $\dfrac{\beta}{\alpha}$, which is $\dfrac{I_1H}{f_e}\div\dfrac{I_1H}{f_0}$ if the angles are small.

So $\dfrac{\beta}{\alpha}=\dfrac{f_0}{f_e}$ and so in normal adjustment

$$m=\frac{\text{focal length of objective}}{\text{focal length of eyepiece}}.$$

This can be put in another form. The disc at D is the image of the objective formed by the eyepiece. Let v be its distance from the eyepiece.

Using $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f},$$

$$u=+(f_0+f_e) \quad \text{(real object)},$$

$$f=f_e.$$

So $$\frac{1}{v}+\frac{1}{f_0+f_e}=\frac{1}{f_e} \quad \text{and} \quad v=\frac{f_e}{f_0}\cdot(f_0+f_e).$$

The magnification of this image is $\frac{v}{u}$, which equals $+\frac{f_e}{f_0}$, the reciprocal of the angular magnification of the instrument.

So angular magnification

$$m=\frac{\text{diameter of objective}}{\text{diameter of real image of objective formed by eyepiece}}.$$

If the image is at the near point the angular magnification will be increased in the ratio $\left(1+\frac{25}{f_e}\right):\frac{25}{f_e}$, if the eye is close to the eyepiece; and if the eye is at the eye-ring little difference will be observed between this and m for normal adjustment.

For a near object the magnification is best worked out from first principles as a separate problem.

Measurement of magnification.—For a model telescope made of two thin lenses the magnification in normal adjustment may be found by finding the focal lengths of eyepiece and objective separately and calculating the ratio, $\frac{f_0}{f_e}$.

It may also be determined accurately for a permanent instrument by measuring with a sextant the angle subtended by the extremities of a distant object and finding with a goniometer (a telescope focussed to receive parallel light and moving over a graduated arc) the angle between the two beams of parallel light emerging through the eyepiece from the extremities of its virtual image. Another method is to measure the diameters of the objective and its real image at D and take their ratio.

For near objects, when not in normal adjustment, the telescope can be focussed on, say, a brick wall and adjusted so that one eye looking through the telescope sees an image which can be fused with that seen directly by the other; the magnification is obtained from direct comparison of the two images. It can be verified by drawing a rough diagram that α will be less and β greater than for normal adjustment, so that the observed magnification will be greater than that found by other methods for an infinitely distant object and image.

EXAMPLE.—An astronomical telescope in normal adjustment consists of two thin lenses 105 cm. apart and gives a magnification of 20. If the eyepiece is pulled out a distance of 5 cm. to focus on a near object, how far away is the object and what is the magnification in this case ?

Here $\quad 20=\frac{f_0}{f_e}$ and $105=f_0+f_e$,

so that $\quad f_0=100$ cm. and $f_e=5$ cm.

Assuming that the image is formed at infinity in the case above, the 5 cm. shift of the eyepiece means that the real image is formed 105 cm. from the objective.

Use $\quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ for the objective ;

then $\quad v=(+105$ cm.) (real image),

$f=(+100$ cm.).

So $$+\frac{1}{u}=\frac{1}{100}-\frac{1}{105}.$$

$$+\frac{1}{u}=0{\cdot}01-0{\cdot}009524=0{\cdot}000476.$$

So $u=+2101$ cm.; hence the object is about 21 metres in front of the objective.

The new angular magnification will be $\frac{105}{5}$ instead of $\frac{100}{5}$; *i.e.* 21.

Measurement of field of view. *Method* 1.—A divided scale is set up a considerable distance from the instrument, and the length y included in the field noted. If u is the distance of the scale from the objective and α the angle this length y subtends at it, then

$$\tan\frac{\alpha}{2}=\frac{y}{2u}.$$

Method 2.—A converging lens of focal length f is set to give a real image of a distant object on a screen. The telescope is placed with its objective pointing towards the lens, and the length y of the image now formed on the screen noted. Then $\tan\frac{\alpha}{2}=\frac{y}{2f}$; or the telescope may be set with its eyepiece towards the lens and the observation repeated, when $\tan\frac{\alpha}{2}=\frac{y}{2fm}$, where m is the angular magnification.

Method 3.—The telescope is mounted on a divided circle, and the angle through which it is turned to bring a distant object first on one edge of the field and then on the other observed.

Telescope objectives.—These must be achromatic and corrected for spherical aberration. Large aperture is also important with high magnifications on grounds of both brightness and resolving power. For maximum brightness we want the diameter of the objective's image D to be at least as great as that of the pupil, δ; so

$$\delta\,(\text{a constant}) = \frac{\text{diameter of objective}}{\text{magnification}}.$$

Even the rim of a telescope objective produces diffraction effects which limit the detail observable. The least distance at which two points on the image can be separated in monochromatic light of wavelength λ is $\frac{1{\cdot}22\lambda \times f_0}{d_0}$, where d_0 is the diameter of the objective, which should thus be large.

Eyepieces have already been dealt with; the same types are used for telescopes of this kind as for microscopes.

The terrestrial telescope.—The final image of the astronomical telescope is inverted. An *erect image* can be obtained without

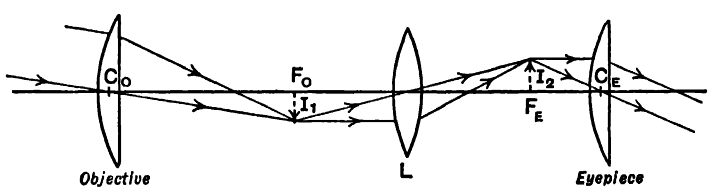


FIG. 189.—Terrestrial telescope.

change of magnification by causing a converging lens, L, to produce a real image, I_2, of I_1 at the minimum distance of four times its focal length, f_L.

Disadvantages of this arrangement are that it increases the length of the instrument by $4f_L$, that an extra lens means loss of light, and that L must be carefully corrected. Also the field of view is reduced unless extra lenses are incorporated. Another method of erecting the image is to use a right-angled prism arranged as Fig. 107, p. 102.

The Galilean telescope.—The objective is a long-focus converging lens, the eyepiece a diverging lens, and they are separated by a distance equal to the (numerical) difference of their focal lengths.

The objective would give a real inverted image of a distant object at I_1 at its principal focus. Before reaching I_1 the rays are intercepted by the eyepiece and made to proceed as parallel again, giving a *virtual erect image* at infinity. Fig. 190 shows the extreme rays and one through the optical centre of the objective traced through the system. From this we see that the disc at D is

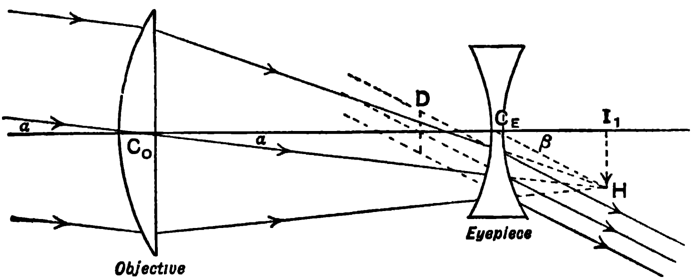


FIG. 190.—Galilean telescope.

a virtual image of the objective, and that the best place for the eye is as close to the eyepiece as possible, as the rays are diverging from the eyepiece and the field of view decreases as the eye recedes from it.

On this account the effective aperture of the eyepiece is that of the pupil, and if the pupil's diameter is δ, only those rays passing through the optical centre of the objective at an angle α less than $\frac{\delta}{2(f_0 - f_e)}$ will enter the eye. As δ is small compared with the diameter of eyepiece or field lens in the astronomical type, the field of view is considerably less with a given objective.

The angular magnification, β/α, can be seen by drawing C_0H, C_EH to be $\frac{I_1H}{f_e} \div \frac{I_1H}{f_0}$, or $\frac{f_0}{f_e}$.

This type of telescope has the advantage of being shorter than the terrestrial telescope and causing less loss of light. If high magnification is not required, $(f_0 - f_e)$ will be small and the field of view not too greatly reduced.

Binoculars.—Short telescopes of the Galilean type are mounted as **binoculars**, chiefly used for opera glasses. **Field glasses** usually consist of astronomical telescopes, in which the image has been inverted by two totally reflecting prisms arranged with their principal sections at right angles so as to produce two lateral inversions in two directions at right angles, and hence the complete inversion of the image that an erecting lens would give.

These prism binoculars enable higher magnification to be used, as the Galilean form has a uselessly small field of view with a high magnification. As field lenses can be used in the eyepieces, the field of view is enlarged. For a given high magnification they will be shorter than a Galilean arrangement. As the objectives are wider apart than the eyes, perspective is heightened.

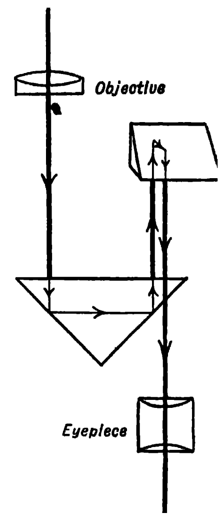


FIG. 191.—Arrangement of erecting prisms in prism binoculars.

Reflecting telescopes.—On p. 211 it was shown that a telescope objective should have a large aperture if fine detail and a bright image are required. It should be clear that no telescope can possibly *increase* the apparent brightness of an object which subtends a finite angle at the objective; the most that can be expected of a well-designed instrument is that it shall not reduce the apparent brightness too much below that observed with the naked eye.

The fixed stars are so distant that they can be taken to be points, and different considerations apply. Their apparent brightness is increased by a telescope, and the gain in brightness, for photographic work at least, is proportional to the *fourth power* of the diameter of the objective. An explanation of this is given on p. 295. The advantages of using an objective of large aperture for astronomical work are thus an increase in the detail observable and an increase in the apparent brightness of the stars.

The largest and most effective astronomical telescopes use a

concave mirror instead of a converging lens as an objective. Fig. 192 (*a*) shows the type of **reflecting telescope** designed by Newton. A paraboloidal mirror is the objective. For astronomical work such a mirror is completely free from spherical aberration for axial points. There can be no chromatic aberration. Manufacture and mounting are easier than for lenses of corresponding very wide aperture. The largest refracting telescope is the 40-inch refractor of the Yerkes Observatory; its focal length is as large as 65 feet

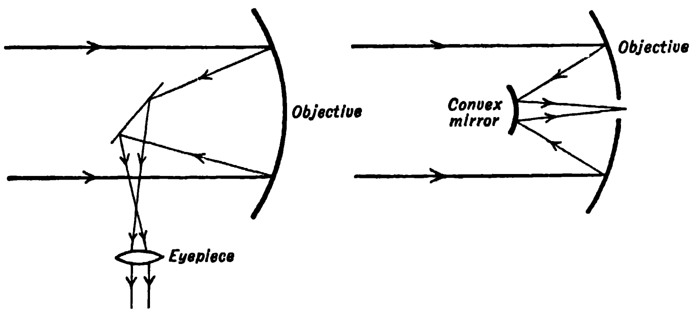


FIG. 192.—Reflecting telescopes.
(*a*) Newtonian reflector. (*b*) Cassegrain reflector.

—to reduce aberrations—and a large dome is needed to house it. As the lens can be supported only at the rim, and therefore nothing can be done to stop the lens bending under its own weight, the construction of refractors of greater aperture is impracticable. There is in addition the problem of obtaining the necessary large mass of glass free from imperfections; the reflecting telescope needs only a perfect surface. With both types thermal expansion is a nuisance, but for the reflector there may be some possibility of progress in this direction with materials of lower coefficient of expansion and better thermal conductivity than those used hitherto.

Newton's first telescope disappointed him, as much light was lost by reflection; for a small-aperture telescope there is no doubt that an achromatic lens is better than a mirror, as it wastes less light. But lenses with an aperture much greater than 40 inches would be so thick that the loss by absorption in the glass would be greater than the reflection loss at a corresponding

mirror, so that even on this account the reflector is the better type for very large telescopes.

Early reflecting telescopes used objectives of speculum metal, a hard alloy of copper and tin difficult to work and polish. The

FIG. 193.—The 100-inch rollector at Mount Wilson Observatory. (*Courtesy of Mount Wilson Observatory.*)

mirror of the 100-inch Mount Wilson reflector is made of glass silvered on its front surface. Silver films tarnish readily but are easily renewed. Aluminium gives a better reflecting surface,

particularly in the ultra-violet, is durable, and scatters very little light. The giant 200-inch reflector in course of construction for the California Institute of Technology has a Pyrex mirror which will be coated with aluminium.

With these great telescopes visual observations may be made in the Newtonian manner at the principal focus of the objective. By intercepting the converging beam on a convex mirror a **Cassegrain system** may be set up for direct photography or for spectroscopic observations at the lower end of the tube; the effective focal length of the objective is thus increased. Also, photographs may be taken with the plate placed at the principal focus of the objective.

The usual form of mounting for an astronomical telescope is the **equatorial mounting.** The instrument is free to rotate about an axis parallel to the earth's axis (the **polar axis**), and about a second axis at right angles to this (the **declination axis**). As very long exposures are desirable for photography, the telescope is turned about the polar axis by clockwork so that the star observed is always in the centre of the field.

Photography brings its own difficulties; an objective which gives a wide sharp field when used with an eyepiece will not usually give a sharp real image on a flat plate except for points close to its axis. It is possible to correct lenses, but not single concave mirrors, for this defect. Reflecting telescopes are thus not satisfactory for photographing large areas of the sky.

Experiments are being made with reflectors of the Cassegrain type, in which the objective proper departs slightly from the paraboloidal form and the convex mirror is very nearly a hyperboloid, the object being to improve definition at points away from the axis. This is known as the Ritchey-Chrétien design; it gives a very short, compact telescope.

QUESTIONS ON CHAPTER XIII

1. Describe the construction of the compound microscope, and illustrate your answer with a diagram showing the paths of rays through the instrument.

What are the chief defects in microscopes, and how are they remedied? (C.W.B.H.S.C.)

2. Draw a diagram to show how two lenses are arranged to form a microscope, tracing the paths to the eye of two rays from a point on the object not on the axis.

Convex lenses, of focal lengths 3 cm. and 5 cm., are used respectively as the objective and eyepiece of a microscope. If the object is 3·5 cm. from the objective and the final image is 25 cm. from the eyepiece, what is the distance between the centres of the lenses ? (C.H.S.C.)

3. Explain with the aid of a diagram how to arrange two lenses to form a microscope. Trace the path of a pencil of rays from a point of the object not on the axis to the eye.

A microscope has an objective of focal length 0·5 cm., and an eyepiece of focal length 2·5 cm. If the final image is 25 cm. from the eyepiece and the magnification is 330, what is the distance between the centres of the lenses ? (C.H.S.C.)

4. A person whose minimum distance of distinct vision is 20 cm. uses a magnifying glass of 5 cm. focal length held close to the eye. What must be the position of the object examined, and what magnification is obtained ? How would you make a compound microscope with two lenses ? Discuss the relations between the position of the object, the distance between the lenses, and the distance of distinct vision. (N.U.J.M.B.H.S.C.)

5. A microscope is set up placing an object-glass of 1·5 in. focal length 12 in. from an eye lens of 2 in. focal length and setting the object so that the final image seen by the observer is 10 in. from the eye lens. Compare the angle subtended by this image at the eye lens with that which the object subtends at a point 10 in. away. Point out the reasons why the optical arrangement of a real microscope is more complicated than that indicated above. (N.U.J.M.B.H.S.C.)

6. Draw a diagram illustrating clearly the passage of rays through an astronomical telescope.

The focal lengths of objective and eyepiece of such a telescope are 12 in. and 1 in. respectively. The telescope is focussed on a scale 6 ft. from the objective, the final image being formed 12 in. from the eye of the observer. Calculate the length of the telescope and the magnification produced by it. (L.H.S.C.)

7. Draw a diagram to show the principle of an astronomical telescope, and point out the factors upon which its magnifying power depends.

An astronomical telescope has an objective of focal length 60 ft. What focal length must the eyepiece have to give a magnifying power of 2500 ? (C.H.S.C.)

8. Describe the optical system used in a simple astronomical refracting telescope, and explain why, in practice, both the object-

glass and the eyepiece of such a telescope are composed of at least two thin lenses.

Describe how you would measure in a laboratory the magnifying power of a telescope for an object distant about 20 ft. from the eye. (O.H.S.C.)

9. A simple astronomical telescope is made of two convex lenses of focal lengths 1 ft. and 2 in. respectively. How far apart will the lenses be when used by a normal eye to look at the moon? How much movement of the eyepiece will be necessary if the observer then focusses on the branch of a tree distant 20 ft. from the objective, without altering the accommodation of his eye?

Give a diagram to show how the images are formed by the telescope. (O.H.S.C.)

10. Describe how you would set up two thin convex lenses of suitable focal lengths to form a telescope, and how you would determine the angular magnification produced by your arrangement.

A simple telescope provided with an object-glass of 50·0 in. focal length and a convex eye lens of 7·5 in. focal length is focussed so that a virtual image of a distant object is formed at a distance of 30·0 in. from the eye lens. Draw a diagram on squared paper showing the paths through the telescope of three rays from a point on the object, one ray passing through the centre of the object-glass and the others through its edge. Explain the reasons for the path indicated for each of the three rays and determine the angular magnification obtained.

[In your diagram use an axial scale of 1 in. = 10 in. and take the diameter of the object-glass, the distance from the axis of the point image which it forms, and the diameter of the eye lens as 2 in., 0·25 in., and 1 in. respectively.] (N.U.J.M.B.H.S.C.)

11. An astronomical telescope consisting of an objective of focal length 60 cm. and an eyepiece of focal length 3 cm. is focussed on the moon so that the final image is formed at the minimum distance of distinct vision (25 cm.) from the eyepiece. Assuming that the diameter of the moon subtends an angle of $\frac{1}{2}°$ at the objective, calculate (*a*) the angular magnification, (*b*) the actual size of the image seen.

How, with the same lenses, could an image of the moon, 10 cm. in diameter, be formed on a photographic plate? (C.H.S.C.)

12. Describe the construction of an astronomical telescope, and give a diagram showing the course of the rays by which two distant stars are seen with it.

Explain what is meant by the magnifying power of the telescope, and show that this is given by the ratio of the focal length of its object-glass to the focal length of its eyepiece. How would you convert the astronomical telescope into one which would be of service terrestrially? (C.W.B.H.S.C.)

13. Describe with the aid of a diagram how two lenses can be arranged to form a simple astronomical telescope.

For such a telescope the best position for the eye is a little behind the eyepiece in a plane where a real image of the object-glass would be formed by the eyepiece ; what is the advantage of this position ?

If the eye is in this position, show that the magnifying power of the telescope (defined as the ratio of the angle subtended at the eye by the final image to the angle subtended by the object at the unaided eye) will not differ much whether the telescope be arranged to give the final image at the near point or at infinity. (O.H.S.C.)

14. An astronomical refracting telescope is adjusted to give a real image of the sun on a screen. Draw a diagram showing the paths of a pencil of rays through the telescope to a point on the boundary of the image. If the focal lengths of object-glass and eye lens are 100 cm. and 2·5 cm. respectively, and the image of the sun on a screen placed 30 cm. from the eye lens is 9·6 cm. in diameter, find the angle which the sun subtends at the centre of the object-glass. (N.U.J.M.B.H.S.C.)

15. Describe with the aid of diagrams the details of construction of the object-glass and eyepiece of an astronomical refracting telescope and state their functions. Show that when an astronomical telescope consisting of object-glass and eye lens is set up with object and image at infinity, its angular magnification is equal to the ratio of the diameter of the object-glass to the diameter of the image of the object-glass formed by the eye lens. (N.U.J.M.B.H.S.C.)

16. A Galilean telescope has an object-glass of 12 cm. focal length and an eye lens of 5 cm. focal length. It is focussed on a distant object so that the final image seen by the eye appears to be situated at a distance of 30 cm. from the eye lens. Determine the angular magnification obtained and draw a ray diagram.

What are the advantages of prism binoculars as compared with field glasses of the Galilean type ? (N.U.J.M.B.H.S.C.)

17. Describe the construction and mode of action of Galileo's telescope (opera glass).

Show by means of a diagram how the concave lens produces a magnified image. (L.H.S.C.)

18. Describe the optical system of a pair of prism binoculars, and explain its advantages over (i) a single terrestrial telescope, and (ii) other types of binoculars. (C.H.S.C.)

19. Obtain a formula for the focal length of an eyepiece composed of two coaxial thin lenses set with their centres a distance d apart.

In the Huygens eyepiece the front plano-convex lens has a focal length three times that of the plano-convex eye lens, and their distance apart is twice the focal length of the eye lens. What advantages has this arrangement over a single lens ? (O.H.S.C.)

20. Give details of the optical systems of reflecting and refracting telescopes. (N.U.J.M.B.H.S.C.)

21. Explain (*a*) why telescope objectives are constructed of two or more lenses made of different kinds of glass, (*b*) why a camera lens working at $f/3 \cdot 5$ is better but more expensive than one working at $f/8$, (*c*) the advantages of prism binoculars as compared with those of the Galilean type. (N.U.J.M.B.H.S.C.)

CHAPTER XIV

WAVE THEORY

Introduction.—Any theory as to the manner in which light is propagated must explain consistently:

(1) Rectilinear propagation in a homogeneous medium.

(2) The laws of reflection and refraction.

(3) The phenomena of interference, diffraction, polarization, and emission to be studied in later chapters.

It must also lead to no conclusions contrary to observed facts and be economical of *ad hoc* hypotheses or modifications made to explain particular observations.

Huygens (1678) suggested that light is propagated as a wave motion in a subtle, imponderable, elastic medium permeating all space, called the **ether**. A point source according to this view emits waves spreading out in three dimensions, as do ripples on the surface of a pond in two dimensions. Each particle of the ether, on being set in motion, acts as a fresh source, giving rise to **secondary waves**, and the envelope of these secondary waves after any interval gives the position of the **wave front** at that instant. The **rays** giving the direction in which the disturbance travels are everywhere normal to the wave front.

Assuming that the *velocity of light is less in dense media than in vacuo*, this simple picture explains the laws of reflection and refraction. With adequate mathematical amplification (Ch. XVIII) it explains rectilinear propagation, interference, diffraction, and polarization.

An alternative theory was proposed by Newton, who objected that Huygens did not explain rectilinear propagation and that this ether would be very *dense*. He suggested that "rays of light" were "very small bodies emitted from shining substances."

These particles travelled in straight lines in a homogeneous medium and were either attracted or repelled when they approached a surface bounding two media. A transparent dense medium attracted the particles, exerting a force normal to the surface; thus the particle was deflected towards the normal to the surface on entering glass or water from air and travelled in the denser medium with *increased velocity*. Reflection of the particles occurred with repulsion, like the bouncing of a ball. The interference phenomena associated with his name were explained by supposing that "rays" of different colours were bodies of different sizes, and that they were subject to periods or "fits" of easy reflection and easy transmission at regular intervals, these "fits" possibly being due to waves which were stirred up by the bodies and travelled with greater velocity. Double refraction was due to some asymmetry of the bodies, like the polarity of a magnet. He also asked, "Are not Gross Bodies and Light convertible into one another, and may not Bodies receive much of their Activity from the Particles of Light which enter their Composition?"

Newton's suggestions were made in the form of modest queries for others to investigate. They were accepted throughout the eighteenth century with little questioning, and experiments which the wave theory explained more satisfactorily were at first received with some hostility. Evidence for the wave theory accumulated and survived detailed criticism. Foucault's experimental proof that the *velocity of light is less in water than in air* discriminated definitely in favour of the wave theory. It was finally established that light is an electromagnetic radiation in the ether similar in nature to the now familiar radio waves but of much shorter wavelength, and the classical electromagnetic theory was able to explain the whole of the phenomena known then.

The simple wave theory deals satisfactorily with the propagation of light. The question of its **emission** at once raises discussion as to the nature of the emitter and problems which are beyond the scope of this book, for which the reader is referred to specialist treatises on atomic physics. Some mention has been made, in the chapter on the spectrum, of the simplest phenomena of emission and their first consistent explanation, for the sake of

completeness in that section. Here it is sufficient to state that Newton's queries are not unlike the questions in process of investigation to-day.

Huygens' construction. Propagation of a spherical wave in a homogeneous medium.—The diagram represents the trace of a developing spherical wave on a plane through the source.

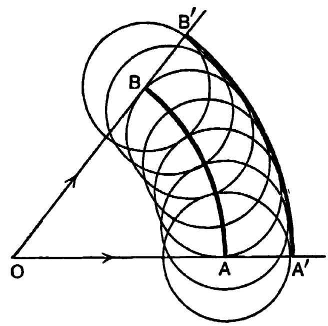


FIG. 194.—Propagation of spherical wave.

Let O be a uniform point source emitting light. Let AB be the position of the wave front at any instant. Let v be the velocity of light in the medium.

Each point on the wave front AB acts as a secondary source. To find the position of the wave front t seconds later, describe a series of circles with centres on AB and radii vt; these are the traces of the secondary waves. The arc $A'B'$ enveloping the small circles is the trace of the new wave front.

At a great distance from O the radius of curvature of the wave front will be large, and the front can be regarded as being *plane*; the *rays*, or normals to the wave front, will then all be *parallel.*

Reflection of a plane wave at a plane surface.—Let XY be the trace of a plane reflecting surface, and AB that of a plane wave striking it at A. Let v be the velocity of light, and t seconds the time for the edge B of the wave front to reach the surface at A'. As soon as the wave first strikes the surface a secondary wave is sent out from A; each point such as M on the wave front gives rise to a secondary wave when it reaches the surface, as at P;

and at the instant at which reflection is complete a secondary wave is about to start from A'. If reflection had not occurred, the point M on the wave front would have reached N; instead a secondary wave of radius $PM' = PN$ has developed. Similarly the reflected wave at A has given rise to a secondary wave of radius $AB' = AD$.

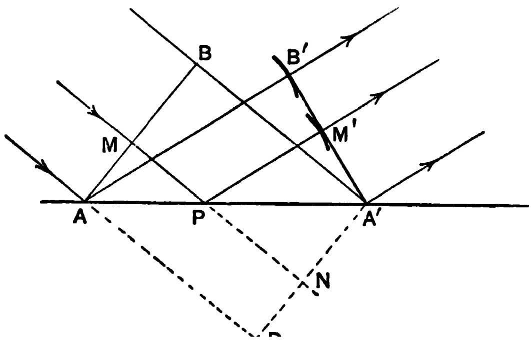


FIG. 195.—Reflection of plane wave.

Through A' draw a line representing the trace of a plane touching the secondary wave diverging from A at B'. Now since $AB' = BA' = AD = vt$, and AA' is common to all of them, the three right-angled triangles $AA'B$, $AA'B'$, $AA'D$ are equal in all respects. So in the triangles $AA'B'$, $AA'D$ the perpendicular PM' dropped to $B'A'$ equals PN; and the reflected wave diverging from P will touch the plane represented by $A'B'$. Similarly the secondary waves from all points on the surface touch the same plane. So $A'B'$ is the envelope of the reflected waves. *A plane wave is thus reflected as a plane wave from a plane surface.*

From the triangles $AA'B$, $AA'B'$, $A\widehat{A'}B = A'\widehat{A}B'$. Now $A\widehat{A'}B$ is the angle between the incident wave front and the sur face, and so equals the angle between the incident rays and the normal, *i.e.* the angle of incidence. Similarly $A'\widehat{A}B'$ is the angle of reflection. The incident and reflected rays clearly lie in one plane. Hence the wave theory accounts satisfactorily for the laws of reflection.

Refraction of a plane wave at a plane surface.—Let XY be the trace of a plane surface separating media of refractive indices μ_1 and μ_2. Let AB be a plane wave front incident from the first medium. Let v_1 and v_2 be the velocities of light in the two media, v_1 being greater than v_2 in the case of the diagram. That the refracted wave front is plane can be proved simply, as in the case of the last paragraph for reflection.

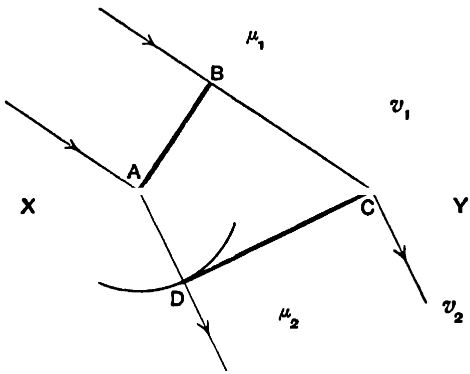


FIG. 196.—Refraction of plane wave.

Secondary waves are emitted from successive points of the wave front as they pass XY. Let t be the time taken for B to reach C; then $BC = v_1 t$. The secondary wave emitted by A into the second medium has travelled for t seconds and that from C is just about to start. The trace of A's wave is thus a circle of radius $v_2 t$, and the trace of the envelope of all the secondary waves a line CD touching this circle in D.

Now, $$BC = AC \sin \widehat{BAC} = v_1 t,$$

$$AD = AC \sin \widehat{ACD} = v_2 t.$$

So, $$\frac{\sin \widehat{BAC}}{\sin \widehat{ACD}} = \frac{v_1}{v_2} = \text{a constant}.$$

Now the angle of incidence is $\widehat{BAC}$ and the angle of refraction $\widehat{ACD}$, so this gives Snell's law, $\frac{\sin i}{\sin r} = \text{constant}$, and also gives a

new interpretation of refractive index, for the constant ${}_1\mu_2$ should be

$$\frac{\text{velocity of light in medium 1}}{\text{velocity of light in medium 2}}.$$

Actual measurements of the velocity of light in dense media give values reconcilable with this.

Total internal reflection.—Considering Fig. 196 again, it may not always be possible to draw the triangle ACD. If $AD > AC$, clearly no tangent from C can be drawn to the circle whose centre is A.

The limiting case will occur when $AD = AC$.

Now, $$AC = \frac{BC}{\sin \widehat{BAC}} = \frac{BC}{\sin i}$$

and $$BC = v_1 t, \quad AD = v_2 t;$$

hence the greatest possible value of i for which a refracted ray is obtainable is given by

$$v_2 = \frac{v_1}{\sin i} \quad \text{or} \quad \sin i = \frac{v_1}{v_2}.$$

It can be seen that this is only possible when $v_2 > v_1$, that is, when the wave front is striking a less dense medium. The limiting value of i is the **critical angle** C, for which we know

$$\sin C = {}_1\mu_2 = \frac{v_1}{v_2}.$$

The construction suggests that refraction is impossible for all angles of incidence greater than the critical angle C. Considered more carefully, since points on the wave front at the surface act as secondary sources, it suggests that while the wave front is totally reflected, an effect *might* be detectable very close to the surface in the second medium, and this is actually found.

Spherical wave fronts and spherical surfaces.—We shall deal with the reflection and refraction of a spherical wave from spherical surfaces ; the simple cases of a plane wave striking a spherical surface or a spherical wave striking a plane surface can easily be deduced from these results.

(*a*) *Reflection at a spherical surface of small aperture.*—Let a point object O lie on the principal axis of a concave mirror with

pole P and centre of curvature C. The aperture of the mirror, LM, is small, so that OL and OM can be regarded as nearly parallel to OP. Let LBM be the trace of a wave front diverging from O and falling on the aperture. At once L and M send out secondary wavelets, and each point of the mirror surface follows in turn as the wave front strikes it. The process of reflection is complete when the wave front reaches P. Consider the reflected wave front at this instant. It will be *spherical*, converging to some point I,

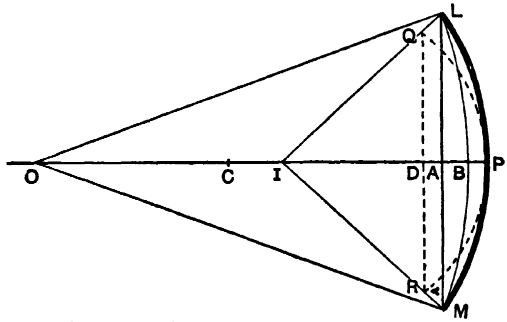


FIG. 197.—Reflection at spherical surface.

enveloping the secondary wavelets given out from the surface. P will be its extreme point, and it will touch the best-developed secondary wavelets from L and M at points Q and R. Let QR cut the axis in D. Let v be the velocity of light, t the time taken to cover the distance BP. Then $BP = vt$. The secondary wavelets, which start from L and M when the incident wave front is at B, will be spheres of radii $LQ = MR = vt$. We are expressly confining our considerations to a surface of small aperture, whence the following approximations can be made :

(1) As LQ is nearly parallel to the axis, $LQ = DA = vt$, and $DQ = LA$.

(2) Both AB and BP are small and BO is nearly equal to PO.

Consider the circular trace with centre O, radius OB.

Then, $LA^2 = AB(2OB - AB) = 2AB \cdot OB$ approximately.

So, $$AB \doteq \frac{LA^2}{2OB}.$$

Similar calculations for the surface of the mirror itself and for the reflected wave front give

$$AP = \frac{LA^2}{2 . CP},$$

$$PD = \frac{QD^2}{2PI} = \frac{LA^2}{2PI}.$$

Now, $$AB = AP - BP,$$

$$PD = AP + AD = AP + BP,$$

whence $$AB + PD = 2AP$$

or $$\frac{LA^2}{2OB} + \frac{LA^2}{2PI} = \frac{2LA^2}{2CP}.$$

Since OB and PO are taken equal, this becomes

$$\frac{1}{OP} + \frac{1}{PI} = \frac{2}{CP},$$

which can be written with the usual notation and convention,

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

With a convex surface a similar discussion follows, except that the wave front meets the pole first. In this figure (Fig. 198) D is

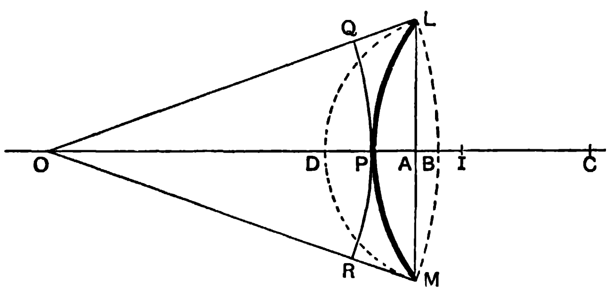


FIG. 198.—Reflection at spherical surface.

the extreme point of the reflected wave front at the instant of completion of the reflection and $PD = vt$.

As $$QL = PD,$$

then $$PB = PD.$$

Now, $$AB = PB - AP,$$

$$AD = PD + AP.$$

So, $$AB - AD = -2AP.$$

Writing as before

$$AB = \frac{LA^2}{2OB}, \quad AD = \frac{LA^2}{2IA} = \frac{LA^2}{2OI},$$

$$AP = \frac{LA^2}{2CP},$$

we get, as OB equals OP very nearly,

$$\frac{1}{OP} - \frac{1}{OI} = -\frac{2}{CP};$$

which becomes, writing v for $(-OI)$, r for $(-CP)$, u for $(+OP)$,

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

(b) *Refraction at a spherical surface.*—Let LPM be the trace of the surface ; let a spherical wave diverge from O on the axis and after refraction appear to diverge from I.

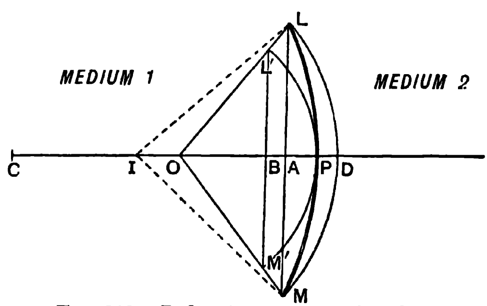


FIG. 199.—Refraction at spherical surface.

Let $L'PM'$ be the trace of the wave front at the instant of first incidence and LDM the trace at the instant when refraction is complete.

While the edge of the wave travels the distance LL' in the first medium, the secondary wave from P goes a distance PD in the second.

Let v_1 and v_2 be the velocities of light in the first and second media and μ_1 and μ_2 their absolute refractive indices.

Then, $$LL' = v_1 t, \quad PD = v_2 t.$$

With similar lettering and the same approximations as before,

$$AB = PB - PA = v_1 t,$$

$$PD = AD - PA = v_2 t.$$

So,
$$\frac{PB - PA}{v_1} = \frac{AD - PA}{v_2};$$

or, since
$$\frac{\mu_1}{\mu_2} = \frac{v_2}{v_1},$$

$$\mu_1 (PB - PA) = \mu_2 (AD - PA).$$

Now,
$$PB = \frac{L'B^2}{2PO} = \frac{LA^2}{2PO}, \quad PA = \frac{LA^2}{2PC}, \quad AD = \frac{LA^2}{2PI};$$

whence
$$\mu_1 \left(\frac{LA^2}{2PO} - \frac{LA^2}{2PC} \right) = \mu_2 \left(\frac{LA^2}{2PI} - \frac{LA^2}{2PC} \right),$$

which, after cancelling, and substituting the usual symbols with their signs, becomes

$$\frac{\mu_2}{v} + \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}.$$

Fermat's Principle. Optical path.—An important generalization, which can be shown geometrically to follow from the laws of reflection and refraction, states that the time taken for light to travel between two given points has a stationary value, usually a minimum. This is Fermat's **Principle of Least Time.** As the wave front at any instant is composed of the *extreme points* of the secondary wavelets from the previous instant, Huygens' construction shows that the direction of propagation is that in which a given distance requires least time for its traversal. "Wave theory" methods of dealing with problems more complicated than those already discussed dispense with the construction and use the principle.

If a distance l is traversed in a medium of absolute refractive index μ in which the velocity of light is v, the time taken is l/v seconds, or $\mu l/c$ seconds if c is the velocity of light *in vacuo*. As this is to be a minimum for the whole wave front it will be constant for all parts of it, so that if different parts of it traverse different media, $\mu l/c$ and hence μl will be the same for each part. The quantity μl is called the **optical path** of the light in the medium of refractive index μ. More generally, for light traversing several

media in succession, the optical path is $\Sigma\mu l$, and for all parts of the wave front $\Sigma\mu l$ is constant.

Analytical treatment of wave motion.—For a full treatment of wave motion reference should be made to works on mechanics. A summary of the results is given here for reference.

The simplest type of transverse wave motion is that in which the particles of the medium execute a simple harmonic motion at right angles to the direction of propagation of the wave.

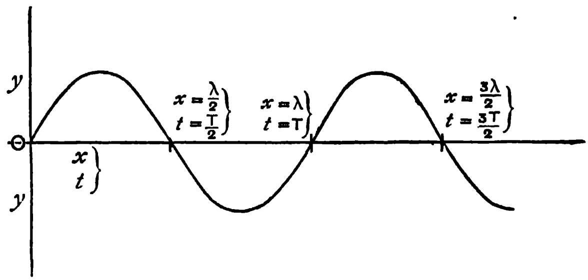


FIG. 200.—Displacement time curve for transverse wave.

An instantaneous picture of such a wave is shown in the figure, the wave travelling in the x-direction while particles of the medium oscillate in the y-direction.

Such a wave may be represented by the equation

$$y=a \sin 2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)$$

for

(a) if x is proportional to t, it represents a disturbance moving outwards with velocity λ/T;

(b) at any given x-position, y executes a simple harmonic oscillation of **amplitude** a and **period** T;

(c) at any given instant t, points on the path of the wave separated by integral multiples of λ have the same value of y; λ is called the **wavelength**.

The term $2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)$ is called the **phase** of the motion.

As has been stated, the wave velocity dx/dt is λ/T; this is the velocity with which the wave travels out through the medium.

The *energy* conveyed to any point in the medium is *proportional to the square of the amplitude* at that point. This may be illustrated by analogy with the transfer of mechanical energy in a material medium. For if the displacement y of a particle of such a medium is given by the wave equation, its velocity dy/dt at time t is

$$\frac{dy}{dt} = -\frac{2\pi}{T} a \cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right). \quad \text{So } \left(\frac{dy}{dt}\right)^2 = \frac{4\pi^2 a^2}{T^2} \cos^2 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

and the maximum kinetic energy of the particle is $\frac{1}{2}m.\frac{4\pi^2 a^2}{T^2}$, if m is its mass, and so proportional to a^2.

The number of waves passing a given point in one second is called the **frequency**, ν. As $\nu = 1/T$, the relation between v, the wave velocity, ν, and λ, is $v = \nu\lambda$. The frequency is really the important physical property of the motion, as it is determined by the source of the waves ; the wavelength depends on the velocity of propagation and will be different in media which the wave traverses with different velocities.

In Chapter XVII we shall consider the resultant amplitude when two such waves of equal amplitude and wavelength, but differing in phase, are superposed. If one has suffered a retardation of δ cm. relative to the other, its equation can be written

$$y' = a \sin 2\pi\left(\frac{x+\delta}{\lambda} - \frac{t}{T}\right).$$

It can be seen that this represents a wave *retarded* by the distance δ, for by time t it has had to travel a distance δ further than the wave for which $x=0$ when $t=0$. This can also be written

$$y' = a \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \frac{2\pi\delta}{\lambda}\right],$$

so that the phase difference for a given path difference δ will be $2\pi\delta/\lambda$.

If $\delta = \lambda$ or any integral multiple of λ, the phase difference is 2π or a multiple of 2π ; if $\delta = \lambda/2$ the phase difference is π, and so on.

The Doppler effect.—Relative motion between source and observer causes a change in the observed wavelength. If they are approaching, more waves reach the observer in any interval of time than would be received if both were at rest ; and if they are

moving apart, fewer. The effect was first remarked on by Doppler in 1842.

The corresponding sound-wave effect is heard as a change of frequency (" pitch "). Now, with sound waves we can easily refer the motion of observer, source, or air to the surroundings, and say which, if any, is fixed for a given relative velocity. With light waves this is really impossible, and the proper treatment is a relativity problem. However, if the relative velocity of source and observer is small compared with the velocity of light, as it is in nearly all optical cases, it is possible to take over the acoustic case as an analogy and obtain a very nearly accurate result by imagining that the observer is fixed and that the light waves are travelling through a stationary medium.

Consider a stationary observer and moving source. Let v be the velocity of light and v' that of the source. Let T seconds be the period of the waves, λ their wavelength. Then $\lambda = vT$. Now in time T the source moves a distance $v'T$ *towards* the observer. The distance by which each wave precedes the next is now not vT but $(v-v')T$, so that the observed wavelength is

$\lambda' = (v-v')T$; or, since $T = \lambda/v$, $\lambda' = \dfrac{v-v'}{v}\lambda$.

Rearranging, $\lambda' - \lambda = -(v'/v)\lambda$. If the source is receding from the observer with velocity v', then the sign of v' is changed, and

$$\lambda' - \lambda = +\frac{v'}{v}\lambda .$$

Thus, spectral *lines*, each of which is characterised by a definite true wavelength λ (p. 255), are shifted towards the violet end of the spectrum if the source is approaching, and towards the red end if it is receding. Since the shift $\lambda' - \lambda$ is proportional to λ, it will be different for different lines in the same spectrum, being nearly twice as great for a red line as for a violet line. The *proportional* change in wavelength, $(\lambda'-\lambda)/\lambda$, is the same in all parts of the spectrum. Since $(\lambda'-\lambda)/\lambda = \pm v'/v$, we can estimate the least value of v' to give a measurable shift in an ordinary spectroscope. Supposing this can just separate the two sodium D lines, which differ by about 1 part in 1000, then the least detectable shift $(\lambda'-\lambda)$ is such that $(\lambda'-\lambda)/\lambda$ is 10^{-3}, whence

$$v' = 10^{-3} \times v = 3 \times 10^{8} \text{ cm./sec.}$$

QUESTIONS ON CHAPTER XIV

1. Light from a point source falls on the plane surface of a transparent medium of refractive index μ. Discuss the ensuing phenomena from the point of view of the wave theory of light. (O. & C.)

2. Obtain from the wave theory of light an equation connecting the distances of a point object and its image from a mirror of given curvature. Derive also from the wave theory the relation between the distances of object and image from a curved refracting surface separating two media of refractive indices μ_1 and μ_2. A thin clock-glass of radius of curvature 1 ft. floats with convex side downwards on water of refractive index $\frac{4}{3}$. Find the apparent position of an object seen through the centre of the glass and distant 2 ft. below it. (O.H.S.C.)

3. Explain, on the wave theory of light, the behaviour of a slender pencil of light proceeding from a point source when it meets a plane surface separating two transparent media. Deduce the relation between the refractive index and the velocities of light in the two media. (C.H.S.C.)

4. How did Huygens explain the reflection of light on the wave theory ? Using Huygens' conceptions, show that a series of light waves diverging from a point source will after reflection at a plane mirror appear to be diverging from a second point, and calculate its position. (C.H.S.C.)

5. Show how the wave theory accounts for (*a*) the refraction, (*b*) the total reflection, of light at a plane surface. What evidence in favour of the wave theory has been obtained by experiments on the velocity of light ? (C.H.S.C.)

6. Give an account of the Doppler effect in the propagation of sound and light. If 5×10^{-9} cm. is the smallest difference of wavelength which can be detected by a spectrometer in the neighbourhood of the wavelength 6×10^{-5} cm., what is the smallest velocity of a star relative to the earth which can be measured ? (O. & C.)

7. What do you understand by Doppler's principle ? Show how the observed frequency of waves will differ from the frequency at which they are emitted if their source is approaching a moving observer. A spectroscopic examination of the light of a certain star shows that the apparent wavelength of a certain spectral line is 5001×10^{-8} cm., whereas the observed wavelength of the same line produced by a terrestrial source is 5000×10^{-8} cm. In what direction and at what speed do these figures suggest that the star is moving relative to the earth ? (O.H.S.C.)

CHAPTER XV

VELOCITY OF LIGHT

Astronomical methods. Römer and Bradley.—Römer in 1673 determined the velocity of light from observations on the eclipses of the satellites of Jupiter. Jupiter has four principal and seven very small satellites. Of the four principal satellites, that nearest to the planet has a period of 1 d. 18 h. 28 m. while the fourth has a period of 16 d. 16 h. 32 m. All revolve in orbits nearly parallel to the plane of Jupiter's orbit, so each satellite is eclipsed once in every revolution as it enters the shadow of Jupiter. Jupiter's "year" is 11·86 terrestrial years. The four chief satellites were known in Römer's time; he observed the eclipses of one with a period of rotation of about 42 hours.

In the figure, E_1 and J_1 represent the positions of the earth and Jupiter when they are at their closest distance, **in conjunction.** After 0·545 year they will be as at E_2 and J_2, **in opposition.** The principle of the method is best explained by supposing that the true period of revolution of one of the satellites is known. Reckoning from the position E_1J_1, and calculating the times of eclipses about six months later, they are found in the position E_2J_2 to be about $16\frac{1}{2}$ minutes or, say, 1000 seconds late. The diameter of the earth's orbit is about 186,000,000 miles, so that if the 1000 seconds represent the time taken by the light to travel across the earth's orbit the velocity should be about 186,000 miles per second, or about 3×10^{10} cm./sec.

The interval between successive eclipses is longer than the average when the earth is receding from Jupiter, and less when the earth is moving towards the planet, a state of affairs analogous to the "Doppler effect" change of frequency to a moving observer discussed in the last chapter. This observation was

what first led Römer to the conception that light travelled with a finite velocity. Actually the determination was made without calculating the true period.

Observations were made of the time T_1 for the n eclipses occurring between the positions E_1J_1 and E_2J_2, and of the time T_2 for the n eclipses occurring between E_2J_2 and E_3J_3 when the earth and Jupiter are closest again.

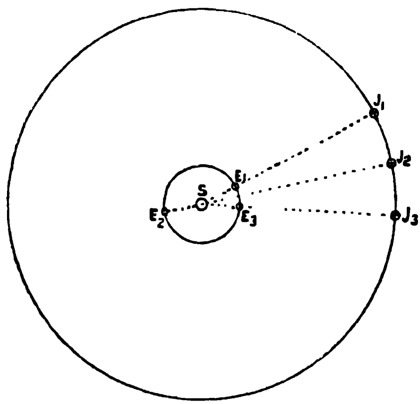


FIG. 201.—Römer's determination.

Let t be the true interval between eclipses, d the diameter of the earth's orbit, and v the velocity of light. Then T_1 is greater than nt by the time taken for light to cross the distance E_1E_2 and T_2 is less than nt by the time taken by light to cover E_2E_3.

So,
$$T_1 = nt + d/V,$$
$$T_2 = nt - d/V,$$
and
$$T_1 - T_2 = 2d/V.$$

$T_1 - T_2$ was found to be 33 minutes, giving $d/V = 16\frac{1}{2}$ minutes.

Contemporary objections to Römer's result were chiefly against the assumption that the satellites actually revolved uniformly, which there was then no method of testing. Its *accuracy* depends on the certainty with which the diameter of the earth's orbit is known; this requires the determination of the radius of the earth, and of the angle this radius subtends at

the sun, both of them difficult astronomical measurements only made roughly in Römer's time.

Bradley in 1726 was able to calculate the velocity of light from the apparent displacement or aberration of the fixed stars, which he discovered. The observing telescope is not at rest, but moving with the velocity of the earth at about $18\frac{1}{2}$ miles per second. Now suppose the telescope is moving in direction AB with a velocity v, and light is coming from a star whose true altitude is θ with velocity V. The velocity of the light relative to the telescope is found by the ordinary rules—compounding V with a velocity v in the reverse direction BA. If OC represents V in size and direction, OD represents v reversed, then the diagonal OE of the parallelogram $OCED$ represents the velocity of the light relative to the telescope. So the telescope will have to be pointed in the direction EO instead of along CO to receive an image of the star on its crosswire.

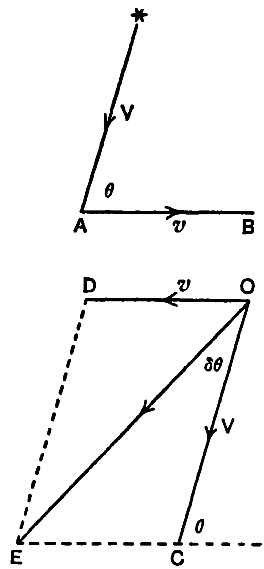


FIG. 202.—Bradley's aberration method.

In Fig. 202, $$\frac{OE}{\sin\theta}=\frac{CE}{\sin\delta\theta},$$

or as OE and OC are nearly equal and $\delta\theta$ is small,

$$\frac{OC}{\sin\theta}=\frac{CE}{\delta\theta}$$

or $$\frac{V}{\sin\theta}=\frac{v}{\delta\theta},$$

whence $$\delta\theta=\frac{v}{V}\sin\theta.$$

The greatest aberration is thus observed when $\sin\theta=1$, when $\delta\theta$ is about 0·0001 radian, giving a value for V of about 185,000 miles per second if $v=18\frac{1}{2}$ miles per second.

Fizeau's method.—AB is essentially an astronomical telescope and W is a toothed wheel whose rim is at the principal focus, F,

of the objective, O (Fig. 203). Light from a source S is reflected at the surface of the glass plate, G, and brought to a focus at F, whence it will emerge from the objective as a parallel beam. This beam, after traversing a distance of several kilometres, falls on a reflector consisting of a converging lens, L, which brings it to a focus on the surface of the concave mirror, R. In order to align the axes of telescope and reflector the latter is temporarily converted into a telescope by substituting an eyepiece with cross-wires for R.

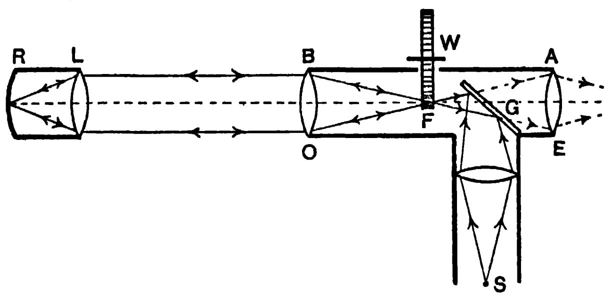


FIG. 203.—Fizeau's toothed-wheel method.

The optical centre of L is at the centre of curvature of R, and so the incident beam of parallel light is returned parallel and falls on O, which forms a real image of the source at F. This is viewed through the glass plate, G, by the eyepiece, E.

As the wheel rotates, its teeth interrupt the beam at F. If it rotates slowly, then the eye at E sees the image appear and disappear as spaces and teeth pass successively before E. As the speed is increased, flickering ceases and a continuous impression is received of an image less bright than that observed directly. If teeth and spaces are of equal width and the teeth blackened and bevelled to prevent reflection there is complete extinction for certain speeds of rotation. This occurs when the light goes from F to R and back in the time taken for a tooth to succeed a space in the position F.

Fizeau's wheel had 720 teeth and the length of the base line between telescope and reflector was 8633 metres. He found that extinction first occurred when the wheel rotated 12·6 times per second. The time taken for the light to travel 2×8633 metres

was thus $\frac{1}{2 \times 720}$ of a revolution or $\frac{1}{12 \cdot 6 \times 2 \times 720}$ seconds, so that the velocity of light in air was $2 \times 8633 \times 12 \cdot 6 \times 2 \times 720$ metres per second, or $3 \cdot 13 \times 10^{10}$ cm./sec.

Foucault's method.—Light from a slit S falls on the converging lens L and afterwards on a plane mirror R, which rotates rapidly about a vertical axis. A concave spherical mirror is placed so that its axis is perpendicular to the axis of rotation of R and its

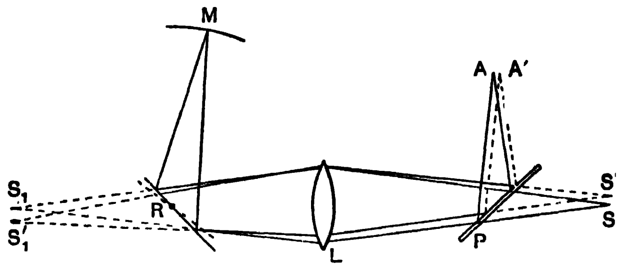


FIG. 204.—Foucault's rotating-mirror method.

centre of curvature lies on this axis. L is adjusted to give an image of S on M, so that the reflected light appears to diverge from this image, strikes the mirror R again, and is brought to a focus at S'. Suppose the mirror R at rest. Then S' and S coincide. Now suppose the mirror rotates. Then during the time taken for light to go from R to M and back again, the mirror R has rotated through an appreciable angle θ. The reflected pencil will not now traverse exactly the same path from R to L, and S' will be displaced in the direction of rotation of the mirror. A piece of plane glass placed at 45° to the axis of the beam reflects part of the light on its return journey and an image of S will be observed at A when R is at rest and at A', distant x from A, when R rotates. The distance x is measured with a micrometer eyepiece.

Let $SL = a$, $LR = b$, $RM = D$. Let the small angle turned through by the mirror in the time taken for light to go from R to M and back be θ radians. Now the light returning from M is reflected from R and appears to come from a point at a distance D behind R, so that the pencils forming the images at S and S' appear to come from sources S_1 and S_1' behind R.

The angle through which the reflected rays are turned is twice the angle moved through by the mirror, or 2θ. So $S_1S_1' = 2D\theta$.

Now, considering L, the angles subtended at its optical centre by S_1S_1' and SS' must be the same.

So, $$\frac{SS'}{a} = \frac{S_1S_1'}{b+D},$$

whence $$SS' = \frac{a \,.\, 2D\theta}{b+D} = x.$$

It remains to express θ in terms of the angular velocity of the mirror and the velocity of light. Let the mirror make n revolutions per second. Its angular velocity is $2\pi n$ radians per second. The time taken for light to make the return journey between R and M is $\frac{2D}{V}$ seconds.

So, $$\theta = 2\pi n \,.\, \frac{2D}{V} = \frac{4\pi n D}{V}.$$

Substituting this value for θ in the previous equation,

$$x = \frac{8\pi n a D^2}{V(b+D)}$$

or $$V = \frac{8\pi n a D^2}{x(b+D)}.$$

The revolving mirror was driven by an air turbine, and its speed measured by a stroboscopic method.

The actual apparatus differed from that shown in Fig. 204. The distance RM was 20 metres, but four fixed mirrors were placed between R and M, so that the apparatus was only about one-fifth of this length. The lens L was placed between R and the first of these mirrors. The displacement observed was small—about 0·7 mm. The result obtained was 298,000 kilometres per second.

The principle of the method involves the assumption that the ordinary laws of reflection hold for oblique reflection at a *rapidly rotating mirror*, and for an image sweeping across a fixed mirror with a *transverse velocity comparable with that of light*. The apparatus was improved by Michelson, who placed the lens L between R and M to obtain a more brilliant reflected image and enable D to be increased.

With an apparatus of laboratory dimensions, Foucault was able to determine whether light travelled with greater or less speed in dense media than in air and so apply the "experimentum crucis" to the two theories of light. A second mirror, M', of the same radius of curvature as M was added, and a tube of water placed between R and M'. Two final images were thus viewed, and that formed by light which had traversed the water was more displaced, showing that *light travelled more slowly in water than in air.* Michelson, with similar apparatus, showed that $\dfrac{\text{velocity in air}}{\text{velocity in water}}$ was equal to 1·33—in good agreement with the value of the refractive index. These experiments are decisive in favour of the wave theory.

Wave velocity and group velocity.—Michelson found that the velocity of yellow light was 1·76 times greater in air than in carbon bisulphide. The corresponding refractive index of carbon bisulphide is 1·64, and this is the value to be expected for the ratio of the velocities.

If $y=a\sin 2\pi\left(\dfrac{x}{\lambda}-\dfrac{t}{T}\right)$ represents a simple harmonic wave of wavelength λ, then the wave velocity for wavelength λ is given by $v=\lambda/T$. Now what is observed in all the methods involving the *observations of a signal* is the velocity with which the *signal*, consisting of a sharply bounded group or train of waves, is propagated. This **group velocity** is not the same as the **wave velocity.**

For, consider two waves of wavelengths λ and $\lambda+d\lambda$ and periods T and $T+dT$, with the same amplitude a.

The displacements in the medium at time t for the two waves are given by

$$y_1=a\sin 2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)$$

and

$$y_2=a\sin 2\pi\left(\frac{x}{\lambda+d\lambda}-\frac{t}{T+dT}\right).$$

The resultant displacement is given by

$$y=y_1+y_2=a\sin 2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)$$
$$+a\sin 2\pi\left(\frac{x}{\lambda+d\lambda}-\frac{t}{T+dT}\right).$$

This resultant disturbance is the signal that is propagated. The right-hand side becomes

$$2a \sin \pi \left[\left(\frac{x}{\lambda} + \frac{x}{\lambda + d\lambda}\right) - \left(\frac{t}{T} + \frac{t}{T + dT}\right)\right] . \cos \pi \left[\left(\frac{x}{\lambda} - \frac{x}{\lambda + d\lambda}\right) - \left(\frac{t}{T} - \frac{t}{T + dT}\right)\right];$$

which is $$2a \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right) \cos \pi \left(\frac{xd\lambda}{\lambda^2} - \frac{tdT}{T^2}\right),$$

or $$2a \cos \pi \left(\frac{xd\lambda}{\lambda^2} - \frac{tdT}{T^2}\right) \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right).$$

So that superposed on the original sine wave travelling with velocity λ/T is a change in amplitude of wavelength $\frac{\lambda^2}{d\lambda}$ and period T^2/dT, whose velocity must be $V' = \frac{\lambda^2}{T^2} \cdot \frac{dT}{d\lambda}$.

Now, $$\frac{\lambda}{T} = V, \quad \text{so} \quad T = \frac{\lambda}{V}$$

and $$\frac{dT}{d\lambda} = \frac{V - \lambda \frac{dV}{d\lambda}}{V^2},$$

whence $$V' = V^2 \left(\frac{V - \lambda \frac{dV}{d\lambda}}{V^2}\right)$$
$$= V - \lambda \frac{dV}{d\lambda}.$$

Carbon bisulphide is *highly dispersive* and so $dV/d\lambda$ is considerable in this case.

If the wave velocity V is calculated from the refractive index and $\lambda dV/d\lambda$ from the dispersive power in the yellow part of the spectrum, the value of V' works out in close agreement with Michelson's measurement.

All the experiments except that of Bradley really measure V'. But as $dV/d\lambda$ is negligible in air and zero in interstellar space, the distinction is not important.

Michelson's determination.—In 1926 Michelson made what is probably the most accurate determination of V. In place of the single rotating mirror he used an equiangular prism, A, with polished steel faces. Several prisms were used in different experi-

ments, one with 8 faces and others with 12 and 16. The distance D, covered twice by the light between its two reflections from A, was 35·4245 kilometres. The number of revolutions of A per second, n, was so adjusted that in the case of the 8-sided prism an angle of 45° was turned through in time $\frac{2D}{V}$.

Then, $\frac{1}{8n}=\frac{2D}{V}$ and $V=16\,Dn$.

The slit C was illuminated by an arc, and light after falling on one face of A was reflected from the plane mirrors D and E on

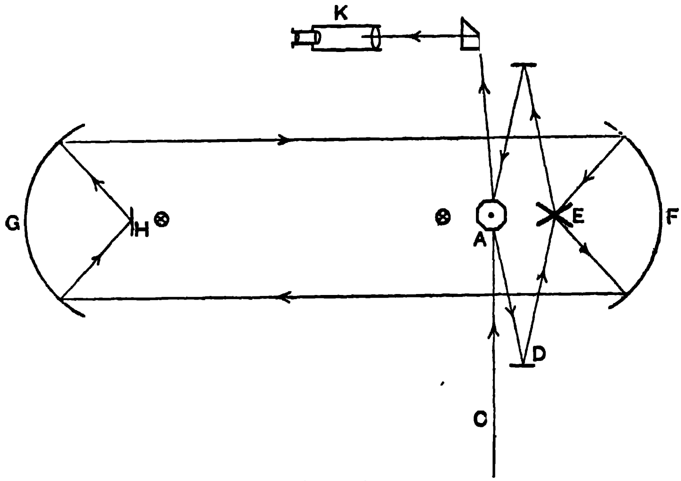


FIG. 205.—Michelson's determination.

to the concave mirror F of 61 cm. aperture and 930 cm. focal length, the distances of C and D being so arranged that F reflects the light from C as a parallel beam. F was set up on Mount Wilson, and another mirror of the same dimensions, G, on Mount St. Antonio. At the principal focus of G an image of C was formed on a small concave mirror H of 930 cm. radius of curvature, and the rays reflected back to G were returned as a parallel beam to F, after which they were reflected from the plane mirrors E and J on to another face of A.

An image of the slit was formed on the crosswire of the micrometer, K. If the octagonal mirror had rotated through exactly

45° in time $2D/V$, then the image would be in exactly the same position as if the mirror were stationary. If the angle differed from 45° by a small amount $\delta\theta$ the rays leaving A for the second time were rotated through an angle $2\delta\theta$, and from the displacement of the image this small correction was calculated.

Michelson's result is given as $2{\cdot}99796 \pm 0{\cdot}00004 \times 10^{10}$ cm./sec. for the velocity of light *in vacuo*.

Conclusion.—The **astronomical methods** give the velocity of light in *empty space* directly. **Terrestrial** methods of course measure the velocity *in air*, from which the velocity *in vacuo* is obtained by multiplying by the refractive index of air at the temperature and pressure of the experiment.

The velocity of light *in vacuo* is a constant of great importance, usually denoted by the symbol c. The following are some of the more important calculations into which it enters:

(1) *Frequency calculations.*—In spectroscopic theory calculations are made in terms of the vibration frequency ν, while all direct optical measurements determine the wavelength λ. The relation between the two is $c = \nu \times \lambda$, the *wavelength in vacuo* being understood by λ. If the wavelength λ is determined in a medium of refractive index μ the relation is $c/\mu = \nu \times \lambda$.

(2) *Energy and momentum calculations.*—The energy, E, conveyed by a given quantity of radiation can be regarded as due to a mass E/c^2 moving with velocity c. The momentum conveyed by it is thus E/c.

(3) *Variation of mass with velocity.*—According to the theory of relativity the effective mass m of a body of mass m_0 moving with a velocity v comparable with c is $m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$.

(4) *Relation between the two systems of electrical units.*—One absolute electromagnetic unit of charge is equal to c absolute electrostatic units. One absolute electromagnetic unit of potential difference equals $1/c$ absolute electrostatic units. Corresponding relations involving some power of c hold for the units ll quantities measured in the two systems. The value of c d by measurements of an electrical magnitude on both son's value ; this is part of the agnetic character to light.

; mirror method was improved 1929–33), using a much shorter

optical path in an evacuated tube about a mile long. It was believed that a straight mile could be measured more accurately than a larger distance, and as the velocity of light *in vacuo* was required, it was more satisfactory to work nearly *in vacuo* than to apply uncertain corrections to observations in the open air.

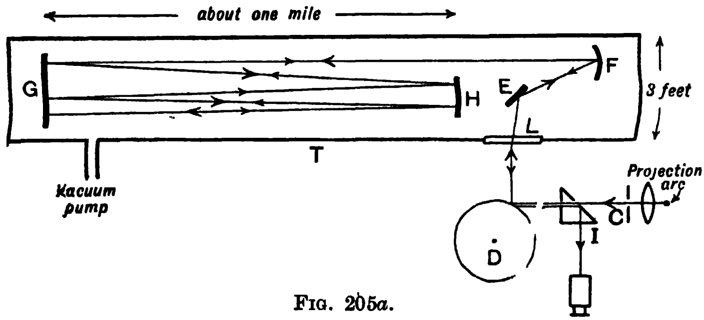


FIG. 205*a*.

The apparatus is shown in Fig. 205*a*. The 32-sided rotating mirror D, driven by an air turbine, reflected a beam from the slit C through a plane glass window L to the plane mirror E and thence to the concave mirror F. The beam was then reflected several times between the plane mirrors G and H, the focusing of F being arranged so that a conjugate image of C appeared either on G or on H, whence the beam retraced its path, a little below the original path, and emerged to strike the lower part of D, thence passing through the prism I to the observing eyepiece. The mile-long vacuum tube T, 3 feet in diameter and made of corrugated steel sheet, was evacuated to a pressure of between 0·5 and 5 mm. of mercury by a large pump driven by a 15 H.P. motor.

The method of measurement is essentially the same as in the 1926 experiment. If the return beam appears, when D makes N revs./sec., to be in the same position as when D is at rest, then the time taken for the light to cover the return optical path of total length d is $\frac{1}{32N}$ sec., or a whole multiple of this. The speed of rotation was selected, and measured with the aid of a specially constructed standard tuning fork, using the previously obtained values of c as a guide. The mean of nearly three

45° in time $2D/V$, then the image would be in exactly the same position as if the mirror were stationary. If the angle differed from 45° by a small amount $\delta\theta$ the rays leaving A for the second time were rotated through an angle $2\delta\theta$, and from the displacement of the image this small correction was calculated.

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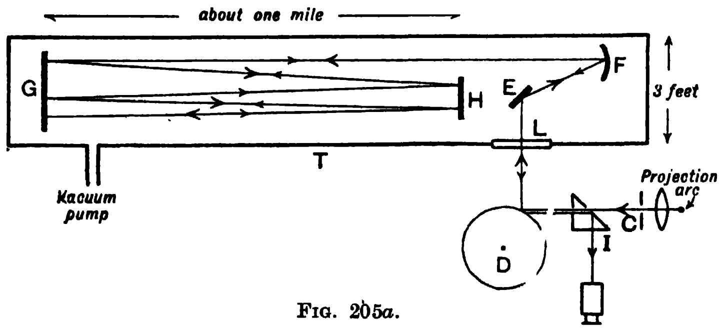
More recent work.—The rotating mirror method was improved by Michelson, Pease, and Pearson (1929–33), using a much shorter

optical path in an evacuated tube about a mile long. It was believed that a straight mile could be measured more accurately than a larger distance, and as the velocity of light *in vacuo* was required, it was more satisfactory to work nearly *in vacuo* than to apply uncertain corrections to observations in the open air.



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thousand separate determinations gave, for the velocity of light *in vacuo*, the value 299,774 km./sec., with a probable error later estimated as ± 4 km./sec.

Kerr cell methods.—It is advisable to refer to Chapter XIX before reading this section.

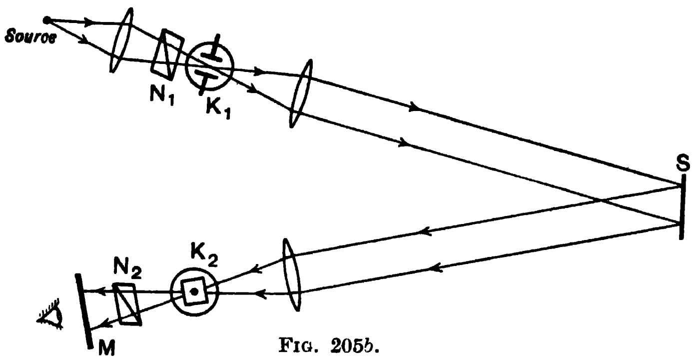


Fig. 205*b*.

The Kerr cell, used in these experiments, consists of two electrodes immersed in nitrobenzene. When an electric field is applied between the electrodes, the liquid becomes doubly refracting; and if a Kerr cell is placed between crossed nicols, the arrangement transmits light only while the field is applied. An alternating voltage applied across the cell should thus cause intermittent transmission of a beam of light directed through it, with results exactly similar to the succession of teeth and gaps in Fizeau's experiment. With modern radio technique, it is possible to obtain, and measure accurately, very high frequencies of alternation. For example, the frequency corresponding to a radio wavelength of 30 metres is very nearly 10^7 cycles per second. A nicol-Kerr cell arrangement in which the output of a short-wave radio oscillator is applied across the cell should thus enable the Fizeau experiment to be repeated with very much shorter optical paths. Some early experiments of this kind were done, but the difficulty of observing complete extinction, a defect of Fizeau's method, was not overcome. The later work of Karolus and Mittelstaedt (1928) and of Anderson (1937–41) avoided this defect.

Fig. 205*b* indicates the apparatus of Karolus and Mittelstaedt. Radiofrequency alternating voltage, of frequency n (which could

be varied) of the order 3 to 7 million cycles per second, was applied synchronously to the two Kerr cells K_1 and K_2, placed so that their electric fields are at right angles to one another. The nicol N_1 was set so that plane polarised light, polarised in a plane at 45° to the cell's electric field, fell on K_1, and the beam emerged from K_1 elliptically polarised. This beam, after reflection from the distant mirror S placed at the end of a base line 41 metres long, or after repeated reflections from intermediate mirrors to give

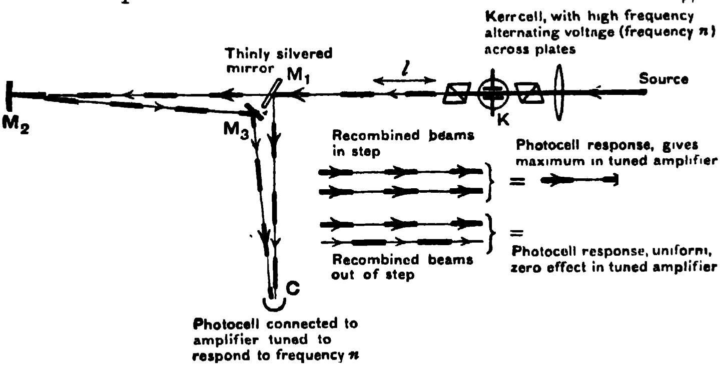


FIG. 205c.

optical paths up to 322 metres, fell on K_2. If the time taken for the light to travel between K_1 and K_2 be a whole multiple of $1/n$ sec., the state of K_2 on its arrival is the same as that of K_1 on its departure, but as the cells are " crossed " the elliptical polarisation is destroyed, and a plane polarised beam emerges from K_2. This beam fell on the nicol N_2, crossed with respect to N_1, and if the above condition was satisfied, complete extinction was observed on the screen M. The frequency was altered until this extinction occurred, and n then measured. Then the velocity of light in air is given by the length of the optical path divided by the appropriate multiple of $1/n$. The result obtained in 1928 for the velocity of light *in vacuo* was 299,778 $\pm$ 20 km./sec.

Fig. 205c shows the principle of the two methods used by Anderson, though both this and the following account have been very much simplified. A Kerr cell K placed between crossed nicols has a steady voltage applied across it so that light is always transmitted through the system. Superposed on this

steady voltage is the output of an oscillator of fixed frequency n, about 14 million cycles per sec. The intensity of the transmitted beam is thus modulated (*i.e.*, waxes and wanes) 14 million times per second, and this is portrayed in the figure by equally spaced thick lines (enhanced intensity) and thin lines (diminished intensity). The " thick lines " are separated in time by one fourteen-millionth of a second ; how far are they apart in space? If the modulated beam falls on a photocell connected to a radio amplifier *tuned to respond to frequency* n, the latter records only the extent of the *variation* in strength, not the absolute intensity of the beam. This is the key point of the method.

The modulated beam falls on a thinly silvered mirror M_1, which reflects part directly to the photocell C, and transmits the rest to the movable mirror M_2, whence it is reflected by M_3 to rejoin the other beam at C. The amplifier response depends on the path or phase difference between the two beams. If they are exactly in step, they reinforce one another, and the amplifier records maximum response. If they are exactly out of step, one waxes as the other wanes, the effect on the cell is that of an unvarying beam, and the amplifier shows no effect, since it is tuned to record *variations*. Let l be the separation in space of the bright intervals (thick lines). Then, if the difference in paths between the distances $M_1 M_2 M_3 C$ and $M_1 C$ is a whole multiple of l, the two beams are in step, while if this difference differs by $l/2$ from such whole multiple, they are exactly out of step. The mirror M_2 is moved until the amplifier response is a minimum, and these two distances are then measured. As c is already approximately known, there is no doubt as to the whole number concerned. Thus l is found, and as the time interval to cover this distance is $1/n$ sec., the velocity is obtained by dividing l by $1/n$. In Anderson's earlier experiments, with $n = 14 \times 10^6$/sec., a distance of about 159 metres gave $7{\cdot}5\,l$. His final result, published in 1941, was $299{,}776 \pm 14$ km./sec., corrected to vacuum.

Great interest has centred for the past fifteen years on the question whether c is really constant in value, or whether the value is decreasing steadily as time goes on. It is true that the values obtained by recent experiments are on the average about 100 km./sec. less than those of the 19th century, and in all the

experiments done since 1928 attempts have been made to detect a change with time. The present position has been summed up by R. T. Birge, who states that the results of all recent experiments are consistent with a constant value of c, this value being 299,776 $\pm$4 km./sec.

QUESTIONS ON CHAPTER XV

1. Explain fully how the velocity of light has been determined from astronomical observations.

Calculate the ratio of the longest to the shortest interval between successive eclipses of one of Jupiter's moons, being given that the orbital velocity of the earth is v cm./sec. and the velocity of light is V cm./sec. (L.H.S.C.)

2. Draw a diagram showing the arrangement of the apparatus and the path of the rays of light in Fizeau's toothed wheel method for measuring the velocity of light. What are the chief difficulties met with in carrying out the experiment ?

If the wheel has 150 teeth and 150 spaces of equal width and its distance from the mirror be 12 kilometres, at what speed, in revolutions per minute, will the first eclipse occur ? (N.U.J.M.B.H.S.C.)

3. Give an account of some method by which the velocity of light has been determined. How has it been shown that the velocity of a light-signal in water is less than its velocity in air ? (O. & C.H.S.C.)

4. Describe in detail *one* method by which the velocity of light has been determined.

Show how the refraction of light from air into water was explained by the emission and by the wave theory of light, pointing out which theory seems the more in agreement with fact. (O.H.S.C.)

5. Describe carefully Fizeau's method of determining the speed of propagation of light by means of a toothed wheel.

Given that the distance of the mirror is 8000 yards, that the revolving disc has 720 teeth, and that the first eclipse occurs when the angular velocity of the disc is $13\frac{3}{4}$ revolutions per second, calculate the speed of propagation of light. (C.W.B.H.S.C.)

6. Give a brief account of the wave theory of light.

Describe *one* method of determining its velocity of propagation. (C.W.B.H.S.C.)

7. Describe Foucault's (rotating mirror) method of measuring the velocity of light, and the effect upon the displacement of the image of putting a column of water between the rotating and the fixed mirrors. (L.H.S.C.)

8. Describe in some detail a method by which the velocity of light has been determined. Mention *briefly* any other methods which have been used, and state which you consider the most accurate method. Indicate *briefly* why an accurate knowledge of this constant is of importance in physics. (O. & C.H.S.C.)

CHAPTER XVI

SPECTRA. THE SPECTROMETER. COLOUR

The complete spectrum.—In Ch. XX and in works on other branches of physics accounts will be found of **X-rays, γ-rays, ultra-violet light, infra-red radiation or radiant heat,** and **electromagnetic waves.** All these, together with visible light, are radiations of the same nature—**electromagnetic vibrations** in the **ether.** All travel in empty space with the same velocity and all under suitable experimental conditions exhibit the characteristics of a transverse wave motion, which we shall study in later chapters. The great differences in their physical behaviour are attributable to differences of **frequency**; provided that there is no change of medium, the wavelength is inversely proportional to the frequency, so that it is often said that the characteristics of the radiations depend on their wavelengths. The frontispiece gives an outline of the more important characteristics and the order of the wavelength range in centimetres. For wavelengths in and near the visible region of the complete spectrum more convenient units are the **Ångström unit** (Å.U.), 10^{-8} cm., and the **micron** (μ), 10^{-4} cm.

The visible spectrum. Production of a pure spectrum.—Newton's first experiments with a prism produced a spectrum from sunlight, using a beam admitted through a small hole. The spectrum, spread out in the plane of a principal section of the prism, was blurred as the patches of light produced by the individual colours overlapped. The effect can be reproduced by looking at a source of light through a prism, though in this case the blue end of the spectrum appears nearest to the vertex of the prism.

Using a narrow slit parallel to the refracting edge of the prism

the blurring is reduced. An additional improvement is to use a lens to give a sharply focussed series of images of the slit S_1 on

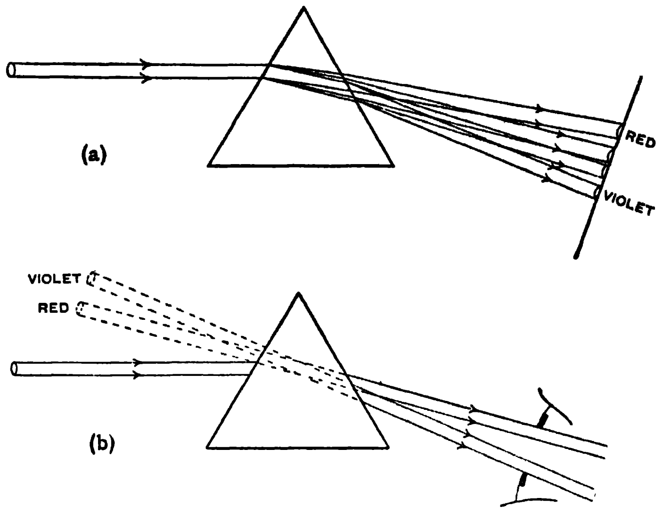


FIG. 206.—Impure spectrum.
(*a*) On screen. (*b*) Viewed through prism.

the screen S_2. Complete sharpness at S_2 is not possible for more than one colour at a time with this arrangement, as the different

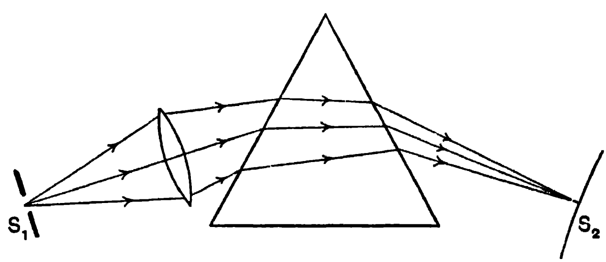


FIG. 207.—Approximation to pure spectrum.

colours have paths of different lengths in the glass. The best results are obtained with the prism set so that the middle of the spectrum undergoes minimum deviation. The other parts of the

spectrum are then near their minimum deviations and their paths through the prism differ least.

A converging pencil of rays striking a prism is made to converge less if the angle of incidence of the central ray is greater than that for minimum deviation, and converges more if the angle of incidence is less. The only way to avoid the focussing difficulty this causes, and obtain *all parts of the spectrum in focus at once*, is to use a **parallel incident beam** and bring the various emergent beams to a focus with a converging lens. It is not *essential* in this case that the prism shall be at minimum deviation for any colour, for a parallel beam will always emerge as a parallel beam, though the prism is usually set at minimum deviation for the middle of the spectrum.

The usual method for the production of a **pure spectrum** is then, as shown in Fig. 208:

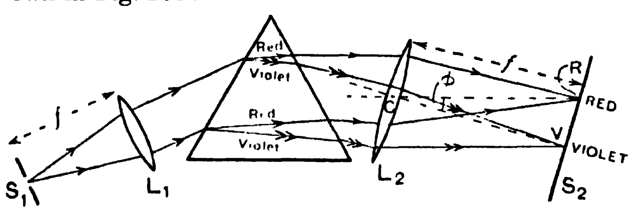


FIG. 208.—Production of pure spectrum; principle of prism spectrometer.

(1) *A narrow slit* S_1 *at the principal focus of a converging lens which should be achromatic.*

(2) *The prism set at minimum deviation for the middle of the spectrum.*

(3) *The emergent beams brought to a focus by an achromatic objective giving a series of images of the slit at* S_2.

With this arrangement the spectrum produced is a series of sharp images of the slit, practically free from overlapping.

The spectrometer.—Fig. 208 embodies the chief features of the spectrometer and Fig. 209 shows the principles of its mechanical arrangement. S_1 is an adjustable slit mounted with L_1 in a tube called the **collimator**, which is fixed to the main frame of the instrument. For visual observation L_2 is made the objective of a **telescope**, crosswires are placed at S_2, and a **Ramsden eyepiece**

(not shown) used to view the spectrum. The telescope carries two verniers, each reading to 1′. The prism is mounted on a table provided with levelling screws, which may be clamped firmly to a 360° scale graduated in degrees and thirds of a degree.

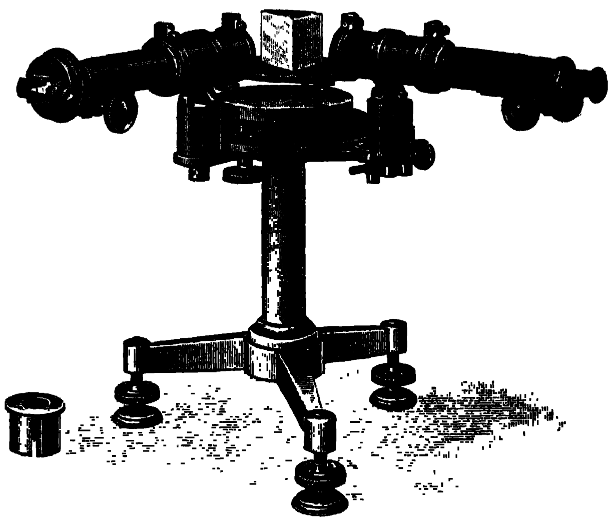


FIG. 209.—A Laboratory spectrometer.
The collimator is on the left of the diagram: the telescope on the right.

The **prism table** and scale are clamped to the main frame of the instrument during most of the observations, and the telescope, with its attached verniers, is free to rotate. During some stages of adjustment the telescope and vernier are clamped to the frame of the instrument and the prism table and scale are free to rotate.

The optic axes of telescope and collimator must intersect on the common axis of rotation of prism and telescope and be perpendicular to this axis. The instrument is usually supplied in this condition. The adjustments usually performed by the student are the *focussing of telescope and collimator*′ and the *levelling of the prism.*

Focussing.—The eyepiece of the telescope is focussed on the crosswires. The telescope is then focussed on a distant object. It is next turned to receive direct light from the collimator, which is focussed until the image of the slit seen in the telescope is sharp. The telescope has been set to receive parallel light, so the collimator now gives it. This method is frequently impracticable and a better method is that due to Schuster.

In **Schuster's method** the telescope is set to receive light at *greater deviation than the minimum* and the slit illuminated with monochromatic light. Two positions of the prism will then give images on the crosswires—one with an *angle of incidence greater* than that for minimum deviation (when the emergent beam will be *more diverging*), and the other with an *angle of incidence less* than that for minimum deviation (when the emergent beam is *more converging* than the incident beam).

In the first position, the telescope is focussed until the best image is obtained; turning the prism into the second position reduces the sharpness, which is restored by focussing the collimator. The two operations are repeated until the image of the slit appears equally sharp in both positions.

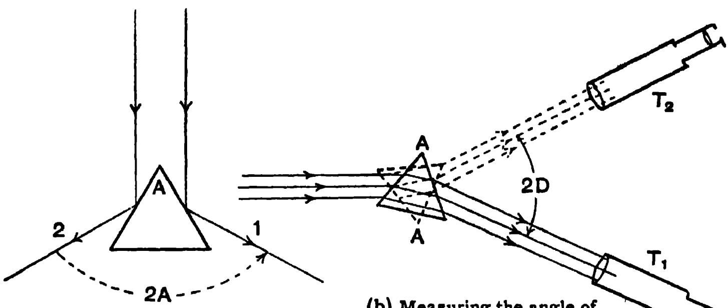
Levelling the prism.—Telescope and collimator are fixed at an angle of about 120° and the prism rotated, so that each of the faces to be used in turn reflects light into the telescope. Levelling screws adjust the top of the table, so that the image in each case is in the centre of the telescope's field.

Experiments with the spectrometer.—(*a*) *Finding refractive index by the method of minimum deviation.*—A sodium flame serves as a monochromatic source of light. The angle A of the prism is measured by allowing the beam from the collimator to be split by the edge at A and picking up the two reflected beams in the directions (1) and (2). The angle between the two positions of the telescope is $2A$, whatever the position of the prism. Alternatively, the telescope may be clamped at an angle of about 120° to the collimator and the prism table, with scale clamped to it, rotated. Readings of the vernier are taken in the two positions at which an image is seen by reflection at the two faces in turn, and the angle turned through by the prism between these two positions is $(180° \pm A)$.

To find the angle of minimum deviation D, the scale is clamped to the frame and the prism table left free. Readings of the telescope's position in the two minimum deviation positions T_1 and T_2 are taken, and D is half the angle between these positions. From the formula

$$\mu = \frac{\sin\frac{(A+D)}{2}}{\sin\frac{A}{2}},$$

${}_{air}\mu_{glass}$ for sodium light is calculated.



(a) Measuring the angle of the prism (b) Measuring the angle of minimum deviation

FIG. 210.—Refractive index by minimum deviation method.

(b) *Finding refractive index by the critical angle method.*—The angle of the prism is determined as before. The other measurement to be made is the angle of emergence, from one face, of the ray which leaves the other at the critical angle. The telescope is first set normal to the face of emergence in the following manner:

The prism is removed, the scale clamped to the instrument's frame, and the telescope set to receive light directly from the collimator to give an image on the crosswires. The telescope is now turned through 90° and clamped to the frame. The prism is placed on the table, the scale released from the frame, and the prism rotated until an image of the slit formed by reflection in one face of the prism is seen on the crosswires. This face is at 45° to collimator and telescope. The prism table and scale are turned through exactly 45°; the prism face is now normal to the axis of the telescope. If during subsequent operations the

prism table is kept clamped to its scale, and the telescope to its vernier, then the reading in this position will always be that when the face and telescope are at right angles.

The bases of many prisms are ground, and with this type the ground base may be illuminated and the telescope set on the boundary between the bright and dimmer parts of the field. Here total internal reflection, the first thing one thinks of in connection with the critical angle, is used. The critical angle boundary is, however, shown up much more sharply by looking through the prism at a diffuse source of light, such as the sky or brightly illuminated white paper or ground glass; it is then a boundary dividing light from *complete darkness*, for rays are striking the far face of the prism at all angles, but none can be transmitted at an angle greater than the critical angle. The best way of doing the experiment is, then, to place a piece of ground glass close to the far face of the prism with a sodium flame behind it, and set the telescope crosswires on the boundary seen through the face for which the adjustments have been made. The difference between the vernier reading in this position and that when the telescope is set normal to the face gives the angle of emergence θ, from which μ can be found from the formula

$$\mu = \sqrt{1 + \left(\frac{\sin\theta + \cos A}{\sin A}\right)^2}.$$

(*c*) *Dispersion measurements. Spectroscopy.*—Discharge tubes containing gas at a pressure of the order of a millimetre give spectra consisting of a number of bright lines accompanied by fainter lines and bands. The wavelengths of the principal lines in the spectrum of hydrogen are:

Red (C) - - - -	6563 Å. U.
Blue (F) - - - -	4861 Å.U.
Violet (G) - - - -	4340 Å.U.

Helium gives seven or eight bright lines well distributed throughout the spectrum:

Red - - - - -	7065 Å.U. 6678 Å.U.

Yellow - - - - -	5876 Å.U.
Green - - - - -	5015 Å.U. 4921 Å.U.
Blue - - - - -	4713 Å.U.
Violet - - - - -	4471 Å.U.

With a hydrogen tube the refractive indices μ_{blue} and μ_{red} for the blue and red lines may be determined, and the dispersive power $\frac{\mu_{\text{blue}} - \mu_{\text{red}}}{\mu - 1}$ used in the discussion of achromatism (p. 165) calculated.

With the helium tube, the prism may be set at minimum deviation for the yellow line and a series of values of deviation θ against wavelength λ obtained, from which a θ/λ curve for the prism is drawn. This enables the spectrometer to be used as a *spectroscope*; it is a calibration curve, from which the wavelengths of unknown lines can be read at once after their angles of deviation have been measured. Alternatively, instead of keeping the prism fixed, the angle of minimum deviation for each line may be found, μ calculated for each, and a μ/λ curve obtained. The quantity $d\mu/d\lambda$, obtainable for any wavelength from the slope of the μ/λ curve, is of particular interest as it enables us to calculate the resolving power of the prism and also the velocity of a light-signal in its material.

Discussion of the spectrum produced by a prism.—When dealing with the thin prism we defined the term **dispersion between two colours** as the angular separation between them, and **dispersive power** as the ratio $\frac{\text{dispersion}}{\text{mean deviation}}$. The dispersion and dispersive power are clearly greatest in those parts of the spectrum for which the $\mu - \lambda$ curve is steepest.

Fig. 211 shows that the dispersion decreases as we go towards the red end of the spectrum. The colour distribution in the spectrum of a typical glass prism is shown in Fig. 212. The actual length of the spectrum will depend on the angle ϕ between the extreme ends and the focal length f of the telescope objective. Referring back to Fig. 208, and considering the rays going

through the optical centre of the lens, it is seen that the length of the spectrum is $f \times \phi$.

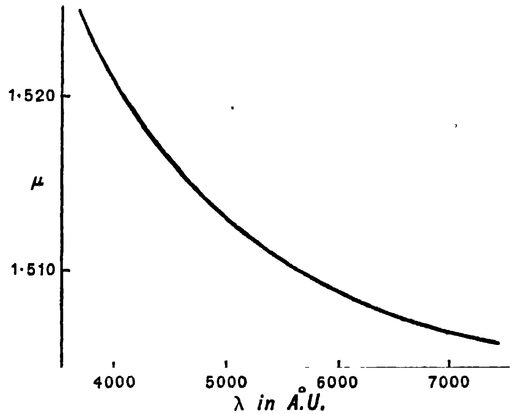


FIG. 211.—μ/λ curve for prism.

The *amount of detail* visible in the spectrum is called the **resolving power.** If two wavelengths, λ and $\lambda + d\lambda$, can just be resolved as separate, this is given by $R = \frac{\lambda}{d\lambda}$. It is shown later that this can be expressed as either $D\frac{d\theta}{d\lambda}$, where D is the aperture of the telescope, or $t\frac{d\mu}{d\lambda}$, where t is the difference in paths traversed in the prism by the edges of the beams.

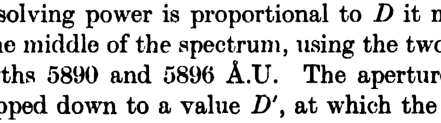


FIG. 212.—Spectrum produced by prism.

As the resolving power is proportional to D it may be determined for the middle of the spectrum, using the two sodium lines of wavelengths 5890 and 5896 Å.U. The aperture of the telescope is stopped down to a value D', at which the lines are just indistinguishable.

Then the reduced resolving power, $R', = \frac{\lambda}{d\lambda} = \frac{5890}{6} =$ approximately $1000 = D'\frac{d\theta}{d\lambda}$.

And the full resolving power, $D\frac{d\theta}{d\lambda} = 1000 \times \frac{D}{D'}$.

The direct vision spectroscope.—A train of crown and flint glass prisms designed to give dispersion with no deviation of the middle of the spectrum is used in the direct vision spectroscope, in which the spectrum is formed as in Fig. 206*b*. As the instrument is made as short as possible an eyepiece is needed to bring

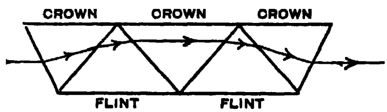


FIG. 213.—Path of rays for middle of spectrum in direct-vision spectroscope.

the virtual images of the slit in focus. Types employing a diffraction grating instead of prisms give about twice the dispersion and are less expensive, but with prisms greater brilliance is obtained.

Emission spectra.—An incandescent **black body** emits a spectrum **continuous** in the visible regions and extending continuously into the infra-red and ultra-violet regions.

Line spectra are emitted by vaporised substances in flames, and by gases in discharge tubes. Arcs and sparks give lines characteristic of the electrodes. Each element exhibits its own line spectrum, whether in combination or not. *The line spectrum is thus a property of the atom.* Elements of the same chemical family show similar spectra, with a general displacement of each line towards the violet end of the spectrum with increasing atomic number.

This suggests that that part of the atom responsible for the chemical behaviour, the outer shell of electrons, is responsible for its visible spectrum. The source of the energy is easily seen. In flames, the atoms acquire energy by collision in the course of thermal agitation; and in discharge tubes, arcs and sparks by collision with ions moving rapidly in the electric field. The energy thus acquired puts the atom into an **excited state**, in which electrons are moved from their normal orbits into orbits of **higher energy**. When the electrons return to their normal orbits, or to intermediate ones, their energy is emitted as radiation. Why

this radiation takes the form of *sharp lines* of definite wavelength was explained by Bohr in terms of the quantum theory. First, the possible orbits in both normal and excited atoms are supposed limited to certain definite ones by quantum principles, each possible orbit corresponding to a definite energy level. Secondly, the motion of an electron to its normal orbit with energy E_0 from one of higher energy E thus liberates a definite quantity or quantum of energy $E - E_0$. Thirdly, this energy will be emitted as radiation of a definite frequency ν given by the quantum equation

$$(E - E_0) = h\nu,$$

where h is a universal constant known as Planck's constant.

The line spectra of nearly all elements can be classified into **series**, each E_0 giving lines corresponding to the transitions $(E_1 - E_0)$, $(E_2 - E_0)$, $(E_3 - E_0)$ ... and so on, where E_1, E_2, E_3 ... are the possible orbits one, two, three ... energy levels above E_0.

The series relationship was first worked out for the lines of the visible spectrum of hydrogen by **Balmer**, the wavelengths of which can be written in the form

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{K^2}\right),$$

where $K = 3, 4, 5, 6$ for the four visible lines and R is a constant called the **Rydberg constant.** Lines corresponding to larger values of K are observable in the ultra-violet, converging towards a limit at 645·6 Å. Other series for hydrogen, expressible in similar form, are the ultra-violet Lyman's series given by

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{K^2}\right.$$

and the infra-red Paschen's series

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{K^2}\right),$$

K being 2, 3 ... in the first case and 4, 5, 6 ... in the second and R the same constant.

The limit of each series in the short-wave side will correspond to the transition in which an electron has fallen to the E_0 orbit from outside the atom. Theory enables the energy cor-

responding to the **series limit** to be calculated ; for example, in the case of mercury it corresponds to the work done in moving an electron through 10·39 volts. The **ionization potential**, or the potential difference through which an electron must fall in order to ionize an atom by collision—that is, to give the atom potential energy by removing an electron from its normal orbit to outside the atom and so put it into a state in which it is ready to emit the series limit line—was shown by *direct electrical methods* to be 10·39 volts for mercury. The energy corresponding to the emission of individual lines can also be calculated. The ultra-violet mercury line of wavelength 2537 Å.U. needs for its emission energy equal to that possessed by an electron that has fallen through 4·9 volts. *Direct experiments* show that electrons which have fallen through this potential difference lose all their energy when they collide with mercury atoms (giving the atoms just the right amount of energy to produce the line), and that the 2537 Å.U. line is then emitted. The single line thus excited is called a **resonance line** and the potential difference through which an electron must fall to excite it its **resonance potential.**

Observations with instruments of exceptionally high resolving power show that single spectral lines are split up into groups when the source is in a strong electrostatic field (**Stark effect**) and strong magnetic field (**Zeeman effect**), this latter affording the astronomer a means of measuring the magnetic fields in stellar bodies. If the atom emitting light is moving relatively to the observer in the line of sight the Doppler effect changes the observed frequency of the lines, altering their positions in the spectrum. Ordinary molecular velocities in flames and discharge tubes suffice to give the lines a finite breadth from this cause, and the motion of stars and rotation of planets give displacements which enable these motions to be measured spectroscopically.

The spectrum of an ionized atom differs from that of the normal atom ; for example, that of ionized helium resembles closely that of hydrogen with alternate lines missing. In strong electric fields such as are produced by sparks in the laboratory the spectra of ionized atoms are obtained. The same occurs at very high temperatures.

Gases whose molecules contain more than one atom give a **band** spectrum in the red and infra-red. The source of this is the molecule.

Bohr's calculation of the wavelengths of the Balmer series.— According to the Rutherford-Bohr picture, the atoms of all elements consist of positively charged nuclei surrounded by a "planetary system" of electrons. In the normal atom the nuclear charge just balances the sum of the charges of the electrons. The simplest atom is that of hydrogen, which has only one electron; the mass of the nucleus is about 1800 times that of the electron, so that it can be regarded as a fixed centre. The simplest orbit is a circle.

The ordinary mechanics and electricity on which the following calculation is based will be well within the reach of the student, who is advised to refresh his memory by looking up the formulae in mechanics for the behaviour of a particle describing a circle and also the law of inverse squares and the meaning of potential in electrostatics. The statements of the quantum theory will be treated as assumptions, which justify themselves by results.

Suppose that an electron of mass $m = 0{\cdot}899 \times 10^{-27}$ gm. and charge $e = 4{\cdot}774 \times 10^{-10}$ e.s.u. is rotating about the heavy nucleus. Certain orbits are possible in which it will rotate without radiating, and these are such that the angular momentum is an integral multiple of $h/2\pi$, where h is **Planck's Constant** ($h = 6{\cdot}55 \times 10^{-27}$). (*First assumption.*) Let r be the radius of one of these orbits, and ω the uniform angular velocity of the electron.

Then $mr^2\omega = nh/2\pi$, where n is an integer. ....(1)

The charge on the nucleus is also e, and the attraction between the two charges provides the force towards the centre which maintains the motion, so $e^2/r^2 = mr\omega^2$;

$$\therefore\ mr^3\omega^2 = e^2. \quad \text{.................................(2)}$$

Dividing the corresponding sides of (1) and (2),

$$r\omega = \frac{2\pi e^2}{nh};$$

$$\therefore\ \text{kinetic energy} = \tfrac{1}{2}mr^2\omega^2 = \frac{2\pi^2 me^4}{n^2h^2}. \quad \text{..............(3)}$$

There is still the potential energy to consider. The potential at distance r from the nucleus is e/r; the work required to bring

the electron to this distance from infinity is then $-e^2/r$. This is the electron's potential energy. To substitute for r, square both sides of (1), and divide both sides of (2) by the resulting terms :

$$m^2r^4\omega^2 = n^2h^2/4\pi^2,$$
$$mr^3\omega^2 = e^2;$$
$$\therefore \frac{1}{r} = \frac{4\pi^2me^2}{n^2h^2};$$
$$\therefore \text{ potential energy, } -\frac{e^2}{r}, = -\frac{4\pi^2me^4}{n^2h^2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The total energy, E, is, therefore, the sum of (3) and (4);

$$\therefore E = \frac{2\pi^2me^4}{n^2h^2} - \frac{4\pi^2me^4}{n^2h^2}$$
$$= -\frac{2\pi^2me^4}{n^2h^2}.$$

As this is negative, the greatest (that is, least negative) energy will belong to those orbits for which n is greatest.

For $\quad n=1, \quad E = -\frac{2\pi^2me^4}{h^2}\cdot\frac{1}{1^2}$

$$n=2, \quad E = -\frac{2\pi^2me^4}{h^2}\cdot\frac{1}{2^2}$$
$$n=3, \quad E = -\frac{2\pi^2me^4}{h^2}\cdot\frac{1}{3^2}$$
$$n=4, \quad E = -\frac{2\pi^2me^4}{h^2}\cdot\frac{1}{4^2}, \text{ and so on.}$$

Energy is liberated when an electron jumps from one orbit to another of less energy, and appears as monochromatic radiation of frequency ν, where $h\nu$ = energy liberated. (*Second assumption.*) For the Balmer series, the final orbit has $n=2$; and electrons falling into this orbit from those for which $n=K$ (where K stands for an integer greater than 2) will lose energy

$$\frac{2\pi^2me^4}{h^2}\left(\frac{1}{2^2}-\frac{1}{K^2}\right);$$
$$\therefore h\nu = \frac{2\pi^2me^4}{h^2}\left(\frac{1}{2^2}-\frac{1}{K^2}\right)$$

and, as
$$\nu = \frac{c}{\lambda},$$
$$\frac{1}{\lambda} = \frac{2\pi^2 me^4}{ch^3}\left(\frac{1}{2^2} - \frac{1}{K^2}\right).$$

The value of the Rydberg constant (p. 256) should thus be $\frac{2\pi^2 me^4}{ch^3}$. The experimental value from the wavelength measurements is 109,678, and that calculated from the constants e, m, c, and h agrees very closely with this. The agreement between the theory and experiment thus justifies the assumptions made.

If the final orbit has $n = 1$, the wavelengths of Lyman's series can be calculated ; $n = 3$ gives Paschen's series, while $n = 4$ and $n = 5$ correspond to other known infra-red series. Fig. 213a illustrates the way in which all these series are emitted.

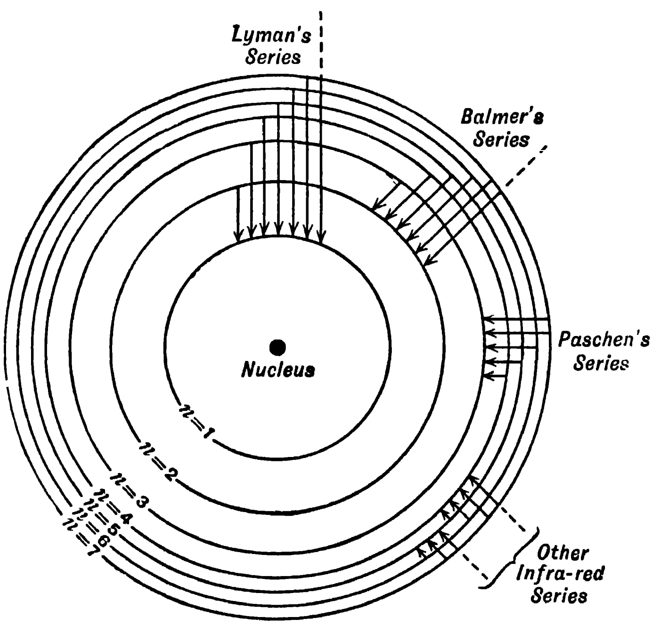


FIG. 213a.

The general case of the inverse-square orbit is an ellipse.

Several ellipses of different eccentricities can be shown to be equivalent to each of the circular orbits of Fig. 213*a*. While the shape of the orbit need not affect the result of the previous calculation, it can be used to explain the slight differences in wavelength of the many fine lines of which a single line is really composed ("fine-structure"), which are seen with spectroscopes of very high resolving power, and also explains the Zeeman effect.

In the unexcited hydrogen atom, the electron occupies the orbit $n=1$. Going through the periodic table, each succeeding element increases in nuclear charge by one unit, and adds on one more electron in the normal state. These electrons fill up the various possible orbits; the orbit $n=1$, holding 2 electrons, is called the K-shell, that for $n=2$, filled with 8 electrons, is the L-shell, and so on. With a heavy element, visible radiation is produced by the outer electrons, while the counterparts of the Lyman and Balmer series, involving the $n=1$ and $n=2$ orbits as the final ones, are the K-series and L-series X-ray spectra. If N is the atomic number, the nuclear charge is Ne; a calculation of the wavelengths of the lines, assuming everything to be as simple as in the hydrogen atom except for the increased nuclear charge, gives

$$\frac{1}{\lambda}=\frac{2\pi^2me^4}{ch^3}N^2\left(\frac{1}{1^2}-\frac{1}{K^2}\right) \text{ and } \frac{1}{\lambda}=\frac{2\pi^2me^4}{ch^3}N^2\left(\frac{1}{2^2}-\frac{1}{K^2}\right),$$

for the K-series and L-series. For each series $1/\lambda \propto N^2$, or the square root of the frequency is proportional to N, *cf.* Moseley's result (p. 344).

Newer ideas have been added to this simple picture. Experiments have shown that *electrons are diffracted like waves*. In light, wave calculations tell us all we need about the *distribution* of energy, but the energy itself appears in *quanta*. The new wave theory of matter, called quantum mechanics, originated by de Broglie in 1924 and developed by Heisenberg, Schrödinger, and Dirac, is used to calculate the *distribution* of electrons, which when encountered individually appear as *charged corpuscles*. This enables a mathematical explanation of Bohr's quantum assumptions to be given, and also deals satisfactorily with more complicated atoms.

For a fuller discussion of spectral theory, reference should be made to one of the standard text-books on atomic physics.

Absorption spectra. Fraunhofer lines.—Light from a source giving a continuous spectrum is found after passing through a coloured transparent liquid or vapour to give a spectrum crossed by dark bands and lines. These bands and lines are called an **absorption spectrum** and are characteristic of the absorbing substance, changing considerably with temperature and pressure. Solids in the ordinary way absorb large portions of the spectrum; red glass, for example, has an absorption band embracing the green, blue, indigo, and violet. Most colourless gases have strong absorption bands in the infra-red and ultra-violet. Ångström suggested that there was a connection between emission and absorption spectra and Kirchhoff verified that **a substance which emits at a certain temperature light of a given wavelength possesses also the power at that temperature of absorbing light of the same wavelength.** This is the explanation of the bands and lines, and one which nowadays we should deduce from the reasoning of p. 256, since an atom could be put into an excited state ready to emit a quantum of energy of a certain wavelength by absorbing a quantum of energy of the same wavelength.

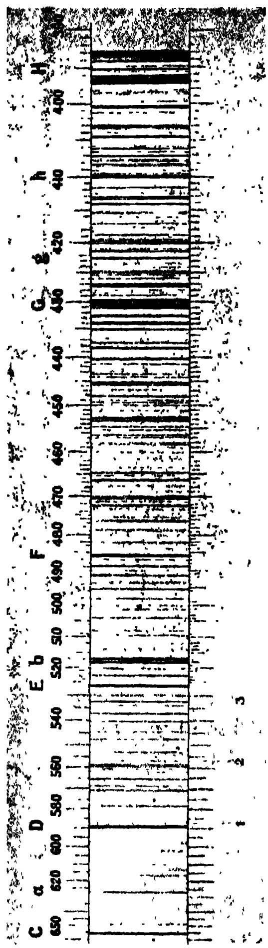


FIG. 214.—Fraunhofer lines. (*From "An Introduction to the Study of Spectrum Analysis," by W. Marshall Watts. Longmans, Green & Co., Ltd.*)

The sun's spectrum is found to be crossed by a very large number of fine dark lines known as **Fraunhofer lines**, after their first systematic investigator. Thousands of these lines have been mapped, and the more important ones labelled with

letters as shown in the diagram. Fraunhofer had remarked that the intense double dark line D coincided with the place in the spectrum of the lines emitted by sodium. Kirchhoff, in the course of checking the coincidence, observed that when bright sunlight was viewed through a sodium flame the D lines stood out darker than before, and they also appeared when a continuous source was examined through a sodium flame.

Kirchhoff's experiment may easily be performed with a direct vision spectroscope, using a bright electric lamp as the continuous source.

Kirchhoff and Bunsen established that the *Fraunhofer lines are the absorption spectrum of vapours through which the sun's light has passed*, either on the sun's "atmosphere" or our own. Coincidences were found for four hydrogen lines (the intense lines labelled C, F, H, h), some 450 lines in the iron-arc spectrum, and those of many other elements. The sun is entirely gaseous in physical constitution, the density increasing from the surface downwards. The outer layers, down to a depth of about 10,000 miles, form the **photosphere**, or visible surface of the sun. This emits a continuous spectrum which, in passing through the slightly cooler outer region of incandescent vapours, the **reversing layer** of the **chromosphere**, is robbed of the lines in the absorption spectrum of those elements therein.

Some lines in the Fraunhofer system become much stronger when the sun is low in the heavens and thus shines through a greater thickness of our atmosphere, and many of these are stronger when the relative humidity of the atmosphere is high. Janssen found that these lines were much less distinct when observations were made at a height of 9000 feet, that they could be reproduced in the spectrum of an incandescent flame viewed from a distance of 13 miles, and nearly all of them in a similar spectrum examined through 118 feet of aqueous vapour. He showed that these lines were due to the earth's atmosphere, and many of them to its water vapour; they are known as **telluric lines.** A group of lines in the region of the D lines, extending from 5860 Å to 6030 Å, is called the **rain band.** It is darker when there is much water vapour in the air. An ingenious way of detecting the telluric lines was devised by Cornu. Light from

each edge of the sun's disc in turn was thrown on the spectroscope slit by a lens moving rapidly from side to side; owing to the Doppler effect there is a displacement towards the violet of the lines originating in the sun when the edge rotating towards the observer is viewed, and to the red for that receding. So the true solar lines flickered, while the telluric lines remained steady.

Fluorescence.—When light energy is absorbed it may be converted into heat, initiate chemical change as on the photographic plate, cause electrons to be emitted as in the photoelectric cell, or be reradiated as light. We should expect that in the case of the sodium vapour of Kirchhoff's experiments the absorbed light would be reradiated as light of the same wavelength (but in all directions instead of only towards the spectroscope). Some substances reradiate light of a wavelength *different* from that absorbed; for example, uranium glass and fluorescein solution emit a yellowish-green light when illuminated by ordinary sunlight, numerous substances emit visible light in ultra-violet light and under X-rays. The **fluorescent light,** as it is termed, is always of a longer wavelength than that of the incident radiation. **Fluorescent powders,** on this account, are coming into use with mercury-vapour lamps to convert the intense ultra-violet emission into visible light.

Energy other than radiation may excite what can be regarded as a type of fluorescence; for example, the impact of cathode particles on glass and many other substances and the shaking of mercury in an evacuated tube.

Colour.—(*a*) *Recombining the spectrum.*—Fig. 215 shows an arrangement by which a nearly pure spectrum may be formed at A and recombined at B, where the lens L_2 forms an image $P'Q'$ of the portion PQ of the first face of the prism. Portions of the spectrum at A can be intercepted by small opaque screens. The patch of light at B then shows the colour produced by the rest of the spectrum combined. When the red portion is cut out at A the patch at B appears a "Marina green"; this blue-green is clearly white light minus red, and the two colours, red and this blue-green, which combined together give white, are called **complementary colours.** If the green part of the spectrum is

removed, the complementary colour is a cerise red, and the removal of yellow gives a purplish-blue, and so on. The sensation of white can then be produced in innumerable ways by suitable pairs of complementary colours.

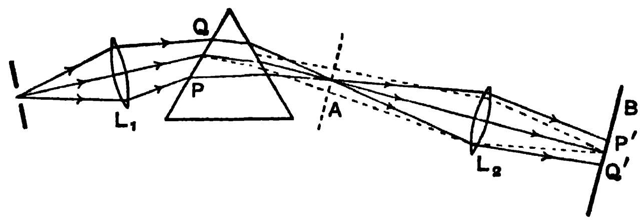


FIG. 215.—Arrangement for recombining spectrum.

(*b*) *Primary colours.*—Three qualities can be distinguished in colour. These are **intensity** or **brightness**, **hue** (which may be defined as that quality in which the colour differs from neutral grey), and **saturation** or **freedom from dilution with white light.**

Any colour can be produced by a suitable combination of only three colours. There are three colours from which nearly all colours can be produced by *addition*, that is, by straightforward superposition of illuminations in different proportions on a screen; they are red of wavelength 6500 Å.U., green of wavelength 5550 Å.U., and blue of wavelength 4800 Å.U. These are called the **primary colours.** The extreme red and violet of the spectrum and other colours not obtainable by addition are matched by subtracting one colour from the mixture of the other two, that is, by superposing the one primary colour on the colour to be matched and then matching this mixture with the mixture of the other two primaries.

When all three primaries are mixed, let us say, for the present purpose, that equal quantities are used if the result is white; this is a rather vague definition of " quantity " but we need not be more precise here. The **colour triangle** sums up in a diagram the results of mixing the primary colours. Red, blue, and green are represented by the corners of an equilateral triangle. Every point in the plane of the diagram represents a colour. To find the result of mixing any two colours, a mass proportional to the quantity of each colour is imagined to be at the point representing

that colour ; the point giving the resulting colour is at the centre of gravity of the masses. White is clearly represented by the centre of gravity of the triangle. Equal quantities of green and red give a " two-unit " yellow which mixed with one part of blue gives white, and so on ; it can be seen that the mid-point of a

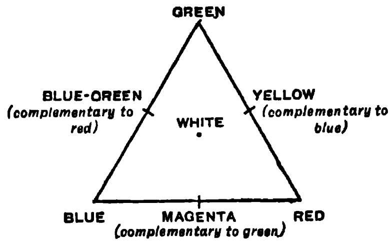


FIG. 216.—Colour triangle.

side gives the colour complementary to the opposite vertex. The mid-points of the sides themselves form a triangle whose centre of gravity is white, so a mixture of the three colours complementary to the primaries in equal quantities also gives white. The nearer a colour is to the centre of gravity of the triangle the less saturated it is. The extreme ends of the spectrum and the very saturated colours lie outside the edges of the triangle.

Colours apparently not represented on a diagram of this kind, such as brown, owe their appearance to their *low intensity* ; brown is given by a very feeble yellow or red.

(*c*) *The colours of natural objects*.—To a normal eye, with reasonably high illumination, the colours of objects are determined by the nature of the light by which they are illuminated and the proportion of the incident light absorbed. A piece of paper appearing white in daylight reflects nearly all of the incident light and so will appear red in red light, blue in blue light, and so on. A matt surface appearing red in daylight does so because it absorbs all the rest of the spectrum ; held in red light it will appear red, but held in blue or green light black. Pigments act, then, not by manufacturing their colour, but by *subtracting* its *complementary* from incident white light. When pigments are mixed, the resulting colour will not be that given by superimposing lights of the same colours. If we could get pigments throwing

back approximately monochromatic light, a mixture of any pair would give a black pigment, and in the usual case of blue and yellow pigments the result is green because this is the only part of the spectrum that neither of them absorbs.

The production of any possible colour using three pigments is to be expected. But they will be pigments which *subtract* the primary colours, that is, their colours will be blue-green, red, and yellow, complementary to primary red, green, and blue-violet. The process of three-colour printing consists of taking three separate photographs through filters each transmitting one of the primary colours, and superimposing the three positives printed each in the appropriate complementary pigment.

Filters, like pigments, subtract the colours complementary to that shown by them. The Lovibond Tintometer compares the colour of the specimen under examination with that of a white plaster plate viewed through three filters, blue-green, red, and yellow, whose absorption can be increased in calibrated steps.

Ultra-violet and infra-red.—We can do no more here than mention the difficulties attaching to investigations in the ultra-violet and infra-red. Glass transmits no wavelengths shorter than 3090 Å.; modern fused quartz is transparent down to 1850 Å. For shorter wavelengths fluorite lenses and prisms and concave reflecting gratings are employed for photographic registration and the spectrograph must be evacuated. Specially treated (Schumann) plates can be used down to 136 Å.

The infra-red region is explored photographically using rock-salt or fluorite in place of glass up to 12,000 Å., and beyond that using sylvine to 24,000 Å.U. with thermal recording instruments.

As for photographic plates, the numerous Ilford infra-red emulsions enable recording to be done from 6400 Å.U. to 12,000 Å.U. Those sensitive to the longest wavelengths are prepared with rather unstable dyestuffs, fog easily, and do not keep well. Ordinary plates treated with fluorescent oil were first used for the ultra-violet. The original Schumann plates were made with specially thin gelatine to reduce absorption of ultra-violet light therein, and in the modern Ilford ultra-violet emulsion the effect of the gelatine is reduced by having the sensitive grains on its surface.

A full account of the ultra-violet and infra-red is given in Ch. XX.

QUESTIONS ON CHAPTER XVI

1. Explain with the aid of a diagram how a pure spectrum may be formed on a screen by means of a prism, a slit, and two convex lenses.

What differences are there between the spectra of the light from an electric lamp, a sodium flame, and the sun ? (C.H.S.C.)

2. Describe briefly how you would project a clear solar spectrum on to a screen, and draw a ray diagram to illustrate the arrangement of the apparatus.

How would you show that the spectrum extends in both directions beyond the visible region ? (N.U.J.M.B.H.S.C.)

3. Explain the formation of a spectrum by a prism, and describe how to obtain a pure spectrum.

If a piece of white paper on a black background is viewed through a prism, the edges of the paper appear coloured but the centre remains white. Explain this. (C.W.B.H.S.C.)

4. Describe the optical system of a spectrometer, and give an account of the adjustments which must be made in setting up the instrument to measure the refractive index of a prism. Prove the formula used in such a determination. (O. & C.H.S.C.)

5. Draw a diagram to show the optical parts of a spectroscope.

Describe the apparatus you would require and the method you would adopt to observe the emission spectrum of a gas such as hydrogen. What modification of the apparatus would be necessary if you wished to observe the ultra-violet portion of the spectrum ? (C.H.S.C.)

6. Describe carefully how you would determine the refractive index of a glass prism.

A glass prism with a refracting angle of 60° has a refractive index of 1·515 for red light and 1·532 for violet light. A parallel beam of white light is incident on one face at an angle of incidence which gives minimum deviation for red light. Determine (*a*) this angle of incidence, (*b*) the angular width of the spectrum, (*c*) the length of the spectrum if it is focussed on a screen by an achromatic lens of 100 cm. focal length. (N.U.J.M.B.H.S.C.)

7. Deduce the relation between the angle of a prism, the minimum deviation produced, and the refractive index of the material.

Describe the optical system of the spectrometer, and the adjustments which must be made when setting up the instrument to measure the refractive index of a prism. (C.H.S.C.)

8. Describe the spectroscope, and explain how you would adjust and use it to measure the refractive index of a prism for light from a sodium flame. How can a direct vision spectroscope be made ? What are the chief uses of the spectroscope ? (O.H.S.C.)

9. Describe briefly (*a*) the optical arrangement of a spectroscope, illustrating with a diagram showing the paths of the rays which form the ends of the visible spectrum when the slit is illuminated with white light; (*b*) how the spectroscope is used to measure the angle of a prism. A spectroscope is set up correctly and the readings of the telescope vernier when the crosswires are set on the ends of the visible spectrum are observed to be 60° 45′ and 63° 15′ respectively. The focal length of the object-glass of the telescope is 27 cm. and that of the eye lens 3·0 cm. Determine the angle which the spectrum seen through the telescope subtends at the eye lens.

(N.U.J.M.B.H.S.C.)

10. In general, the angle of deviation for light going through a prism varies with the angle of incidence. Describe an optical instrument by means of which this variation can be measured with moderate accuracy, and draw a curve illustrating the general nature of the results obtained with a 60° prism. Explain the importance of these results in connection with the production of a pure spectrum of a luminous flame, using a slit, prism, and one lens.

(N.U.J.M.B.H.S.C.)

11. Give a general account of the solar spectrum, and compare the physical properties of the visible and non-visible portions of it.

(O. & C.H.S.C.)

12. What do you understand by the statement that the sun's light is not monochromatic ?

How would you obtain a beam of monochromatic light, and what experiments would you perform in order to determine its wave-length ? (O.H.S.C.)

13. Describe the construction of a spectroscope, and explain in detail how you would adjust it if you wished to show someone the Fraunhofer lines of the solar spectrum. (O.H.S.C.)

14. Discuss *two* of the following observations :

(*a*) A yellow colour can be obtained by mixing red and green light, but not by mixing red and green paint.

(*c*) Two colours which match in artificial light do not always match in daylight. (C.H.S.C., *part question.*)

CHAPTER XVII

INTERFERENCE

Introduction.—If two waves of **equal wavelength and amplitude**, differing in phase by $\frac{2\pi\delta}{\lambda}$, are superposed, the displacements of the individual waves, at time t,

$$y_1 = a \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right),$$

$$y_2 = a \sin 2\pi \left(\frac{x+\delta}{\lambda} - \frac{t}{T}\right)$$

give a resultant displacement, at time t, of

$$y_1 + y_2 = a \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right) + a \sin 2\pi \left(\frac{x+\delta}{\lambda} - \frac{t}{T}\right)$$

or
$$y_1 + y_2 = 2a \cos \frac{\pi\delta}{\lambda} . \sin 2\pi \left(\frac{x + \frac{\delta}{2}}{\lambda} - \frac{t}{T}\right).$$

The sine term of the expression is still that for a wave of wavelength λ and period T.

The amplitude will vary with δ between 0 $\left(\text{when } \delta = \frac{\lambda}{2}, \frac{3\lambda}{2} \ldots (2n+1)\frac{\lambda}{2}\right)$ and $2a$ (when $\delta = 0, \lambda, 2\lambda \ldots n\lambda$). The intensity, which is proportional to the square of the amplitude, will vary between 0 and 4 times the intensity produced by a single wave.

The above discussion is applicable to all waves and is purely mathematical. It explains in terms of the wave theory the **interference phenomena** observed when two trains of light waves of comparable amplitude, and differing in phase, are superposed. The two trains of waves must be coherent, that is, have come origin-

ally from the same wave-train emitted by the same atom or molecule.

Young's experiment.—Monochromatic light from an illuminated slit falls through two narrow parallel slits A and B, at distance a apart, on to a screen C at distance D away. Equidistant bright and dark bands are seen, parallel to the slits, in the region XY, symmetrical about the centre line OO'.

The condition for a bright band is that the path difference δ should be a whole number of wavelengths, and for a dark band that δ should be an odd number of half wavelengths.

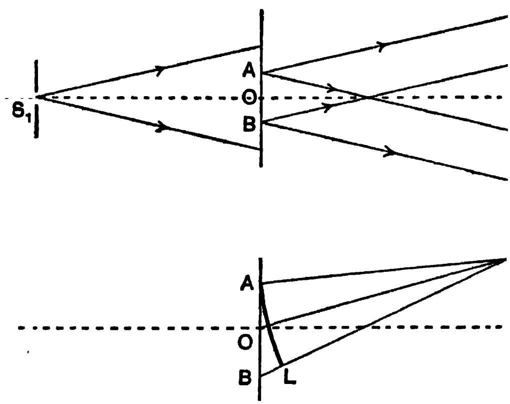


FIG. 217.—Young's interference experiment.

For a point P at distance x from OO' to be on a bright or dark band the calculation is as follows:

Join AP, BP, OP. With centre P and radius PA describe an arc cutting PA in L. As AL is small, we can regard ABL as a triangle.

Now AL is perpendicular to OP, and AB to OO'. The angles $\widehat{ALB}$ and $\widehat{PO'O}$ are right angles. So we have two similar "triangles," ABL, OPO', whence $\frac{PO'}{OO'}=\frac{BL}{AL}=\frac{BL}{AB}$ nearly.

Or,
$$\frac{x}{D}=\frac{BL}{a}.$$

For P to be a **bright band**, $BL = n\lambda$,

so $$x = n\lambda \cdot \frac{D}{a}.$$

For P to be a **dark band**, $BL = (n + \frac{1}{2})\lambda$,

so $$x = (n + \tfrac{1}{2})\lambda \frac{D}{a}.$$

The bright and dark bands are thus *equally spaced* (to a first order of approximation) and the distance between any two bright or dark bands is

$$\Delta x = \frac{\lambda D}{a}.$$

If the incident light is white, the *central band* will be white, for $\delta = 0$ gives a bright band for all wavelengths. The surrounding bands will then be coloured and much less numerous.

Young's experiment has been described, as it illustrates the principles well. It is of great historical importance as being the first of a set of investigations which established the wave theory of light. That the bands were due to interference was at first disputed ; and indeed other bands not due to interference of the two beams of light from both slits are observed. But the true nature of the central pattern of equidistant bands is shown by covering one of the slits, when it vanishes.

A more convincing experiment was performed by Fresnel using instead of actual slits the virtual images of a slit in two mirrors inclined at nearly 180°. The interference pattern is obtainable equally well using a slit parallel to a plane mirror (preferably of blackened glass) as one source and its virtual image in the mirror as the second ; the bands are not symmetrical about the line corresponding to OO'. This arrangement is known as **Lloyd's mirror** ; it introduces a new phenomenon. The central band, for $n = 0$, is *black* and not white. The explanation is that reflection *at a denser medium* causes a phase change of π, or a path change of half a wavelength. This can be proved theoretically, but it is not proposed to do so here. The student will find a complete analogy in the reflection of a sound wave at the open end of a pipe ; a *condensation* is reflected back as a *rarefaction*. No such change occurs at the closed end of a pipe ; and no phase change occurs when light is reflected at a less dense medium.

Lloyd's mirror is interesting also for another reason. If, instead of a slit, a spectrum formed in a plane at right angles to the mirror is set close to the surface, the fringes for all colours can be made to coincide, and so black and white (achromatic) fringes are obtained.

The best laboratory arrangement is **Fresnel's biprism**, in which A and B are the two virtual images formed by a prism of nearly 180° refracting angle of a slit set parallel to its refracting edge. The fringes are so bright that with a Leitz microscope lamp they can easily be projected for class demonstration, and the experiment described below (measuring the separation of the first few red and green fringes) performed roughly with no other measuring apparatus than a metre-stick.

Measurement of the wavelength of sodium light using Fresnel's biprism.—The axis of the biprism P is set parallel to the slit S illuminated by a sodium flame C. Two virtual images of S, A and B, are formed by the prism, and these produce interference fringes in the focal plane F of a microscope M capable of lateral travel on a carriage provided with a vernier scale; or, better still,

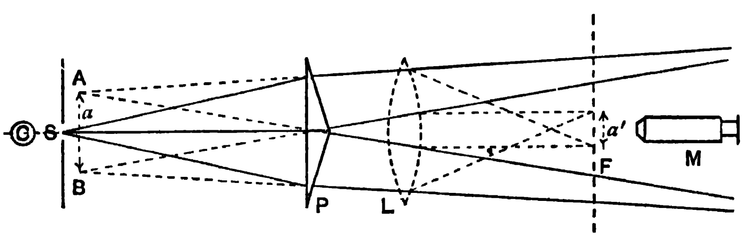


FIG. 218.—Fresnel's biprism.

with a micrometer eyepiece. The distance p between r successive bright bands is noted, and the distance between two successive bright bands, p/r, then equals $\lambda D/a$, where λ is the wavelength of the light, a the distance between A and B, and D the distance from S to F. The distance a may be measured *simply* by putting a lens at L to give real images of A and B in the focal plane F and measuring their distance apart, a', with the microscope, when $a/a' = SL/LF$. For experimental details it is best to consult one of the advanced practical text-books.

For P to be a **bright band,** $BL=n\lambda$,

so $$x=n\lambda\,.\,\frac{D}{a}\,.$$

For P to be a **dark band,** $BL=(n+\frac{1}{2})\lambda$,

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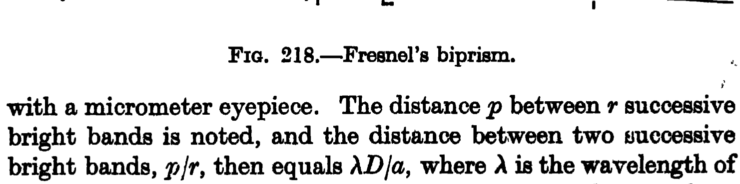

FIG. 218.—Fresnel's biprism.

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Interference in thin plates.—In the diagram, let AB and CD be the faces of a parallel-sided plate of refractive index μ and thickness t, in air.

Consider a ray PQ incident on AB at an angle θ. Part of the energy is reflected at the upper face as QQ', and part refracted at an angle θ, reflected at R and emerges along the path SS' parallel to QQ'. Of course there will be refraction out at R and further internal reflection at S. But we shall consider only the two rays QQ' and SS'.

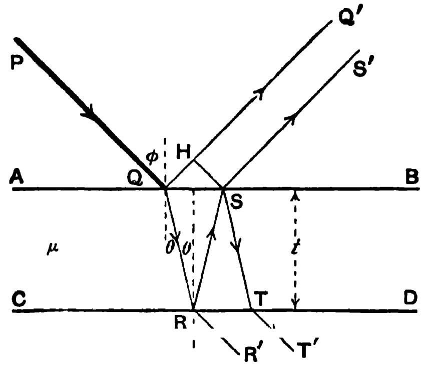

FIG. 219.—Interference in thin film.

Draw SH perpendicular to QQ'. Now while SS' has covered the distance QRS in the medium, QQ' has covered a distance QH in air, so the difference in optical path is

$$\delta = \mu(QR + RS) - QH.$$

Now, $$QR = RS = \frac{t}{\cos\theta}$$

and $$QH = QS \sin\phi = 2QR \sin\theta \sin\phi = 2t \tan\theta \sin\phi;$$

or, as $$\sin\phi = \mu \sin\theta,$$

$$QH = 2\mu t \frac{\sin^2\theta}{\cos\theta}.$$

So, $$\delta = \frac{2\mu t}{\cos\theta} - \frac{2\mu t \sin^2\theta}{\cos\theta} = 2\mu t \frac{(1 - \sin^2\theta)}{\cos\theta} = 2\mu t \cos\theta.$$

We should then expect interference to be possible giving a

dark band when $2\mu t \cos\theta = (n + \frac{1}{2})\lambda$, and a bright band when $2\mu t \cos\theta = n\lambda$. But there is a surface effect we have yet to consider : *reflection at the surface of an optically denser medium causes a path change of half a wavelength*, while an internal reflection, as that at R, does not. So the true path difference is $2\mu t \cos\theta - \lambda/2$ and the conditions will be a **dark band** when $2\mu t \cos\theta = n\lambda$ and a **bright band** when $2\mu t \cos\theta = (n + \frac{1}{2})\lambda$ for reflection.

Now consider the ray RR' and that emerging after two internal reflections, TT'. The same geometrical reasoning applies, but in this case there is no question of reflection at the surface of the denser medium. The conditions will thus be a **dark band** when $2\mu t \cos\theta = (n + \frac{1}{2})\lambda$ and a **bright band** when $2\mu t \cos\theta = n\lambda$ for transmission.

Interference pattern when t is constant and θ varies.—The preceding discussion is purely mathematical. The two rays QQ' and SS' are separated from one another by a lateral distance of SH and will *never interfere unless brought together*; and for interference bands to be observable the amplitudes of the two wave-trains must be comparable, as in fact they are. In the case of a perfectly parallel-sided plate illuminated by a parallel beam of light, since t and θ are both constant for the whole field, no pattern should be observable, for the appearance should be either wholly bright or wholly dark in monochromatic light. In a parallel beam of white light a parallel-sided film also shows no *pattern* and if thin is *coloured* (p. 276); if the transmitted light is viewed through a spectroscope the spectrum is crossed with a *dark line* wherever λ is such that $2\mu t \cos\theta$ is an odd multiple of $\lambda/2$. (This is a convenient way of obtaining a series of reference marks at equidistant wavelength intervals, and is used in Edser and Butler's method of calibrating a spectrometer.)

With a diverging or converging beam incident on a parallel-sided plate, varying path differences in different parts of the field will be produced by differences in θ, and we shall observe **curves of equal inclination.** The interference takes place *wherever the wave-trains are recombined*, either on the retina or in the focal plane of the observing telescope. The fringes themselves appear to be *at infinity*, since the interfering beams are parallel to one another and so are seen only when eye or telescope is focussed

for infinity. Fig. 220 shows how a point P in the pattern is produced.

Note that an *extended source* is used, and that from each point on it only rays striking the plate at the same angle ϕ and refracted

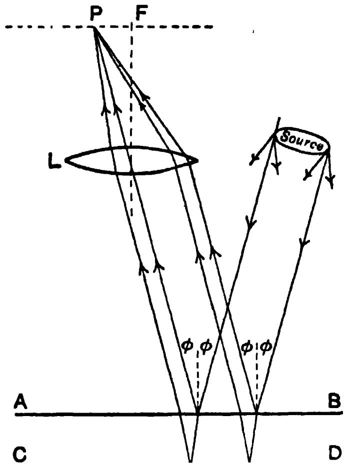

FIG. 220.—Interference with parallel-sided film and extended source.

at the same angle θ reach P. The pattern observed with this arrangement is a series of concentric circles whose common centre is the principal focus of the lens L.

Interference pattern when θ is constant and t varies.—If the

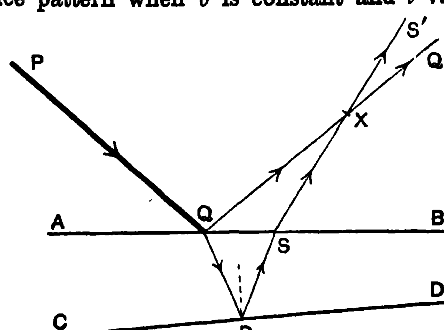

FIG. 221.—Interference with wedge-shaped film.

thickness t varies and θ is constant, interference occurs in exactly

the same manner. Here, as the two sides of the plate are inclined, QQ' and SS' are not parallel but appear to diverge from a point X, which will be practically at the upper surface of the plate. As before, interference occurs when the two wave-trains are brought together by eye or telescope objective. In this case the lens has to be focussed on X, and so the fringes appear to be at X. Further, for the reunion of QQ' and SS' to be possible, both must, of course, be within the aperture of the observing lens. Consideration of Fig. 221 will show that the angle SXQ' increases as the angle of incidence increases, so the effect will be best observed with small angles of incidence. The bands seen are **curves of equal thickness.** If AB and CD are both planes the pattern seen will be a series of parallel bands parallel to the line of intersection of the surfaces.

Newton's rings.—A convex lens of long focal length resting on a plane glass plate gives a series of concentric coloured rings, with

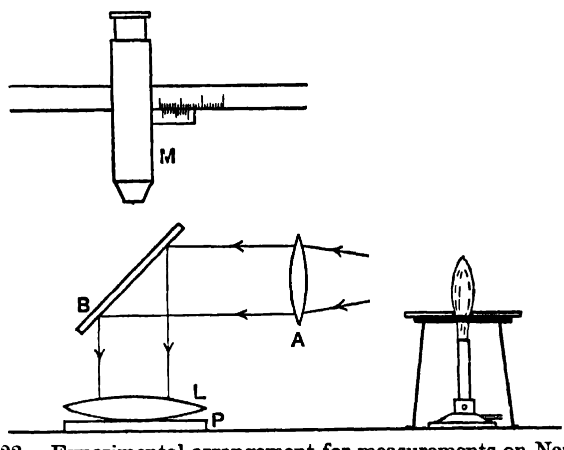

FIG. 222.—Experimental arrangement for measurements on Newton's rings.

the point of contact as centre. The contact spot is black by reflected and white by transmitted light. With monochromatic light many more rings are observed, alternately dark and bright.

The " thin plate " is the film of air between the glass surface

and the lens, and the rings are "curves of equal thickness." Viewed normally, θ is 0, and as $\mu=1$ we can write as the conditions for bright and dark rings:

$$\left.\begin{array}{ll}\textbf{Bright ring} & 2t=(n+\tfrac{1}{2})\lambda \\ \textbf{Dark ring} & 2t=n\lambda\end{array}\right\}\textbf{Reflected light.}$$

$$\left.\begin{array}{ll}\textbf{Bright ring} & 2t=n\lambda \\ \textbf{Dark ring} & 2t=(n+\tfrac{1}{2})\lambda\end{array}\right\}\textbf{Transmitted light.}$$

We can express t in terms of the radius r_n of the nth ring and the radius of curvature ρ of the under surface of the lens. For $r_n^2=t(2\rho-t)$; or, as t is small,

$$r_n^2=2\rho t \text{ or } t=\frac{r_n^2}{2\rho}.$$

So that the above conditions can be written:

$$\left.\begin{array}{ll}\textbf{Bright ring} & r_n^2=\rho(n+\tfrac{1}{2})\lambda \\ \textbf{Dark ring} & r_n^2=\rho n\lambda\end{array}\right\}\textbf{Reflected light.}$$

$$\left.\begin{array}{ll}\textbf{Bright ring} & r_n^2=\rho n\lambda \\ \textbf{Dark ring} & r_n^2=\rho(n+\tfrac{1}{2})\lambda\end{array}\right\}\textbf{Transmitted light.}$$

Determination of the wavelength of sodium light, using Newton's rings.—The arrangement of Fig. 222 is used for determining the wavelength of sodium light. An intense sodium flame is made by soaking a piece of asbestos board in strong brine, boring a $\frac{1}{2}$ in. hole in the centre, and placing this on a tripod over the bunsen flame. Parallel light from the lens A is reflected at the plane sheet of glass B held at 45° to the vertical and falls on the lens L and plate P. The rings are viewed through B by the travelling microscope, M, *focussed on the air film.* Measurements of the diameters of the nth and $(n+m)$th dark rings are made, and since $r_n^2=n\rho\lambda$ and $r_{(n+m)}^2=(n+m)\rho\lambda$, $r_{(n+m)}^2-r_n^2=\rho m\lambda$,

so $$\lambda=\frac{r_{(n+m)}^2-r_n^2}{m\rho}$$

The radius of curvature of the lens surface may be found by Boys' method or using a spherometer.

Colours of thin films.—While the interference pattern with monochromatic light is a series of bright and dark rings and bands, these rings and bands show a succession of brilliant colours with white light. Newton investigated the spectral composition of

these colours in the Newton's rings experiment by casting light from each portion of the spectrum in turn on the lens and

The thickness of coloured Plates and Particles of

		Air.	*Water.*	*Glass.*
Their Colours of the first Order,	Very black	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{10}{31}$
	Black	1	$\frac{3}{4}$	$\frac{20}{31}$
	Beginning of black	2	$1\frac{1}{2}$	$1\frac{2}{7}$
	Blue	$2\frac{2}{5}$	$1\frac{4}{5}$	$1\frac{11}{22}$
	White	$5\frac{1}{4}$	$3\frac{7}{8}$	$3\frac{2}{5}$
	Yellow	$7\frac{1}{9}$	$5\frac{1}{3}$	$4\frac{3}{5}$
	Orange	8	6	$5\frac{1}{6}$
	Red	9	$6\frac{3}{4}$	$5\frac{4}{5}$
Of the second Order,	Violet	$11\frac{1}{6}$	$8\frac{3}{8}$	$7\frac{1}{5}$
	Indigo	$12\frac{5}{6}$	$9\frac{5}{8}$	$8\frac{2}{11}$
	Blue	14	$10\frac{1}{2}$	9
	Green	$15\frac{1}{8}$	$11\frac{2}{3}$	$9\frac{5}{7}$
	Yellow	$16\frac{2}{7}$	$12\frac{1}{5}$	$10\frac{2}{5}$
	Orange	$17\frac{2}{9}$	13	$11\frac{1}{9}$
	Bright red	$18\frac{1}{3}$	$13\frac{3}{4}$	$11\frac{5}{6}$
	Scarlet	$19\frac{2}{3}$	$14\frac{3}{4}$	$12\frac{2}{3}$
Of the third Order,	Purple	21	$15\frac{3}{4}$	$13\frac{11}{20}$
	Indigo	$22\frac{1}{10}$	$16\frac{4}{7}$	$14\frac{1}{4}$
	Blue	$23\frac{2}{5}$	$17\frac{11}{20}$	$15\frac{1}{10}$
	Green	$25\frac{1}{5}$	$18\frac{9}{10}$	$16\frac{1}{4}$
	Yellow	$27\frac{1}{7}$	$20\frac{1}{3}$	$17\frac{1}{2}$
	Red	29	$21\frac{3}{4}$	$18\frac{5}{7}$
	Bluish red	32	24	$20\frac{2}{3}$
Of the fourth Order,	Bluish green	34	$25\frac{1}{2}$	22
	Green	$35\frac{2}{7}$	$26\frac{1}{2}$	$22\frac{3}{4}$
	Yellowish green	36	27	$23\frac{2}{9}$
	Red	$40\frac{1}{3}$	$30\frac{1}{4}$	26
Of the fifth Order,	Greenish blue	46	$34\frac{1}{2}$	$29\frac{2}{3}$
	Red	$52\frac{1}{2}$	$39\frac{3}{8}$	34
Of the sixth Order,	Greenish blue	$58\frac{3}{4}$	44	38
	Red	65	$48\frac{3}{4}$	42
Of the seventh Order,	Greenish blue	71	$53\frac{1}{4}$	$45\frac{4}{5}$
	Ruddy white	77	$57\frac{3}{4}$	$49\frac{2}{3}$

noting the positions of the bright rings in red, orange, yellow,

green, blue, indigo, and violet light. The *spectral* colours transmitted or reflected at any distance from the centre of the system, and so for any thickness of the air film, could thus be stated. The table on page 275, taken from Book II of the *Opticks* (Fourth edition, Bell's 1931 reprint), shows the *observed* colours for different thicknesses of air, water, and glass in reflected light. The unit of thickness is one-millionth of an inch.

It is to be noticed that in the second and third set of rings most colours were clearly observed. Beyond the seventh set no colouring is seen, as there is overlapping of sufficient colours to produce white. The table gives the thicknesses for *normal incidence*. The colours seen in thin oil films on puddles, etc., are "curves of equal inclination," the films being practically parallel-sided and of constant thickness.

Fig. 222(a) shows Newton's diagram for calculating the spectral composition of the observed reflected colours at any thickness. Distances in the direction YH represent what we call wavelengths, and distances in the direction $H\theta$ the thickness of the film—the actual scale depending on its refractive index. The full lines $2\lambda K$, $6\mu N$, $10\nu Q$, ... give the loci of the maxima of the first, second, third ... series or order of rings seen *in the individual spectral colours when the system was observed in monochromatic light of these colours*, and the dotted lines on either side give the extreme limits of these rings. Thus the first, second, and third rings for extreme violet extend from thickness $A1$ to $A3$, $A5$ to $A7$, and $A9$ to $A11$, while for extreme red the corresponding rings go from HI to HL, HM to HO, and HP to HR. If a ruler is held parallel to AH, at a distance from AH corresponding to the thickness of the film, its intersections with the spaces bounded by dotted lines show the spectral colours reflected for that thickness.

Let us move the ruler outwards from AH. For thicknesses less than $A1$ there is no reflection at all—the central black ring. As the ruler moves from 1 to L the first order colours are shown; at 1 the extreme violet, from I to 3 the whole spectrum (white), and from 3 to L the red part. There is a distinct gap between L and 5, the start of the second order. In the second order the

colours are more separated, as the limit of the second order violet at 7 is nearly passed before the extreme red of the second order is reached at M. The line $\tau\rho\sigma\phi$ shows the green of the

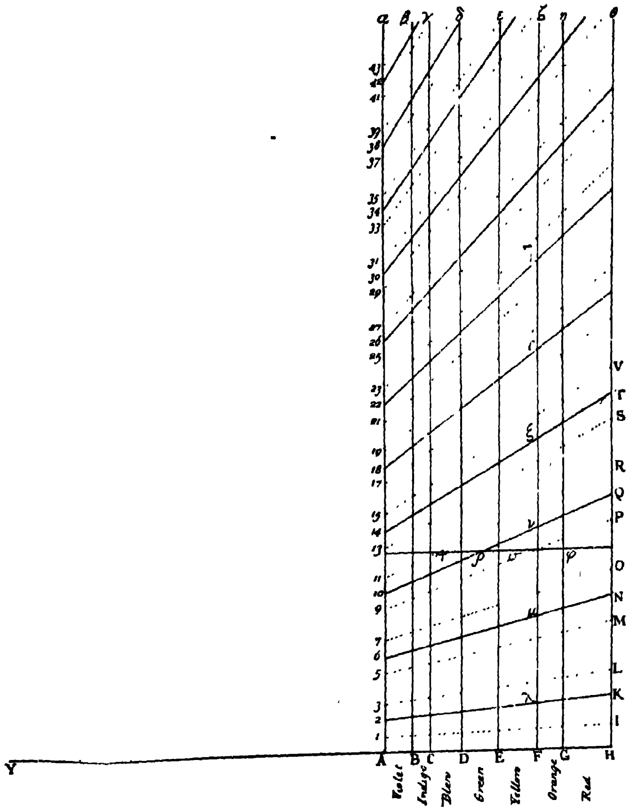

Fig. 222(a). Spectral composition of rings in reflected light.

From Newton's "Opticks". By courtesy of Messrs. G. Bell and Sons, Ltd.

third order, containing in addition to the maximum of spectral green (ρ), fainter blue from ρ to τ and yellow from ρ to σ. The ruler reaches 9 before 0, so the blue of the third order overlaps the red of the second. The fourth order colours as the ruler goes

green, blue, indigo, and violet light. The *spectral* colours transmitted or reflected at any distance from the centre of the system, and so for any thickness of the air film, could thus be stated. The table on page 275, taken from Book II of the *Opticks* (Fourth edition, Bell's 1931 reprint), shows the *observed* colours for different thicknesses of air, water, and glass in reflected light. The unit of thickness is one-millionth of an inch.

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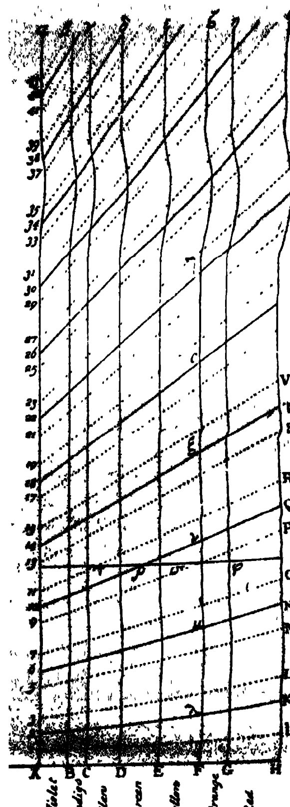

ı of rings in reflected light.

ourtesy of Messrs. G. Bell and Sons, Ltd.

to the maximum of spectral and yellow from ρ to σ. The e of the third order overlaps order colours as the ruler goes

from 13 to beyond 15 overlap the red and orange of the third order, and so on (in the table of observed colours the "third order" ends with bluish red). Towards the right hand of the diagram, the ruler cuts many different orders, showing that there is a sufficiently representative selection of spectral colours to give white; thus at 41 intense rings of blue, green, yellow and red from four different orders overlap.

If Newton's rings are observed in *transmitted* light by a spectroscope so that the slit receives light from a diameter of the ring system, the appearance is exactly *complementary* to that of Fig. 222(*a*). Where Fig. 222(*a*) indicates lines of maximum brightness, *dark bands* running obliquely across the spectrum are observed. A similar effect is shown with a draining soap film—see Sir Charles Boys's "Soap Bubbles" (S.P.C.K.), p. 149.

Interference with large path difference.—Fizeau observed that the sharpness of Newton's rings decreased at first as n increased, reached a minimum at about the 500th ring, and then increased again. The explanation is that the light from a sodium flame is not really monochromatic but consists of two yellow spectral lines of wavelengths 5890 and 5896 Å.U., a difference which in round numbers can be taken as 1 part in 1000.

Each wavelength produces its own set of rings, which for small values of n will nearly coincide. The effect of a single set of sharp rings is seen if n is small. But when $n \times 5896 = (n + \frac{1}{2}) \times 5890$, or n is about 500, the dark rings due to one line lie on the bright ones due to the other, so that uniform illumination will be observed for values of n near this. When $n \times 5896 = (n + 1) \times 5890$ the two sets of rings again coincide, and for values of n round about 1000 the rings are seen as sharply as in the centre of the system.

The two sodium lines can be resolved with an ordinary spectrometer; "fine structure" in apparently homogeneous spectral lines *beyond* the limits of resolution of a spectrometer can be made evident by the fluctuations in the intensities of the interference fringes produced with large path difference.

Interferometers.—Let us consider the case of page 270 again, but suppose now that the surfaces of the plate are partially silvered and that two more emergent rays, VV' and WW', are of amplitude comparable with QQ' and SS'. The path difference between VV' and QQ' will be $4\mu t \cos \theta$, and between WW' and QQ' $6\mu t \cos \theta$.

Now when all four rays are recombined, if QQ' reinforces SS', so must WW' and VV', so that the bright fringe will be more intense. Its intensity, instead of falling off gradually on either side, will cease more sharply, for whereas QQ' and SS' did not interfere with one another for path differences that were fractions of $n\lambda$, QQ' will interfere with VV' when $4\mu t \cos \theta = n\lambda$ or $2\mu t \cos \theta = n\lambda/2$, and with WW' when $6\mu t \cos \theta = n\lambda$ or $2\mu t \cos \theta = \frac{n\lambda}{3}$.

The effect of multiplying the number of interfering beams by silvering the surfaces partially in Newton's rings experiment is

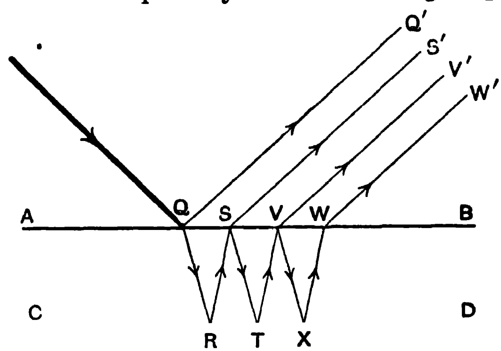

FIG. 223.—Many interfering rays with partly silvered plates.

thus to make the bright rings *more intense* and *sharper*. By combining the two principles of *large path difference* and *many coherent interfering beams*, " fine structure " is revealed in separate ring systems for each wavelength instead of a mere periodic blurring. As the ray QQ' is much more intense than all the rest if the surfaces are partly silvered, the effect is best observed in transmitted light.

In the Fabry-Perot interferometer a thick parallel-sided air film is enclosed between two half-silvered glass surfaces. In the Lummer-Gehrcke plate, multiple reflections inside a parallel-sided glass plate at very nearly the critical angle give rise to a large number of emergent rays of nearly equal amplitudes nearly grazing the face of the plate ; the light enters the plate by means of a right-angled prism cemented to one face. The multiplex of Gehrcke and Lau uses a nearly parallel-sided glass block placed between two Fabry-Perot plates, securing a greater path difference between successive interfering beams. These instruments are used for investigating the " fine structure " of spectral lines.

Interference in thick plates.—A thick parallel-sided plate behaves in exactly the same way as a thin one except that the lateral displacement is considerable—for a glass plate 1 cm. thick a ray incident at 45° will be displaced through about 1·5 cm. The two rays QQ' and SS' can thus be experimented on separately before being made to interfere.

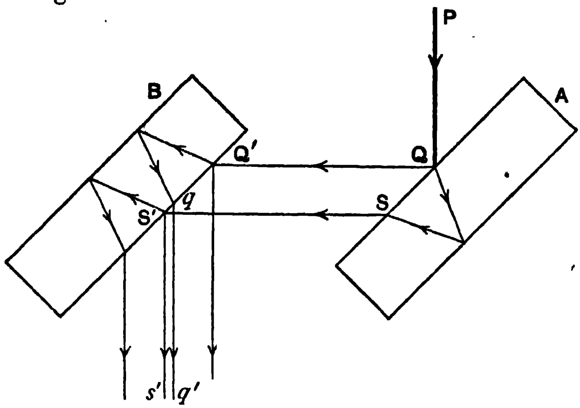

FIG. 224.—Jamin refractometer.

One application of interference in thick plates is the Jamin refractometer. The two rays QQ' and SS' emerging from the first plate A are split up again at the second plate. The rays $S's'$ and qq' will give interference fringes if the plates are inclined to one another at a very small angle. If QQ' passes through an evacuated tube of length l with optically worked plane parallel ends, and SS' through a similar tube containing gas of refractive index μ, the path difference by which SS' is retarded with respect to QQ' is $(\mu-1)l$. Monochromatic light of wavelength λ is used. Both tubes are initially evacuated, and the number of interference bands, n, crossing the field of view as the gas is introduced into one tube are noted. Then $(\mu-1)l=n\lambda$. This apparatus is employed in gas analysis, and has been used widely for measurements on the refraction and dispersion of gases and vapours.

In the Michelson interferometer a partially silvered thick plate is used to split a coherent beam of light into two components of equal amplitude at right angles and subsequently recombine them. Both components are returned along their own paths by means of plane mirrors. One of these mirrors is fixed and movement of the other changes the path difference between the two beams, causing interference bands to cross the eyepiece of the

observing telescope. If the motion of the mirror is d cm. and n bands are observed, then $2d = n\lambda$. Michelson succeeded in evaluating the metre in terms of the wavelengths of three lines in the spectrum of the cadmium arc using this apparatus.

QUESTIONS ON CHAPTER XVII

1. Describe an experiment to illustrate the interference of light, and explain in detail, with the necessary theory, how you would use the experiment to determine the wavelength of light. (C.H.S.C.)

2. Explain the phenomenon of the interference of light, and point out the conditions which must be fulfilled in order to demonstrate the phenomenon.

Describe, and give the theory of, some experiment based on this phenomenon for determining the wavelength of sodium light. (C.H.S.C.)

3. Give an account of an interference method of measuring the wavelength of sodium yellow light. (O. & C.H.S.C.)

4. Under what conditions are interference effects obtained in light and in sound ?

In an experiment with Fresnel's mirrors the interference fringes were observed through an eyepiece fitted with a crosswire. It was found that the distance between successive bright fringes was 0·3 mm. In order to find the position and the distance apart of the two virtual sources a thin lens of focal length 16·0 cm. was used to form two real images of the virtual sources on the crosswires of the eyepiece. The distance apart of the centres of the real images was 0·80 cm. and the distance from the crosswires to the lens was 80 cm.

Calculate the distance apart of the virtual sources, and hence obtain a value for the wavelength of the light used. (O.H.S.C.)

5. Explain the colours of thin films. (O. & C.H.S.C.)

6. Describe how (a) parallel and (b) circular interference fringes can be obtained using two plane glass plates. What is the effect of partially silvering the plates ? Discuss carefully the question of the localization of the fringes in the two cases, and illustrate diagrammatically a simple arrangement by means of which the circular fringes might be photographed. (C.S.)

7. Given two thin partially silvered flat plates of glass, the inclination and distance apart of which can be varied, describe how you could obtain (a) straight, (b) circular, interference fringes. Draw diagrams to illustrate the methods of illumination and observation in each case. Describe and explain what happens when (a) the angle between the plates, (b) the distance apart of the plates, is changed. (C.S.)

8. Discuss the interference effects observed in the neighbourhood of two similar small sources of monochromatic light. A biprism is placed at a distance of 5 cm. in front of a narrow slit, illuminated by sodium light, and the virtual images of the slit formed by the biprism are ·05 cm. apart. Find the width of the fringes formed on a screen placed 75 cm. in front of the biprism. (Wavelength of sodium light $= 5{\cdot}89 \times 10^{-5}$ cm.) (C.S.)

9. Explain what is meant by optical interference. Some of the energy from a wireless transmitter is sent up into the higher regions of the atmosphere, and is there reflected downwards by a horizontal reflecting layer. At the receiving station these downcoming waves combine with the directly transmitted waves to produce interference effects. Describe what you would expect to observe at a receiving station distant 80 km. from the transmitter when the wavelength of the transmitter is slowly changed from 395 to 405 ms. Assume the height of the reflecting layer to be 80 kms. (C.S.)

10. White light is passed normally through two plane glass plates arranged accurately parallel to one another at a short distance apart. When the emergent light is viewed through a spectrometer, the spectrum is seen to be crossed by a number of dark bands. How do you explain this ? Show how the number of dark bands between two given wavelengths varies with the thickness of the film. (C.S.)

11. What is meant by interference of light ? Describe how interference of two streams of light may be shown by an experiment with Fresnel's biprism, and explain how a value for the wavelength of the light used may be deduced from the experiment. (C.S.)

12. Describe a method of finding the refractive index of a gas. The length of the gas-filled tube of a Jamin refractometer is 20 cm. When the pressure is changed by 50 cm. of mercury, 50 fringes pass the crosswires of the observing telescope. Find the refractive index of the gas for sodium light at a pressure of 76 cm. of mercury and at the temperature of the experiment. You may use the relation $(\mu - 1) = \kappa\rho$, where $\rho =$ density. Mean wavelength for sodium D lines $= 5893$ Ångstroms. (C.S.)

13. Give an account of the interference fringes which may be formed by a wedge-shaped film of air enclosed between two flat glass plates. Describe an experiment for the production of these fringes. If fringes of this kind are produced by white light, what will be seen in a spectroscope viewing them if its slit is set parallel to the edge of the wedge ? (C.S.)

CHAPTER XVIII

DIFFRACTION

Introduction.—The simple discussion of the wave theory given in Chapter XIV offered no explanation of the rectilinear propagation of light. The usual two-dimensional demonstrations with a ripple tank show that waves of the kind we picture should certainly be expected to bend round the edge of an obstacle. This does occur, but the departure from the rectilinear path is very small on account of the very short wavelength of light. Grimaldi discovered that with a small source of light the shadow of a small object was larger than that given by geometrical construction and was surrounded by iris-coloured fringes. Newton investigated this effect and Fresnel showed that it did not depend on the nature or shape of the edge of the obstacle. The phenomenon is known as **diffraction.**

Fresnel explained the appearance of the shadow cast by a point source and also the rectilinear propagation of light by treating points on the wave front, which Huygens had imagined as secondary sources, to be sources whose wave-trains were *in a condition to interfere*, and applying the principles of Chapter XVII to find what the resultant effect of the wave at any point would be. Naturally the mathematical treatment is difficult, for we have to consider not *two* but a *very large number* of sources sending out waves which all differ in phase when they reach the point whose illumination is required. The following discussion is not offered as complete. A full treatment will be found in the advanced treatises.

Fresnel's treatment of the wave front. Rectilinear propagation.—Let $ABCD$ represent a section of a plane wave front coming from a very distant point source and travelling from right to

left in the direction of the arrows. It is required to find the effect of the wave at some point P ahead of it.

If we imagine two elements of the wave front selected by slits in a manner similar to Young's experiment, we should expect to find dark lines at points whose distances from the two elements differed by an odd number of half wavelengths, and bright ones where the difference was an even number of

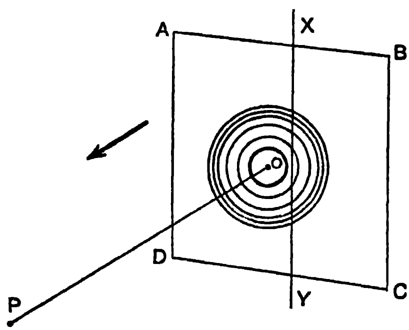

FIG. 225.—Wave front divided into half-period elements for a point P.

half wavelengths. The screen with two slits in it does not *produce* the interference, which is due to the path difference between the waves from the two secondary sources. It simply *selects* a sample of what must be happening with *any two elements* of the wave front. *Slits or no slits,* interference between the waves sent out from points on the wave front at $ABCD$ is to be expected at P.

Let PO be the perpendicular from P to $ABCD$. Let its length be p. Let the wavelength of the light be λ.

With centre P and radii $p+\lambda/2$, $p+\lambda$, $p+3\lambda/2$, and so on describe spheres tracing circles on the plane $ABCD$.

The wave front is thus divided into a series of rings or zones, whose edges differ successively in their distances from P by $\lambda/2$.

The quantity of light each zone contributes at P will be proportional to the *number* of secondary sources on it, that is, to its *area.*

The radius of the outer ring of the nth zone is given by

$$r_n{}^2 = \left(p + \frac{n\lambda}{2}\right)^2 - p^2$$

or $r_n{}^2 = pn\lambda$ if $n\lambda$ is very small, so that $n^2\lambda^2$ can be neglected.

The radius of its inner ring is

$$r_{n-1}^2 = \left(p + \frac{(n-1)\lambda}{2}\right)^2 - p^2$$

or $$r_{n-1}^2 = p(n-1)\lambda$$

with the same approximation.

So the area of the nth zone is

$$\pi r_n{}^2 - \pi r_{n-1}^2 = \pi p\lambda.$$

As λ is small always, this means that for small values of n the area of each zone is the same and so is the energy each emits.

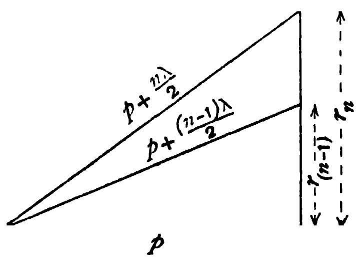

FIG. 226.

Now regarding each zone as a source of light emitting uniformly in all directions, the *energy* received at P from the nth zone will be proportional to $\dfrac{1}{\left(p + \frac{n\lambda}{2}\right)^2}$ by the inverse square law.

So the *amplitude* of the disturbance received at P from the nth zone will be proportional to $\dfrac{1}{\left(p + \frac{n\lambda}{2}\right)}$, since the energy is proportional to the square of the amplitude. With the same approximation, if $n\lambda$ is small, $\left(p + \frac{n\lambda}{2}\right)^{-1}$ becomes $\left(p - \frac{n\lambda}{2}\right)$. The numerical

left in the direction of the arrows. It is required to find the effect of the wave at some point P ahead of it.

If we imagine two elements of the wave front, selected by slits in a manner similar to Young's experiment, we should expect to find dark lines at points whose distances from the two elements differed by an odd number of half wavelengths, and bright ones where the difference was an even number of

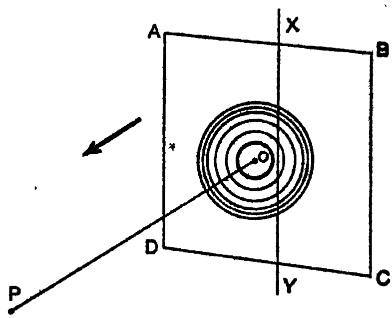

Fig. 225.—Wave front divided into half-period elements for a point P.

half wavelengths. The screen with two slits in it does not *produce* the interference, which is due to the path difference between the waves from the two secondary sources. It simply *selects* a sample of what must be happening with *any two elements* of the wave front. *Slits or no slits*, interference between the waves sent out from points on the wave front at $ABCD$ is to be expected at P.

Let PO be the perpendicular from P to $ABCD$. Let its length be p. Let the wavelength of the light be λ.

With centre P and radii $p+\lambda/2$, $p+\lambda$, $p+3\lambda/2$, and so [illegible] describe spheres tracing circles on the plane $ABCD$.

The wave front is thus divided into a series of zones, whose edges differ successively in their dist[illegible] P by $\lambda/2$.

The quantity of light each zone contributes at P [illegible] portional to the *number* of secondary sources on it, th[illegible] area.

The radius of the outer ring of the nth zone is given by

$$r_n^2 = \left(p + \frac{n\lambda}{2}\right)^2 - p^2$$

or $r_n^2 = pn\lambda$ if $n\lambda$ is very small, so that $n^2\lambda^2$ can be neglected.

The radius of its inner ring is

$$r_{n-1}^2 = \left(p + \frac{(n-1)\lambda}{2}\right)^2 - p^2$$

or $$r_{n-1}^2 = p(n-1)\lambda$$

with the same approximation.

So the area of the nth zone is

$$\pi r_n^2 - \pi r_{n-1}^2 = \pi p\lambda.$$

As λ is small always, this means that for small values of n the area of each zone is the same and so is the energy each emits.

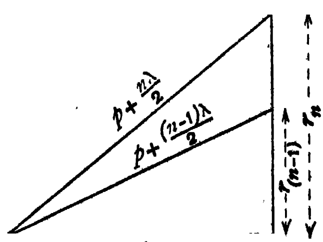

FIG. 226.

Now regarding each zone as a source of light emitting uniformly in all directions, the *energy* received at P from the nth

values of the contributions of each zone to the resulting amplitude at P thus form an arithmetical progression with terms proportional to p, $p-\frac{\lambda}{2}$, $p-\frac{2\lambda}{2}\ldots$, and so on.

Now the average path difference between secondary sources on adjacent zones is $\frac{\lambda}{2}$. So that if the contribution of the first zone to the amplitude at P is $+a_1$, that of the second is $-a_2$, that of the third $+a_3$, and so on.

We can then write for the resulting amplitude at P,

$$A = a_1 - a_2 + a_3 - a_4 + a_5 \ldots + a_n$$

or

$$A = \frac{a_1}{2} + \left(\frac{a_1}{2} - a_2 + \frac{a_3}{2}\right) + \left(\frac{a_3}{2} - a_4 + \frac{a_5}{2}\right) \ldots + \frac{a_n}{2}.$$

As a_2 is the arithmetic mean between a_1 and a_3,

a_4 ,, ,, ,, a_3 ,, a_5,

and so on, each of the terms within the brackets vanishes. **The resultant amplitude at P is then half the sum of the effects of the first and last zones.**

If the wave front is not limited by any obstacle a very large number of the zones will be able to contribute their effects to P, so that a_n will be very small and the resultant amplitude at P is half that contributed by the first zone. The same is true if the wave front approaches P through an aperture so large that a very large number of zones are transmitted, as in the simple experiment with numerous collinear apertures to show that "light travels in straight lines."

On the other hand, if part of the wave front is intercepted by an opaque screen whose edge is at XY and only a few complete zones for P pass it, n will be small and a_n will not be negligible. The resultant amplitude at P will depend on whether an odd or even number of zones is transmitted. For an odd number, a_n and a_1 will differ in path by an even number of half wavelengths and P will be bright. For an even number, a_n and a_1 will differ in path by an odd number of half wavelengths, so that P will be dark—not completely dark as a_n is less than a_1, but showing a minimum of brightness. So dark bands should be discernible beyond the geometrical boundary of the obstacle's shadow.

Similarly, if XY intercepts the first few zones for P so that P appears to be in the geometrical shadow, portions of the zones for which n is larger will be able to contribute some light to P. These are now decreasing as n increases, so what is observed is a gradual falling off of illumination as P moves into the shadow region.

Thus the shadow of XY should be bounded by light and dark bands and light should be observable within the geometrical shadow. With white light the coloured bands observed by Grimaldi should be given.

The zones are also called **half-period elements,** since the average phase difference between successive zones is π.

Cylindrical wave front.—So far we have dealt in general terms with the diffraction of plane waves. We will now discuss the practical case of the wave front coming from a series of point sources arranged on a straight line, that is, from an *illuminated narrow slit.* At all points near the line perpendicular to the slit through its centre the wave front will be a *cylinder* with its axis parallel to the slit. The diagrams will represent sections of the wave front within this region by planes perpendicular to the slit. The half-period elements or zones will be strips parallel to the slit.

Diffraction at a straight edge.—Let O be the line source, AB

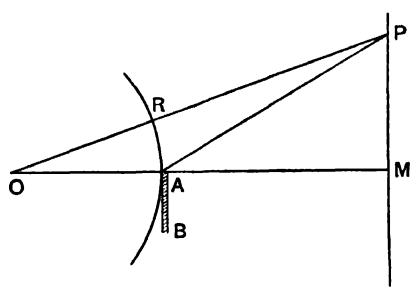

FIG. 227.—Diffraction at a straight edge.

the obstacle, the point at which OA produced meets a screen normal to it marking the geometrical boundary of the shadow.

Consider the point P on the screen. Join PO, and let it cut the wave front in R. The effect of the wave at P is confined to

a limited number of half-period elements round R; and if A is so far from R that none of the important elements is intercepted, then the screen will have no effect on the illumination at P.

If the screen intercepts some of the important elements then P will be a maximum or a minimum, according as RA contains an odd or even number of half-period elements. Complete darkness at a minimum is not to be expected, because the whole portion RS of the wave front has been left intact. If $AP - RP$ is an odd number of half wavelengths, RA will contain an odd number of half-period elements. If $AP - RP$ is an even number of half wavelengths, RA will contain an even number of half-period elements.

Call OA a, AM b, and PM x.

Then, by Pythagoras's theorem,

$$AP^2 = b^2 + x^2 = b^2\left(1 + \frac{x^2}{b^2}\right).$$

So, $$AP = b\left(1 + \frac{x^2}{b^2}\right)^{\frac{1}{2}} = b\left(1 + \frac{x^2}{2b^2}\right) \text{ if } x \text{ is small}$$

or $$AP = b + \frac{x^2}{2b}.$$

Similarly, $$OP = (a+b) + \frac{x^2}{2(a+b)}.$$

So, $$RP = OP - OR = b + \frac{x^2}{2(a+b)},$$

and the difference we wish to consider,

$$AP - RP = \frac{x^2}{2b} - \frac{x^2}{2(a+b)} = \frac{x^2}{2} \cdot \frac{a}{b(a+b)}.$$

Then if $$\frac{x^2}{2} \cdot \frac{a}{b(a+b)} = (n + \tfrac{1}{2})\lambda, \text{ } P \text{ will be bright,}$$

and if $$\frac{x^2}{2} \cdot \frac{a}{b(a+b)} = n\lambda, \text{ } P\text{'s illumination will be a minimum.}$$

If P is within the geometrical shadow the most important of its half-period elements will be cut off by the screen. Those not cut off by the screen are diminishing rapidly in importance and becoming nearly equal, so that the effect at P will be that due to the first few elements passing the edge A. The further P is

below M, the less important will these zones be. So in the geometrical shadow there will be no fringes, but a *gradual falling off* of illumination.

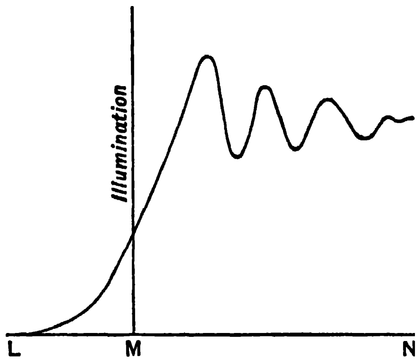

FIG. 228.—Illumination within and beyond geometrical shadow of straight edge.

The distribution of illumination will be as the diagram shows. The portion LM represents the geometrical shadow below M, and between M and N the alternate maxima and minima occur. Beyond N there is uniform illumination, for P is so far from the edge that no important elements are intercepted.

Diffraction round a narrow wire.—If AB be a narrow obstacle of thickness c, and MN its geometrical shadow, diffraction bands

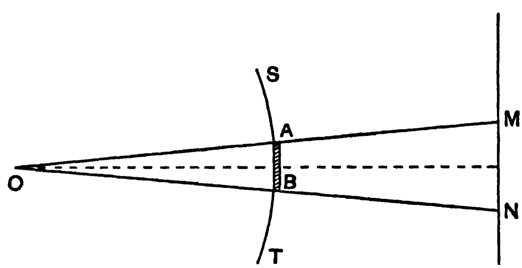

FIG. 229.—Diffraction round narrow wire.

will be observed outside M and N as in the last case, whatever the value of c. If, however, c is so small compared with the distance from obstacle to screen that the light bending into the

shadow (the part MN of Fig. 229) on both sides is superposed on the screen, interference fringes will be produced just as if A and B were two separate slits. These fringes are equidistant and a dark band occurs when $AP-BP=(n+\frac{1}{2})\lambda$, a bright band when $AP-BP=n\lambda$. The distances x of the fringes from the normal to the screen through O are given by

$$x=\frac{u}{a}(n+\tfrac{1}{2})\lambda \text{ for a dark band}$$

and
$$x=\frac{u}{a}n\lambda \text{ for a bright band.}$$

Notice again that these are ordinary Young interference fringes. They can be distinguished from the diffraction bands proper by covering one edge of the wire, when the diffraction bands on the other side remain but the fringes in the centre disappear.

Diffraction through a narrow slit.—As before, MN is the geometrical " shadow " of the slit AB on a screen, light is diverging

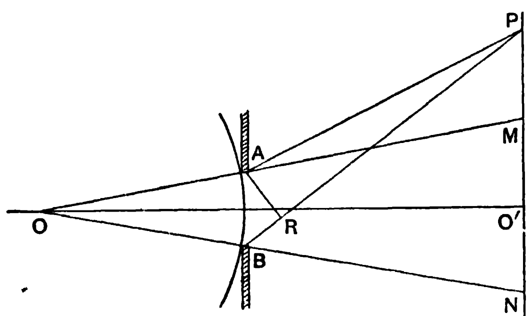

FIG. 230.—Diffraction through narrow slit.

from a slit at O, and OO' is the axis of the slit. To find the illumination at any point P at distance x from OO' divide the wave front at AB into half-period elements for P.

If the difference in path $BP-AP$ equals an even number of half wavelengths there will be an even number of half-period elements in AB and P will be dark, while if $BP-AP$ equals an odd number of half wavelengths AB has an odd number of half-period elements for P and so P is bright.

Fringes will be produced in the shadow beyond M and N (where with a single edge A or B there would be no fringes), and these will be sets of equidistant interference bands.

If d is the distance of the screen from the slit and c the width of the slit, then $x = \frac{d}{c} n\lambda$ **for a minimum at P** and $x = \frac{d}{c}(n + \frac{1}{2})\lambda$ **for a maximum at P.**

If the screen is so far from the slit that $BM - AM$ is less than half a wavelength then all the fringes are outside the region MN. More detailed investigation shows that the illumination at the screen due to a narrow slit would then be somewhat as shown in Fig. 231.

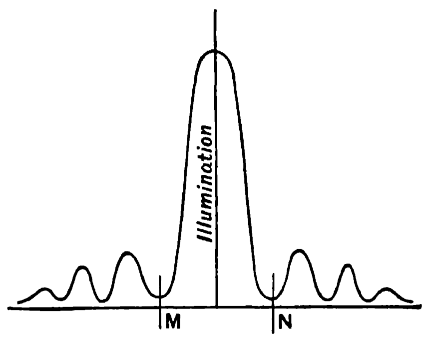

FIG. 231.—Illumination due to narrow slit.

Spherical wave front.—The case of a spherical wave diverging from a point source and falling on either a circular obstacle or a circular aperture can also be treated simply; exactly the same argument and the same geometrical figures as for the cases of cylindrical wave striking a wire or parallel-sided aperture parallel to the slit from which it emerges are used. In this case, of course, the fringes will be circular.

The centre of the shadow cast by a small circular obstacle should be as bright as if the obstacle were not there. The illumination at the centre of the shadow is provided by the first half-period element which passes the disc, and the Fresnel calculation of p. 282 applied to this shows that the effect at the centre of the shadow will be that due to half the amplitude of the first

Elementary theory of the grating.—Consider a small element of the grating. AB, $A'B'$, $A''B''$ are three opaque lines each of width a, and the slits between them are each of length b.

Parallel light falls normally on the grating and is diffracted by each slit. The light from one slit will reinforce that from the other slits when the light from the edges B, B', B'' differs in path by an even number of half wavelengths, for then so do the waves travelling from any other corresponding points on the slits.

Let this occur for light of wavelength λ in a direction θ to the normal to the grating.

Draw a line BM, perpendicular to the rays, meeting that from B' in M. The difference in path between the rays from B and B' is then

$$B'M = BB' \sin\theta = (a+b)\sin\theta.$$

Reinforcement occurs when

$$n\lambda = (a+b)\sin\theta,$$

where n is 0, 1, 2, 3, ...

If $n=0$, $\sin\theta=0$; thus for all values of λ this will be a maximum, so the centre of the pattern is white with white light.

If $n=0$, 1, 2, 3, ... , and so on, $\sin\theta$ is proportional to λ; so that, with white light, spectra, called spectra of the first, second, third, ... order, are formed.

If parallel light, instead of falling normally on the grating, is incident at an angle i, the equation for the path difference between corresponding points on successive slits becomes

$$n\lambda = (a+b)(\sin\theta + \sin i).$$

The deviation for light of wavelength λ in the nth spectrum is

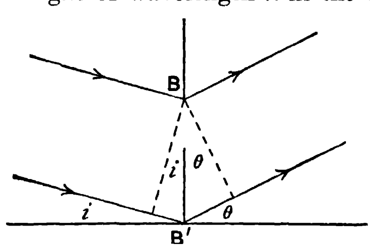

FIG. 233.—Minimum deviation.

given by $D = i + \theta$. This will be a minimum when $dD = 0$ or $di + d\theta = 0$. Now from the above equation $\cos\theta\, d\theta + \cos i\, di = 0$,

If $di = -d\theta$, $\cos\theta = \cos i$, and as each must necessarily be an acute angle, $\theta = i$. This is the condition for **minimum deviation.**

Spectrum produced by a grating.—For the nth order spectrum $n\lambda = (a+b)\sin\theta$ and $n\,d\lambda = (a+b)\cos\theta d\theta$. So the ratio of the small change in deviation $d\theta$ to the corresponding small change in wavelength $d\lambda$ is

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}.$$

Defining $\frac{d\theta}{d\lambda}$ as the **dispersive power** it is seen that this is greatest in those spectra for which n is highest, and is increased in a given order by making $(a+b)$ small.

Neglecting for the moment that θ will be different for the same order and wavelength with different gratings, we can say that the dispersive power at a given wavelength depends only on n and $(a+b)$. So *doubling the number of lines to the inch means that the spectra produced are exactly doubled in length, and the dispersion in each part of the spectrum is doubled.* The ratio of the widths occupied in the spectrum of a given order by any two regions of it will thus always be the same, and *all diffraction spectra will appear similar.* No two prisms need necessarily give the same spectrum. The spectrum of the diffraction grating is called the **normal spectrum.**

In the grating spectrum, the red end of the spectrum is most deviated, while a prism refracts the violet end most.

Of the numerous spectra formed by the grating, overlapping occurs between the third and second and between higher orders.

Resolving power of optical instruments.—1. *The telescope.*—Con-

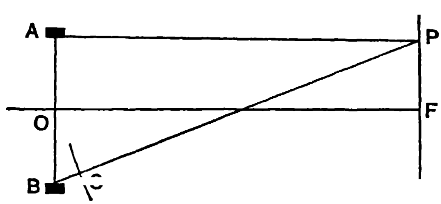

FIG. 234.—Resolving power of telescope.

sider the objective of a telescope forming an image of á distant object in its focal plane. Each point on the object will give rise

to a diffraction pattern, a bright maximum surrounded by alternate maxima and minima. Two points will just be observable as separate when the first maximum of each falls on the first minimum of its neighbour.

Let AB represent the aperture of the telescope, of diameter D. Use the same treatment as for a slit (p. 288). The image of a distant point on the axis will be formed at the principal focus, F. Let P be a point on the first minimum of the pattern surrounding F and let $PF=x$, $FO=f$.

With centre P and radius PA describe an arc cutting PB in C. The condition for P to be on *any* minimum is that the aperture shall have an even number of half-period elements for P, so $BC=n\lambda$. For P to be on the *first* minimum, $BC=\lambda$.

Now,
$$\frac{PF}{PO}=\frac{BC}{AB};$$
and as PO nearly equals FO,
$$\frac{PF}{FO}=\frac{BC}{AB}$$
or
$$\frac{x}{f}=\frac{\lambda}{D}, \quad \text{whence} \quad x=\frac{f\lambda}{D}.$$

Two points on the image at a distance $f\lambda/D$ apart will be just distinguishable as separate. The expression $f\lambda/D$ is called the **resolving power** of the objective. Since D/f is the relative aperture of the lens we can write:

$$\text{resolving power}=\frac{\text{wavelength of light used}}{\text{relative aperture of objective}}.$$

The angle subtended at the objective by two points whose images are at a distance x apart is x/f or λ/D, so the **angular limit of resolution** is λ/D.

The above discussion is rudimentary and overestimates the objective's performance. Rigorous investigation, due to Airy, gives for the resolving power $1{\cdot}22f\lambda/D$. The resolving power is said to be *high* when this expression is *small*.

We shall, however, keep to our simple expression for subsequent argument.

The compound microscope cannot be dealt with properly in this manner, as the object is not self-luminous. Abbe's expression

for the resolving power is $\frac{\lambda}{2\mu \sin \alpha}$, where 2α is the angle subtended at the object by the objective and μ the refractive index of the medium surrounding the object.

One reason for the large aperture of astronomical telescope objectives is the high resolving power they give. An additional advantage, since the objects viewed are luminous points, is the great increase in brightness of the images. The image of a fixed star is not a point, but a series of diffraction rings. The radius of the central *maximum* is given by our simple argument as $f\lambda/2D$, and more accurately as $0{\cdot}61\, f\lambda/D$. Consider this real image as being formed on a screen. Its area is inversely proportional to $(D/f)^2$. The quantity of light energy collected by the objective is proportional to D^2, and most of this light is concentrated on the central bright disc, whose brightness is thus proportional to D^4/f^2. If the real image is an " aerial image " filling a solid angle proportional to $(D/f)^2$ with light, the gain in brightness is proportional to $D^4/f^2 \div (D/f)^2$, or D^2. Note that D/f, besides D, is one of the important factors for brightness when the image is received on a plate. For the 100-inch Mount Wilson reflector D/f is 1/5, and the 200-inch California reflector is being made with the ratio D/f equal to 1/3·3.

The angle subtended at the objective by two distant points can be found by reducing the aperture to a value D', at which they cannot be observed as distinct. Then λ/D' should give the angle in radians. Using this principle in an interferometer Michelson was able to estimate the angular diameter of fixed stars.

2. *Spectroscopes.*—The resolving power of any spectroscope is given by $\lambda/d\lambda$, where $d\lambda$ is the smallest difference in wavelength detectable at wavelength λ.

The observations are made either photographically or with a telescope, and in each case the limiting factor can be regarded as the aperture of the objective D.

The objective will resolve two wavelengths as distinct if they subtend at it an angle $d\theta = \lambda/D$.

So, $$\frac{\lambda}{d\lambda} = D\frac{d\theta}{d\lambda}.$$

In the case of the prism spectroscope, observations are made at minimum deviation θ.

Let the difference in path traversed in glass by the extreme rays be t, represented by BC in the figure.

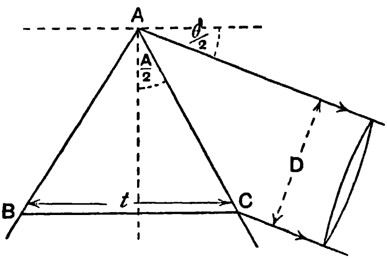

FIG. 235.—Resolving power of prism spectroscope.

Here, $AC = \frac{t}{2}\Big/\sin\frac{A}{2}$; also, $AC = D/\cos\frac{(A+\theta)}{2}$.

So, $$\frac{t}{2}\Big/\sin\frac{A}{2} = D/\cos\frac{(A+\theta)}{2}$$

and $$D = \frac{t}{2}\cdot\frac{\cos\frac{(A+\theta)}{2}}{\sin\frac{A}{2}}.$$

Now, $$\mu = \frac{\sin\frac{(A+\theta)}{2}}{\sin\frac{A}{2}} \text{ for the glass}$$

so $$\frac{d\mu}{d\theta} = \frac{1}{2}\,\frac{\cos\frac{(A+\theta)}{2}}{\sin\frac{A}{2}}.$$

Substituting $\frac{d\mu}{d\theta}$ for its longer equivalent in the expression fo D,

$$D = t\frac{d\mu}{d\theta}.$$

Then, $$\frac{\lambda}{d\lambda}=D\frac{d\theta}{d\lambda}=t\frac{d\mu}{d\theta}\cdot\frac{d\theta}{d\lambda}=t\frac{d\mu}{d\lambda}.$$

If the whole of the prism is in use, t is the length of its base, in this case the full aperture of the telescope is probably not in use, but by D in the preceding argument we can understand the diameter of that part of the aperture actually filled with rays from the prism, bounded by the edges of the prism instead of the rim of the objective. The early spectroscopists used trains of prisms, effectively increasing t to obtain high resolving power. A simple method of measuring the resolving power of a good prism spectroscope has been discussed in Chapter XVI.

In the case of a grating, if N lines are in use, the length of grating used is $N(a+b)$ cm. The angle of diffraction is θ and the telescope aperture D, so that

$$D=N(a+b)\cos\theta.$$

Now if the grating is used with parallel light incident normally,

$$n\lambda=(a+b)\sin\theta.$$

So, $$\frac{d\theta}{d\lambda}=\frac{n}{(a+b)\cos\theta},$$

whence $$D\frac{d\theta}{d\lambda}=N.(a+b)\cos\theta\times\frac{}{(a+b)\cos\theta}=Nn,$$

and $$\frac{\lambda}{d\lambda}=Nn.$$

So the resolving power of a grating spectroscope depends on the total number of lines *in use* and the order of the spectrum.

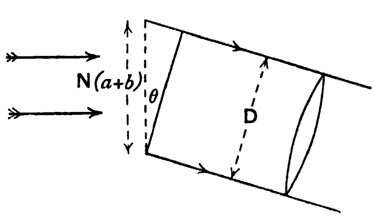

FIG. 236.—Resolving power of grating spectroscope.

Reflection gratings.—Good gratings can be made by ruling very fine parallel grooves on a polished metallic surface. The grooves act as obstacles and light is regularly reflected from the

spaces in between the grooves, behaving as if it had been transmitted through these spaces from the virtual image of the source behind the surface.

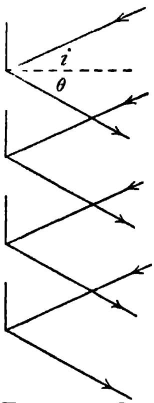

FIG. 237.—Reflection grating.

If i is the angle of incidence and θ the angle of diffraction we get

$$n\lambda = (a+b)(\sin i - \sin \theta),$$

giving the direction θ, in which reinforcement occurs for light of wavelength λ in the nth order spectrum.

The case of a concave reflecting grating may be treated very simply as follows. Rays diverging from a narrow slit parallel to the rulings will be made to converge after reflection and diffraction.

We can regard the grating as a concave mirror, for which the law of reflection $i+\theta=2i$ is replaced by $i+\theta=$ some other constant for a given angle of incidence i and wavelength λ.

Draw a circle with its centre on the axis of the mirror, its diameter, MC, equalling the radius of curvature of the mirror. If L is close to M it can be considered to lie on this circle.

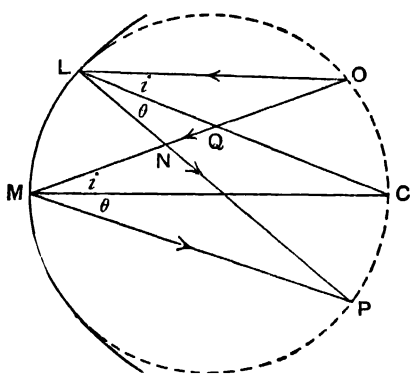

FIG. 238.—Concave grating.

Consider two rays OL, OM from a point O on this circle. After diffraction let them intersect at P. If the lines OM and LP intersect at N, OM and LC at Q, LQO and MQC are similar

triangles, so that $L\widehat{O}M = L\widehat{C}M$. Also LNO and MNP are similar triangles, whence $L\widehat{O}M = L\widehat{P}M$. So that $L\widehat{O}M$, $L\widehat{C}M$, $L\widehat{P}M$ are angles in the same segment of a circle, whence P lies on the circle containing C, O, M.

This argument holds whatever the value of θ, so that if a slit is placed at O the *whole spectrum will be formed along an arc of the circle we have drawn,* whose diameter equals the radius of curvature of the grating face.

This property of the concave grating was used by Rowland, who used a mounting which enabled the spectra to be photographed in the position $\theta = 0$.

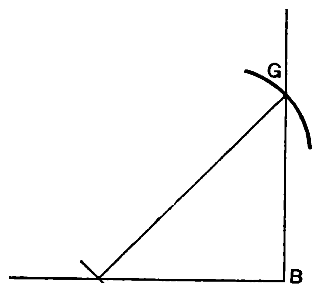

FIG. 239.—Rowland's mounting.

Grating G and plate P were fixed in an arm at a distance apart equal to the radius of curvature of the grating. The ends of the arm were constrained to move along two girders, AB, BC, at right angles, so that for all positions of GP the corner B was on the circle of which GP is a diameter. The slit was placed at B.

An advantage of the concave grating is its complete independence of any focussing system. No absorbing media of any kind need come between grating and plate, so that it is an ideal tool for work in the ultra-violet and infra-red. Also the full resolving power is utilized, for the whole aperture of the grating is available; while, however large a transmission grating may be, its resolving power is limited by the aperture of the spectrometer used.

Michelson's echelon.—Both dispersion and resolving power are proportional to n, the order of the spectrum. With ordinary gratings high order spectra are faint. Michelson designed a grating which would produce bright spectra of very high orders.

The necessary large path difference between beams from successive elements of the grating was secured by placing together plates of parallel-sided equal slabs of glass in the form of an **echelon**, as Fig. 240.

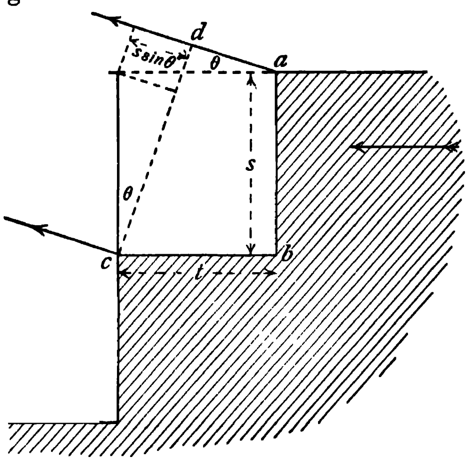

FIG. 240. -Michelson's echelon.

Suppose a beam of parallel monochromatic light is incident normally on the plates. Let

t be the thickness of the plates, bc;
s the width of the steps, ab;
μ the refractive index of the glass at wavelength λ;
n the wavelength retardation between rays emerging from corresponding points as a and c, at an angle θ.

Then,
$$n\lambda = \mu t - ad$$
$$= \mu t - t \cos \theta + s \sin \theta.$$

We shall only deal with very small values of θ, so we can write

$$n\lambda \doteq (\mu - 1)t + s\theta,$$

whence
$$\frac{d\theta}{d\lambda} = \frac{1}{s}\left[n - t\frac{d\mu}{d\lambda}\right].$$

Michelson's echelon.—Both dispersion and resolving power are proportional to n, the order of the spectrum. With ordinary gratings high order spectra are faint. Michelson designed a grating which would produce bright spectra of very high orders.

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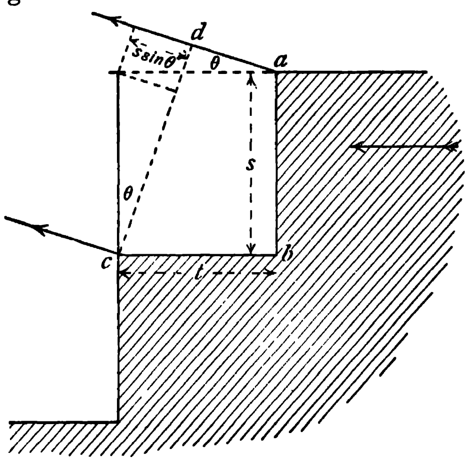


FIG. 240.—Michelson's echelon.

Suppose a beam of parallel monochromatic light is incident normally on the plates. Let

t be the thickness of the plates, bc;
s the width of the steps, ab;
μ the refractive index of the glass at wavelength λ;
n the wavelength retardation between rays emerging from corresponding points as a and c, at an angle θ.

Then,
$$n\lambda = \mu t - ad$$
$$= \mu t - t\cos\theta + s\sin\theta.$$

We shall only deal with very small values of θ, so we can write

$$n\lambda = (\mu - 1)t + s\theta,$$

whence
$$\frac{d\theta}{d\lambda} = \frac{1}{s}\left[n - t\frac{d\mu}{d\lambda}\right].$$

Since θ is small, we can put approximately

$$n = \frac{(\mu - 1)t}{\lambda} \text{ in this equation.}$$

So,
$$\frac{d\theta}{d\lambda} = \frac{t}{s\lambda}\left[(\mu - 1) - \frac{\lambda d\mu}{d\lambda}\right].$$

The whole of the expression in the square bracket is a constant for the glass used for any given wavelength and can be written as k.

$$\frac{d\theta}{d\lambda} = \frac{kt}{s\lambda},$$

giving the dispersion. Differentiating the original expression with respect to n,

$$\frac{d\theta}{dn} = \frac{\lambda}{s}.$$

Put $dn = 1$. Then the angular separation between successive orders of spectra, $d\theta$, $= \frac{\lambda}{s}$.

As with a prism or ordinary grating, the resolving power depends on the effective aperture. If D be the width of the aperture and we have N elements each of thickness all being utilized, $D = Ns$.

Now,
$$\frac{d\theta}{d\lambda} = \frac{kt}{s\lambda}.$$

So the resolving power
$$\frac{\lambda}{d\lambda} = D\frac{d\theta}{d\lambda} = Ns \cdot \frac{kt}{s\lambda}$$

or
$$\frac{\lambda}{d\lambda} = \frac{Nkt}{\lambda};$$

or, putting for t its approximate value, $\frac{n\lambda}{(\mu - 1)}$,

$$\frac{\lambda}{d\lambda} = \frac{Nnk}{(\mu - 1)}.$$

With a 20-plate echelon, for which $t = 10$ mm. and $s = 1$ mm., the dispersion works out to ·37 minute per Å. in the middle of the spectrum. Successive orders of spectra are separated by only 2 minutes of arc. The resolving power works out as ·027 Å.

The echelon would be useless for ordinary spectroscopic work on account of the large overlapping of different orders, and is used for exploring small regions of the spectrum in detail.

QUESTIONS ON CHAPTER XVIII

1. Describe the appearance of the shadow of a ball cast on a white screen by (*a*) the light from an electric lamp, (*b*) sunlight diverging from a pinhole in an opaque screen. What evidence does the second shadow give in support of the theory that light is a disturbance which travels in the form of waves? (O.H.S.C.)

2. Explain the construction and action of a diffraction grating. Describe how you would use it to find the wavelength of one of the green lines of the spectrum produced by a bunsen flame coloured by copper.

How many lines per cm. are there in a grating which gives a deflection of 20° for light of wavelength 6×10^{-5} cm.? (O.H.S.C.)

3. Describe and explain the spectra produced by a plane diffraction grating. What would be the effect if the distance between the rulings were (1) very large, (2) very small compared with the wavelength? (C.S.)

4. Explain the action of a diffraction grating, and describe how you would use it to measure wavelengths. Why is there a lower limit to the difference of wavelength which can be detected by a grating spectrometer? (C.S.)

5. What do you understand by the resolving power of optical instruments? How does it differ from the dispersive power in the case of a prism spectroscope? (O.S.)

6. Discuss under what conditions optical diffraction effects and interference fringes are obtained.

Describe *one* method of obtaining straight line interference fringes and *one* method of obtaining circular fringes. Explain in each case how the fringes are produced and how the appearance of the interference system changes if white light is substituted for light from a sodium flame. (O.H.S.C.)

CHAPTER XIX

POLARIZATION

Introduction. Longitudinal and transverse waves.—If a long spiral spring is held firmly fixed at one end and the other end be moved to and fro along the axis of the spiral, waves travel along the spring. Any section of the spring is executing approximately a simple harmonic motion, the displacements being *in the direction of propagation* of the disturbance, which is therefore called a **longitudinal wave.** Only *one* direction is involved, the *direction of propagation.*

If the end of the spring held in the hand is moved from side to side in a direction at right angles to the axis of the spiral, waves of another type are sent along the spring. Each section of the spring executes an approximate simple harmonic motion but this time the displacements are *at right angles to the direction of propagation,* and all in the plane containing the spring and the hand's motion. The disturbance sent down the spring is called a **transverse wave.** *Two* directions are needed to specify it completely—the *direction of propagation* and that of the *plane in which the displacements of the particles of the spring take place.*

Sound travelling in any medium is propagated as a longitudinal vibration; ripples on the surface of water are an example of transverse vibration. We had no need to enquire in discussing interference and diffraction which kind of wave we dealt with, as both types show these effects and the mathematical treatment in the two cases is identical.

An experiment with two thin plates of tourmaline, cut with their faces parallel to the axis of the crystal, shows a new phenomenon.

A beam of light transmitted by one plate is transmitted fully by the second only when the two are set with their axes parallel. As one is rotated relatively to the other, the intensity of the beam transmitted by both is reduced, and when their axes are at right angles complete extinction occurs.

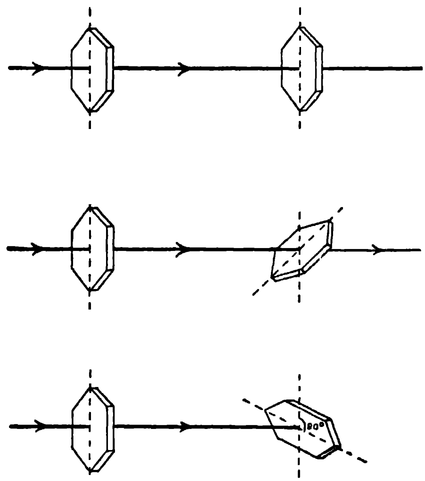


FIG. 241.—Parallel and crossed tourmalines.

The light transmitted by the first crystal differs from ordinary light. It will only be fully transmitted by a second plate when the axis of the second plate is in a certain direction at right angles to the direction of propagation.

This suggests that *light is a transverse wave motion*, and that although in ordinary light vibrations in all possible planes may be present, a tourmaline crystal transmits only those vibrations, or those components of all the vibrations, in *one particular plane*. The transmitted light is said to be **polarized** in that plane.

For the moment we will think of the plane of polarization as being some plane containing the direction of propagation and making some definite angle with the axis of the crystal. Then a second crystal whose axis is not parallel to the first will transmit only that component of the vibration of the polarized light which is in the plane containing its own axis and the direction of propagation.

Polarization by reflection.—A parallel beam of monochromatic light reflected from a sheet of glass at an angle of about 56° appears to the eye to have suffered no change on reflection.

Examination of the reflected light through a tourmaline plate shows, however, that the reflected light is plane polarized. Extinction occurs when the axis of the tourmaline is in the plane of incidence. We say that the reflected light is **polarized in the plane of incidence,** and with this convention fixing our ideas about the

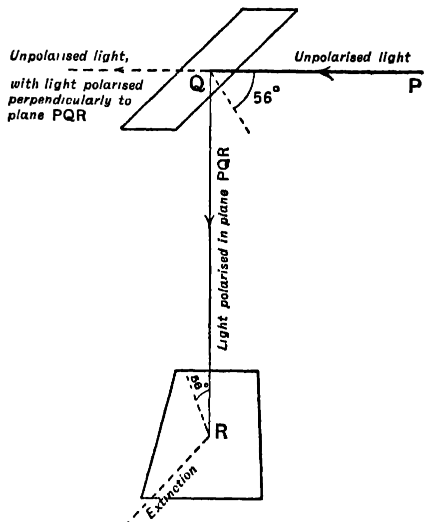


FIG. 242.—Polarization by reflection.

plane of polarization we can restate the conclusion of the last paragraph more precisely and say that the axis of the tourmaline crystal is at *right angles* to the plane of polarization of the light it transmits.

A second plate of glass, set to receive the polarized light at an angle of about 56°, with the plane of incidence at right angles to the plane of polarization, reflects no light. Biot's apparatus for demonstrating the extinction of light by reflection, and also for examining crystal plates in plane polarized light, is shown in principle in Fig. 242.

The angle of incidence at which complete plane polarization

occurs is called the **polarizing angle.** Brewster showed that for any transparent medium the polarizing angle occurs when the reflected and transmitted rays are at right angles to one another, so that $\sin i = \cos r$ and $\tan i = \mu$. For glass of refractive index 1·53 the polarizing angle is 56° 49′. We should not expect complete polarization for all colours at the same angle of incidence on a dispersive medium, and when the experiment is performed with white light the second mirror delivers a faint coloured beam instead of giving complete extinction.

The light transmitted at Q contains ordinary light and that polarized at right angles to the plane of incidence. Reflection at the back surface of the plate occurs at the polarizing angle for light travelling from glass to air, so that the second reflection reinforces the first and the transmitted beam is proportionately richer in light polarized at right angles to the plane of incidence. By using many parallel sheets of glass it is possible to separate the incident light into two completely plane-polarized beams—a reflected beam polarized in the plane of incidence and a transmitted beam polarized perpendicularly to the plane of incidence.

Double refraction in uniaxial crystals.—Iceland spar or calcite, a crystalline form of calcium carbonate, is found in Nature in

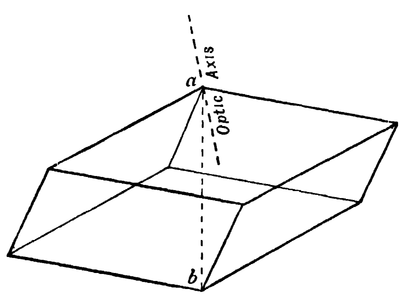


FIG. 243.—Crystal of calcite.

several forms, all of which give a rhombohedron on cleavage. The faces of the rhombohedron are parallelograms, each having angles of 101° 55′ and 78° 5′. Two of the solid angles, a and b, are contained by three obtuse angles. The direction making equal angles with the three faces which meet at these points is called the **optic axis** of the crystal. It should be emphasized that

the *direction* is the optic axis and not any particular *line* in that direction, and that the line *ab* joining the two corners will only be parallel to the optic axis if by accident all the edges of the rhomb are equal. Calcite has only one optic axis and is called a **uniaxial** crystal. Other crystals of the same type are quartz and tourmaline. Any plane parallel to the optic axis and perpendicular to two opposite faces of the crystal is called a **principal section.**

If a narrow parallel beam of light be allowed to fall on one face of a calcite crystal in a principal section two beams emerge, and on a screen behind the rhomb two light spots will be seen. This phenomenon is called **double refraction.** As the rhomb is rotated about the direction of the incident light one of these spots remains unchanged in position, as would happen with the single image given by an ordinary plate of glass. The other spot rotates with the principal section of the crystal, so that both refracted beams are always in a principal section.

The rays forming the stationary spot obey the ordinary laws of refraction; the others do not. Thus we can consider each ray on striking the first face of the rhomb to be split up into an *ordinary ray which obeys the laws of refraction* and an *extraordinary ray which does not.* The lateral displacement of the ordinary ray is greater than that of the extraordinary ray.

If in the path of the emergent beam we place a sheet of glass inclined to it at the polarizing angle, receive the reflected beam on a screen, and rotate the glass sheet about an axis in the direction of the emergent beam, the ordinary image is extinguished in one position. This occurs when the plane of incidence on the glass plate is at right angles to the principal section of the crystal. Thus the **ordinary ray is plane polarized in the principal section.** The relative brightness of the two images changes as the glass is rotated, and when it is at right angles to its first position the extraordinary image is extinguished. So the **extraordinary ray is plane polarized in a plane at right angles to the principal section.**

If the original beam, instead of being ordinary unpolarized light, is plane polarized in some plane making an angle θ with the principal section, we can calculate the intensity of the ordinary and extraordinary rays. For, dealing as we did on p. 283 with

the vector quantity amplitude, if the amplitude of the incident light is a, represented by the line OA (Fig. 244), its component in the direction OY of the principal section's plane is $a \cos \theta$, and in the direction OX is $a \sin \theta$, by the ordinary rules for resolving vectors. The intensity of the ordinary ray is $\propto a^2 \cos^2 \theta$ and that of the extraordinary ray to $a^2 \sin^2 \theta$, and their sum $\propto a^2$, the intensity of the incident ray. From these considerations the student can work out for himself the appearance of the screen if a second crystal of calcite is placed after the first with its principal section containing the direction of the beams, and is rotated about this direction as axis.

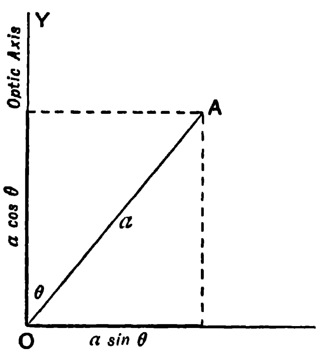


FIG. 244.

Further investigation. Huygens' constructions.—So far we have considered a particular case—the incident light travelling in a principal section. Also, only one angle of incidence has been taken. This might lead us to conclude that, apart from polarization, the only difference between the extraordinary and ordinary rays was that the refractive index of calcite was greater for the latter. More general experiments show the term "extraordinary" to be well deserved. We have stated that it does not obey the laws of refraction. In the general case the extraordinary ray will *not* lie in the plane containing the incident ray and the normal at the point of incidence, and the ratio $\frac{\sin i}{\sin r}$ is *not* a constant for all angles of incidence. If the plane of incidence happens to be that of a principal section the first law is obeyed, but still $\frac{\sin i}{\sin r}$ is not constant. If the rhomb is cut so that the optic axis is parallel to the face of the crystal and perpendicular to the plane of incidence, both laws are obeyed.

According to Huygens, the wave propagated in a uniaxial crystal consists of two surfaces, one within the other. That cor-

responding to the ordinary ray is a sphere, and that for the extraordinary ray an ellipsoid. The two wave surfaces touch at two points, and the line joining these points is the optic axis. In calcite the *ordinary* ray travels *more slowly* than the extraordinary ray in all directions but one, and the wave surface is as shown in Fig. 245.

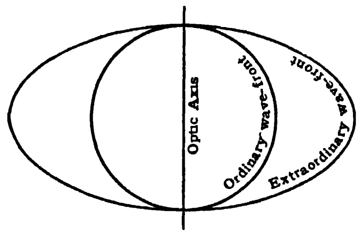


FIG. 245.—Ordinary and extraordinary wave surfaces in calcite.

Calcite and all crystals with this type of surface are called **negative crystals.** In **positive crystals** the ordinary ray travels faster than the extraordinary and the ellipsoid is inside the sphere. Quartz is a positive crystal.

Huygens' construction for the refraction of a plane wave incident on a calcite crystal from air in a principal section is shown in Fig. 246.

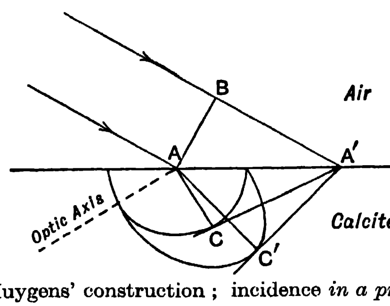


FIG. 246.—Huygens' construction ; incidence *in a principal section.*

AB is the trace of the incident wave front, $A'C$ that of the ordinary and $A'C'$ the extraordinary wave fronts. In this case C and C' lie in the plane of the paper, but BA'/AC' will depend on the angle of incidence and not be constant. The *least* value of BA'/AC' is when the rays are incident perpendicular to the optic axis, and we can call this the **extraordinary refractive index,** μ_e.

Fig. 247 shows the construction for the case when the optic axis lies in the surface perpendicular to the plane of incidence. The section of the double surface by the plane of the diagram is now two concentric circles. As before, the traces $A'C$ and $A'C'$ give the traces of the wave fronts corresponding to the ordinary and extraordinary rays. The ratio BA'/AC will be constant as

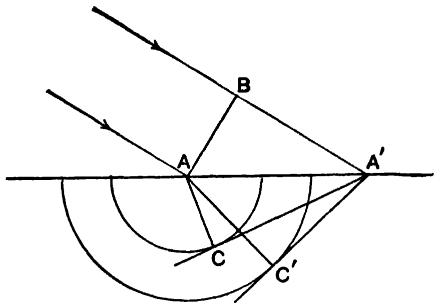


FIG. 247.—Huygens' construction ; optic axis *in the surface normal to the plane of incidence.*

before, giving the same refractive index for the ordinary ray, and $BA'/A'C'$ will also be constant, the same as the minimum (μ_e) of the previous case. *In this case only* both rays obey the laws of refraction.

There will be one direction, and one only, in a uniaxial crystal in which extraordinary and ordinary rays have the same refractive index, and that is **when the optic axis is perpendicular to the surface and in the plane of incidence and the light is incident normally.**

Double image prisms.—A simple calcite rhomb separates the ordinary and extraordinary rays, but the separation can be increased by the use of a calcite prism cut so as to take advantage of the foregoing facts. In **Rochon's prism** a parallelopiped is made of two prisms, A and B, cut with the optic axes as shown. Light incident normally on A, in the direction in which μ is the same for both ordinary and extraordinary rays, is undeviated until it reaches the oblique face of B ; here the ordinary ray is undeviated, but as the refractive index of B is less than that of A for the extraordinary ray, that is deviated towards the vertex of B. In **Wollaston's prism** the ordinary ray in A becomes the extraordinary ray in B. In Rochon's prism only the image formed by the extraordinary rays is coloured ; both are coloured

with Wollaston's. Double image prisms are used in many modern types of spectrophotometer and pyrometer.

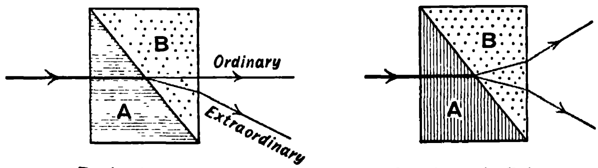


FIG. 248.—Double image prisms.

For calcite the ordinary refractive index is 1·658, so that the critical angle at an air boundary for the ordinary ray is 37° 6′. The minimum μ_e is 1·486, giving a maximum critical angle of 42° 18′. It should thus be possible to separate the two rays using total internal reflection.

The Nicol prism.—If either the ordinary or extraordinary ray could be extinguished, a rhomb of calcite would give a beam of plane polarized light. The **Nicol prism** removes the *ordinary ray* by total reflection at a film of Canada balsam. A rhomb about three times as long as wide is cut through along a plane parallel to the long diagonal of the end faces and perpendicular to the principal section. The cut faces are then cemented together with a Canada balsam film. The refractive index of Canada balsam is about 1·55, which is greater than that of calcite for the extraordinary ray travelling down the prism, so that this will be transmitted.

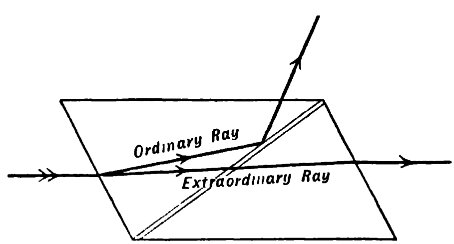


FIG. 249.—Nicol prism.

It is considerably less than that of calcite for the ordinary ray, which may thus undergo total reflection. With the prism as constructed, the ordinary ray from a ray originally incident parallel to the long faces of the rhomb will strike the film of

balsam at an angle greater than 69° 30′, the critical angle for calcite to film, and will be totally reflected. The Nicol thus **transmits only** the **extraordinary ray** and delivers light polarized in a plane perpendicular to the principal section. It is clearly necessary to work with a *nearly parallel incident beam,* for with a diverging beam a considerable portion of the "ordinary ray" light might be transmitted.

Tourmaline also transmits only the extraordinary ray. In this case the ordinary ray is extinguished by *absorption.* A crystal behaving in this way is said to be **dichroic.**

Elliptical and circular polarization.—From Fig. 247 we concluded that when light was incident on a plate of a uniaxial crystal cut parallel to the axis, both ordinary and extraordinary rays

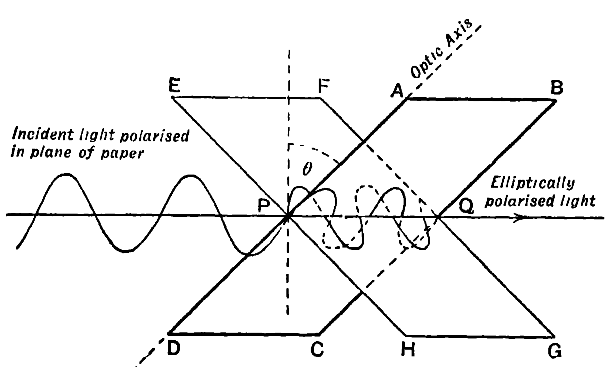


FIG. 250.—Conditions for production of elliptically polarized light, when $\theta = 45°$.

obeyed the ordinary laws of refraction. Hence if unpolarized light is incident normally on the plate, ordinary and extraordinary rays travel along the same path with different velocities, c/μ_0 and c/μ_e, where c is the velocity of light *in vacuo.*

Now consider a plane polarized ray incident normally on a thin plate, its plane of polarization making an angle θ with the principal section $ABCD$ at the point of incidence. If the amplitude of the incident ray is a, the amplitudes of its components in the principal section, and in the plane $EFGH$ at right angles to it, are $a \cos \theta$ and $a \sin \theta$ respectively.

As the two components travel with different velocities there will be a difference in phase between the two components when they emerge from Q. In the general case the transmitted ray will be split up into two parts differing in amplitude and phase.

If it happens that the plane of polarization of the incident light makes an angle of exactly 45° with the principal section, then $\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$. The two parts of the emergent ray will now be *polarized in perpendicular planes* and *differ in phase*, but *will have equal amplitudes*.

In dealing with linear simple harmonic motion in mechanics the student will learn that the path of a particle executing two S.H.Ms. of *equal amplitudes* at *right angles* is an *ellipse* with its axes inclined at 45° to the directions of the vibrations. Particular cases of this are the circle when the phase difference is $\frac{\pi}{2}$ or any odd multiple of it, and a straight line when the phase difference is any multiple of π.

So when the phase difference between the two parts of the emergent ray is $\pi/2$ or any odd multiple of it (that is, the extraordinary ray has gained a quarter of a wavelength on the other in a crystal of the calcite type in passing through the plate) the light is said to be **circularly polarized**, and the crystal plate is called a **quarter-wave plate** for that particular wavelength. If the phase difference is any multiple of π the light will be **plane polarized**, and for any other phase difference **elliptically polarized.**

Calculation of the thickness of a quarter-wave plate for any wavelength.—Let μ_0 and μ_e be the ordinary and extraordinary refractive indices, λ the wavelength of the light *in vacuo*, and t the thickness of the plate. The frequency of the light vibration will be the same whatever medium it travels in. Since velocity = frequency × wavelength we can write

$$\frac{\text{wavelength in plate}}{\text{wavelength } in\ vacuo} = \frac{\text{velocity in plate}}{\text{velocity } in\ vacuo},$$

so that for the ordinary ray the wavelength in the plate is $\frac{\lambda}{\mu_0}$ and for the extraordinary ray $\frac{\lambda}{\mu_e}$.

Suppose we have a crystal of the calcite type in which $\mu_0 > \mu_e$.

As the plate is to produce a path difference of a quarter of a wavelength, let there be n extraordinary and $(n+\frac{1}{4})$ ordinary wavelengths in thickness t.

Then, $$t=\frac{n\lambda}{\mu_e}=(n+\tfrac{1}{4})\frac{\lambda}{\mu_0}.$$

So, $$n\lambda=\mu_e t=\mu_0 t-\frac{\lambda}{4},$$

whence $$t=\frac{\lambda}{4(\mu_0-\mu_e)}.$$

For crystals of the quartz type, in which $\mu_e>\mu_0$, the same calculation gives

$$t=\frac{\lambda}{4(\mu_e-\mu_0)}.$$

For quartz, for sodium light of $\lambda\,5{\cdot}9\times10^{-5}$ cm., $\mu_e=1{\cdot}553$, $\mu_0=1{\cdot}544$, so

$$t=\frac{5{\cdot}9\times10^{-5}}{4(1{\cdot}553-1{\cdot}544)}=0{\cdot}0016 \text{ cm.}$$

If t is any odd multiple of this thickness the phase difference between the two components will be an odd multiple of $\pi/2$. Alternate even multiples of t give phase differences which are odd multiples of π; the plate is then a "half-wave plate."

Detection of circularly and elliptically polarized light.—Circularly polarized light cannot be discriminated from ordinary unpolarized light by a Nicol prism alone. For as the intensity of one component is diminished by rotating the Nicol, that of the other is increased by the same amount. A quarter-wave plate placed in the path of the beam before the Nicol will cause a relative phase change of $\pi/2$, so the phase difference is now π, the light is plane polarized, and one position of the Nicol will extinguish it. The quarter-wave plate would, of course, cause no change in the case of *unpolarized* light.

Elliptically polarized light will change in intensity when viewed through a rotating Nicol, just as if it were a mixture of plane polarized and unpolarized light. Again, if a quarter-wave plate is set in one particular position so that its axis is parallel to one of the axes of the ellipse or at 45° to the axis of the plate originally responsible for giving the elliptically polarized light, plane polarized light is given which the Nicol will detect.

The colours of crystalline plates in polarized light.—We have seen how a plate of uniaxial crystal cut parallel to the optic axis produces a phase difference between the emerging ordinary and extraordinary rays. The two rays do not, however, give rise to interference, for they are polarized in two different planes at right angles. But *two rays of light polarized in the same plane interfere, and two rays polarized at right angles and afterwards brought to the same plane of polarization interfere if they originally belonged to the same beam of light.* These facts were established by Fresnel and Arago.

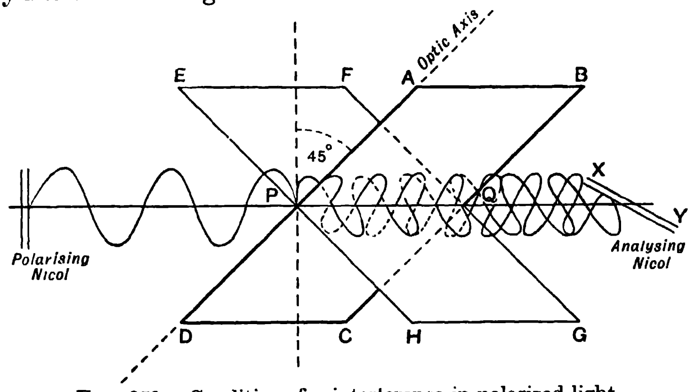


FIG. 251.—Conditions for interference in polarized light.

Consider Fig. 251. Light polarized at an angle of 45° to the optic axis of the crystal AD gives an ordinary ray polarized in the plane $ABCD$ and an extraordinary ray polarized in the plane $EFGH$. Suppose the thickness AB is such that for wavelength λ it is a whole-wave plate; there is then a path difference of one wavelength between ordinary and extraordinary rays. Suppose now that a Nicol prism is placed so as to transmit only light polarized in the plane through XY, at right angles to the original direction of polarization. Both ordinary and extraordinary rays are polarized at 45° to this plane.

Their amplitudes, a, are equal, and the resolved parts of these amplitudes in the direction XY are each equal to $a/\sqrt{2}$. So the Nicol now transmits two rays whose waves have the *same amplitude*, are *in the same plane*, and which come from the *same*

beam of polarized light, therefore being in a condition to interfere. As there is a phase difference of 180° between them, shown in fig. 252, they *will* interfere and no light will be seen through the Nicol. This is not very surprising at first, for after all the Nicol was set so that it would extinguish the whole of the original incident beam. But complete extinction occurs only for that particular wavelength λ. Some light of all other wavelengths can

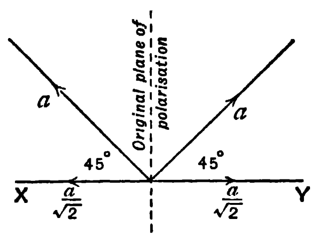


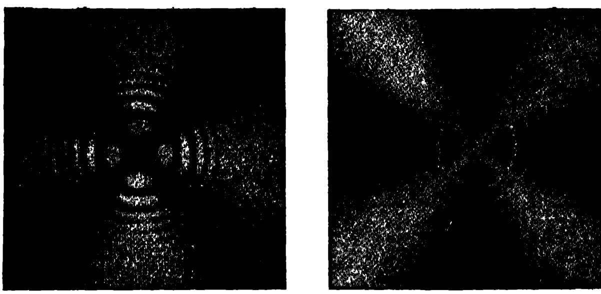
FIG. 252.

be transmitted, with the result that with white illumination the plate seen through the second Nicol presents a uniform hue which is not a pure spectral colour, but the kind of hue observable when Newton's rings are viewed in white light. Indeed Newton's rings can be reproduced by polarization using a plate of **selenite** cut with its axis parallel to the surface and grinding one face concave. (The first Nicol is usually called the **polarizer,** the second the **analyzer.** As set here they are said to be **crossed.**)

As the second Nicol is rotated, the transmitted beam of wavelength λ will, of course, increase in intensity, and when it is at right angles to the position giving extinction, wavelength λ is fully transmitted. In fact if the Nicol is rotated through 90°, it is clear that for all wavelengths the plane polarized light that is *extinguished* in the first position is *transmitted* in the second. So rotating the second Nicol through 90° changes the hue observed to its **complementary** colour. (The Nicols are then said to be **parallel.**)

The single colour observed with a parallel-sided plate cut parallel to the axis is the counterpart of the "equal thickness" phenomena in straightforward interference. A plate cut with its optic axis at right angles to the surface shows coloured rings cor-

responding to "curves of equal inclination" when viewed in divergent plane polarized light through an analyzing Nicol. For each angle of incidence (*a*) the thickness of plate traversed and (*b*) the difference in velocities of ordinary and extraordinary rays have definite values, so that the interference effect should consist of concentric rings. Fig. 253 shows the appearance of such a plate of a uniaxial crystal with the Nicols crossed and parallel. The light and dark crosses are explained simply in more advanced text-books but it is not proposed to discuss them here.



(*a*) Nicols crossed. (*b*) Nicols parallel.
FIG. 253.—Calcite viewed in diverging light through Nicols.

Rotation of the plane of polarization. Optical activity.—When a plate of quartz cut with the optic axis normal to its faces is placed between crossed Nicols and viewed in parallel light, no colour effect is shown, but the analyzer transmits some light. Extinction is restored by rotating the analyzer. This shows that the quartz plate has had the effect of **rotating the plane of polarization** of the light it transmits. The angle of rotation is proportional to the thickness of the quartz plate, being 21·72° per millimetre for sodium light. The rotation may be either in a clockwise or anticlockwise direction, looking towards the source. In the former case the quartz is said to be **right-handed** and in the second case **left-handed** in its **optical rotatory power, or optical activity.**

Compounds whose molecules are asymmetric and cannot be superposed on their mirror images rotate the plane of polarization of light. Examples which will already have come to the notice of the student are sugars and tartaric acids in aqueous solution

and turpentine. The **specific rotatory power,** S, is defined by $S=\frac{\alpha}{lD}$, where α is the observed rotation, l the length of liquid traversed in decimetres, and D the density.

For solutions the formula is $s=\frac{\alpha}{lc}$, where c is the concentration of the solution in gm. per 100 c.c.

Laurent's half-shade polarimeter.—Sodium light from the diaphragm (7) is made parallel by the lens (6) and traverses the polarizing Nicol (5). Plane polarized light strikes the analyzing

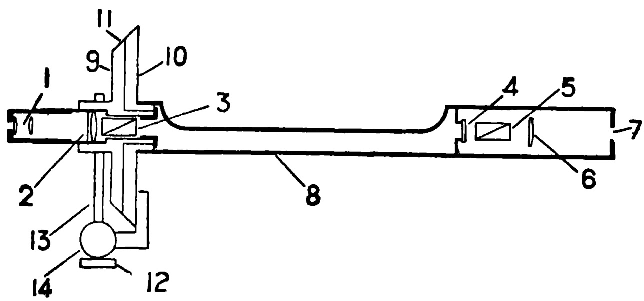


Fig. 254.—Laurent's half-shade polarimeter.

Nicol (3), which can be rotated about the horizontal axis of the instrument, and whose position can be read by a vernier moving over a fixed scale.

If the polarizer and analyzer are crossed so as to produce complete extinction, and an optically active substance is placed between the two, some light is transmitted. The analyzer must be rotated to restore darkness through an angle equal to that through which the substance has turned the plane of polarization of the light going through it.

To make the setting depend on the judgment of *equal brightness*, a half-wave plate of quartz or mica (4) covers half the aperture of the polarizing prism, the shaded part of the circle of Fig. 255 (*a*).

Light polarized in the direction AC will then be resolved by the plate into an ordinary ray polarized in the plane AB and an extraordinary ray polarized in the plane XY, if the principal axis

of the quartz is parallel to AB. If the components on entering the plate are represented by AB, AX, then on leaving it they will be represented by AB, AY. The new plane of polarization is then AC'. It is clear that there will be two positions of the analyzer, (*b*) and (*c*), differing by an angle equal to CAC' (called the **half-shadow angle**), in each of which one-half of the field appears completely dark. Midway between these positions is one in which both halves of the field appear equally bright (Fig. 255*d*).

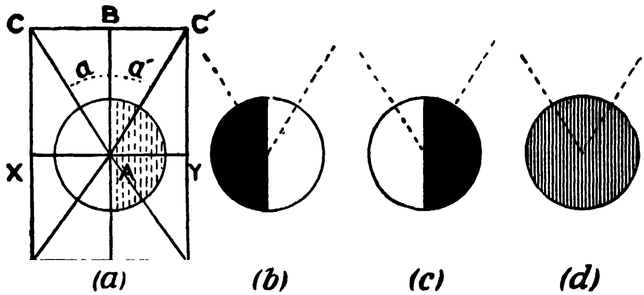


FIG. 255.—" Half-shadow " setting.

The half-shadow angle will be small when the plane of polarization of the light striking the plate makes a small angle with the direction AB of the axis of the plate ; in this case also the illumination of both halves of the field when a match occurs will be small. There is thus an *optimum* half-shadow angle at which, while the angle is small, sufficient light is transmitted to make an accurate match possible.

Practical considerations.—Isotropic substances such as glass and celluloid become doubly refracting under stress, and from the appearance of a specimen between crossed Nicols the nature of the stress can be deduced. The science of **photoelasticity** enables engineering problems on stresses to be solved by examining small transparent models in this way.

The light of the **blue sky** is strongly polarized in a plane containing the sun and the ray coming from the sky. The blue colour and polarization were both reproduced by Tyndall in a famous experiment, in which a mixture of butyl nitrite vapour and hydrogen chloride at low pressure were exposed to the light from an

arc lamp. The growing particles behaved at an early stage of their development just like an "artificial sky." The blue colour was explained as a diffraction effect by Rayleigh, who showed by considering the theory of dimensions that the energy scattered by small particles should be *inversely proportional to the fourth power* of the wavelength, so that the blue end of the spectrum undergoes the greatest scattering. The "particles" in the sky are probably the irregularities in molecular distribution which are to be expected statistically at very low pressures.

"**Cellophane**" behaves in a similar manner to a plate of uniaxial crystal cut with its optic axis parallel to the surface. Two directions at right angles, a **fast axis** and a **slow axis**, in its surface can be distinguished. Half- and quarter-wave plates of "Cellophane" can be made inexpensively, and demonstrations of the "colours of thin plates" made with numbers of strips superposed.

An important new invention is the **polarizing screen** due to E. H. Land, in which minute dichroic crystals are arranged with their polarizing axes parallel in a thin film of plastic. One substance used is herapathite, a compound of quinine sulphate and iodine, and more transparent films have been made using instead a cobalt salt. It has been suggested that if all motor-car headlamps were fitted with such films, and motor drivers wore goggles with similar but "crossed" films, the "dazzle" problem of the roads might be solved effectively. In photography, such screens can be used to cut down light from the sky and also to reduce light reflected from transparent surfaces, so that photographs can be taken obliquely through glass and water.

The polarimeter is used for the quantitative estimation of sugar, when it is known as a saccharimeter.

Microscopes for petrological work are equipped with a polarizing Nicol and quarter-wave plate below the stage and an analyzer over the eyepiece, so that thin transparent rock sections may be examined in plane or elliptically polarized light. Differences in material and structure are shown up brilliantly in different colours. Such a microscope, used with a powerful source as a micro-projector, is an ideal instrument for demonstrating the material of pp. 312-317 on a small scale.

CHAPTER XX

THE ULTRA-VIOLET, THE INFRA-RED, AND X-RAYS

The ultra-violet and infra-red.—The eye is sensitive to radiations of wavelengths between about 4000 Å.U. and 7500 Å.U., the violet and red ends of the spectrum respectively.

The region of the spectrum between 4000 Å.U. and about 136 Å.U. is called the **ultra-violet.** Ultra-violet light affects a photographic plate, causes many substances to fluoresce, ionizes gases, and causes metals to emit electrons. Glasses are practically opaque below 3000 Å.U., with the exception of the Jena "Uviol," which transmits down to 2800 Å.U. Quartz is transparent down to 1850 Å.U., and modern fused quartz is highly transparent at this wavelength and probably considerably beyond it. Fluorite (calcium fluoride) is transparent down to 1200 Å.U.

The behaviour of polished metal surfaces in ultra-violet light is interesting. Silver, which reflects about 95% of the incident light in the visible part of the spectrum, reflects only 4% at wavelength 3160 Å.U., though for shorter wavelengths its reflecting power rises and fluctuates; in fact, a very thin film of metallic silver can be used as a filter to cut off visible light and *transmit* only ultra-violet. Nickel and magnalium (an alloy of magnesium and aluminium), and many other alloys, are more effective than silver, while one of the most useful reflecting surfaces is made by depositing a thin film of aluminium on glass or quartz.

The region between 7500 Å.U. and about 400,000 Å.U. is called the **infra-red.** The existence of this part of the spectrum was first demonstrated by Sir William Herschel (1738-1822), who observed that a thermometer indicated a rise in temperature when its blackened bulb was held beyond the red end of the solar spectrum produced with a glass prism. Infra-red radiation possesses no

outstanding properties which distinguish it from other forms of radiant energy ; when it is absorbed the absorbing body is heated, and on this account it is known as " thermal radiation," or " radiant heat."

Experiments which establish that " radiant heat " is, in fact, of the same nature as visible light are described fully in text-books on heat. The instruments used for detecting and investigating infra-red radiation are thermal instruments, of which **Langley's bolometer** (which is in principle a very sensitive platinum resistance thermometer of low thermal capacity) and the **thermopile** are the two most important.

The following table shows the limits in the infra-red of the transmitting power of substances of optical interest :

Glass - - - - -	30,000 Å.U.
Quartz - - - - -	40,000 Å.U.
Calcite - - - - -	50,000 Å.U.
Fluorite - - - -	90,000 Å.U.
Rock salt - - - -	150,000 Å.U.
Sylvine - - - -	230,000 Å.U.

Sources of ultra-violet and infra-red radiation.—Although only approximately realizable in practice, the conception of an ideal radiator capable of emitting all wavelengths in the visible, ultra-violet, and infra-red parts of the spectrum has been of great importance in theoretical work on radiation. Such a body should also be capable of absorbing all wavelengths, and so is termed a **perfectly black body**. For such a radiator the energy emitted per sq. cm. per sec. is proportional to the fourth power of the absolute temperature (**Stefan's Fourth Power Law**), and the distribution of this energy between the various parts of the spectrum is as shown in Fig. 256. The wavelength most strongly represented, λ_{max}, and the absolute temperature, T, are connected by the relation $\lambda_{max} T = \text{constant}$ (**Wien's Displacement Law**). The curves of Fig. 256, taken from Preston's *Theory of Heat*, were obtained experimentally by Lummer and Pringsheim for the radiation from the interior of a heated cylinder and only go up to 1646° absolute. The maximum for the highest temperature is well in the infra-red, and it can be seen from the general trend of the

curves that at higher temperatures the wavelength λ_{max} would be shorter, the proportion of visible and ultra-violet radiation emitted would be greater, and the absolute amounts of all radiations would be greater.

The distribution curves for such radiators as incandescent platinum surfaces, while they do not agree perfectly with those for an ideal black body, are of the same general type. Any surface which behaves approximately like a black body will, if raised to a sufficiently high temperature, be an effective source of both infra-red and ultra-violet.

The temperature of the sun is about 6000°, and the solar spectrum has been shown to extend from 2000 Å.U. to 400,000 Å.U. This range would be much greater were it not for the atmosphere's absorption.

Mercury vapour lamps and hydrogen discharge tubes, usually in quartz envelopes, are used in the laboratory as sources of both ultra-violet and infra-red. Arcs and sparks between electrodes of various metals are used for the ultra-violet, and the shortest wavelengths of all were obtained by Millikan from sparks passed at very high voltages between metal electrodes *in vacuo*.

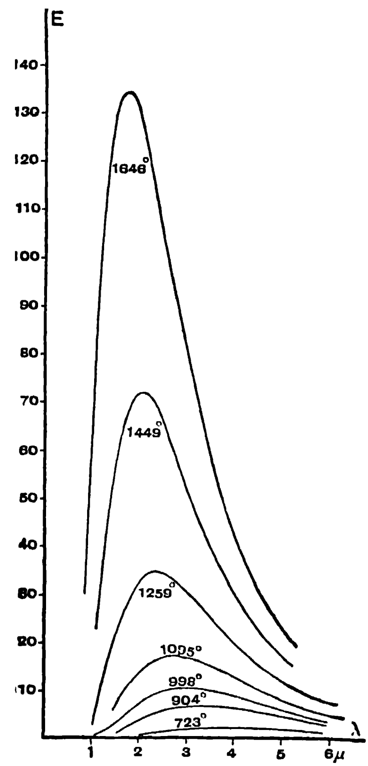


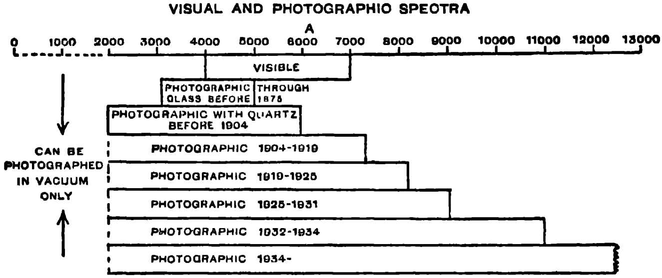
FIG 256.—Distribution of energy in the spectrum of a black body. ($1\mu = 10{,}000$ Å.U.)

Photography in the ultra-violet and infra-red.—Fig. 257, reproduced by permission from an article in *Nature* of May 2, 1936, by Dr. C. E. K. Mees, shows approximately the extent of the part

of the spectrum which it is now possible to photograph. The earliest dry plates were sensitive only to the blue and violet parts of the visible spectrum and the nearer ultra-violet ; and in any case it is impossible to photograph below 3000 Å.U. through glass. Two factors have contributed to the extension of this range. The first is the use of special plates, and the second is the introduction of optical parts of quartz and fluorite and the employment for the extreme ultra-violet of a Rowland concave grating *in vacuo.*

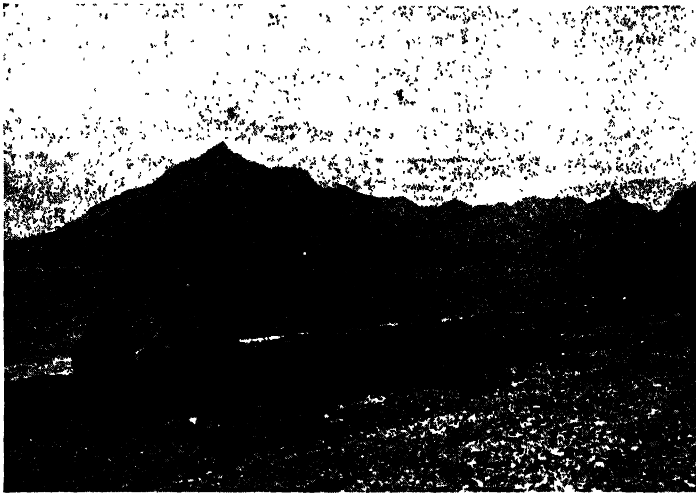
In the ultra-violet there is no difficulty about the sensitiveness of silver bromide, but the gelatine of the emulsion absorbs

VISUAL AND PHOTOGRAPHIC SPECTRA



strongly below 2265 Å.U. Ordinary plates treated with oil are more effective, as the oil fluoresces, giving visible light which can then act on the plate. The difficulty was overcome by **Schumann,** who devised a method of holding the sensitive grains to the plate with the minimum of gelatine ; in the Ilford " Q " ultra-violet plate the sensitive grains are on the surface of the gelatine.

The principle of sensitization to the green and yellow parts of the spectrum was discovered in 1873 by H. Vogel of Berlin. Dyes incorporated with the gelatine and actually adsorbed on the surface of the sensitive grains were effective for just those parts of the spectrum which they absorbed. Green-sensitive or **orthochromatic** plates were made with red dyes—eosin, erythrosin, and ethyl red. Dyes of the chemical family known as cyanines, many of them blue, were later used to make **panchromatic** plates sensitive to the whole visible spectrum ; and modern research on this type of dye has produced kryptocyanine, neocyanine, and



(*a*) Taken with ordinary plate in visible light.

(*b*) Taken in infra-red light.

FIG. 258.—Ordinary and infra-red photographs of the same scene.

(By courtesy of Mr. C Waller and Messrs. Ilford, Ltd.)

tricarbocyanine sensitizing to 7500 Å.U., 9000 Å.U., and 11,000 Å.U. respectively, and recently tetra- and pentacarbocyanine, which sensitize to beyond 13,000 Å.U.

It is well known that ordinary photographs can be taken entirely by infra-red light, through filters which cut out the visible spectrum ; and photographs of this kind are now a feature of many journals. On account of its long wavelength, infra-red light is scattered by the atmosphere to a less extent than is visible light, so that clearly defined pictures are secured through great thicknesses of air, and even through haze and mist. Coloured objects are rendered in an unfamiliar way in the prints ; for example, grass and foliage appear white and the blue sky (which owes its natural colour to the short wavelengths which it scatters) appears black. Fig. 258 shows two photographs of the same view, one taken in visible light with an ordinary plate and the other an infra-red photograph.

Ultra-violet photomicrography.—The compound microscope can be used to project a real image of the slide on a screen ; no alteration of lenses or tube-length is necessary, but the distance of the object from the objective will be a little greater than the working distance for visual observation. Photographs can thus be taken through the microscope if a camera is arranged so that the eyepiece of the microscope takes the place of the camera lens.

The use of ultra-violet light for this purpose offers two advantages. First, as has been stated on p. 295, the resolving power depends on the wavelength of the light used, and is improved by using light of short wavelength. Secondly, structural details in the specimen are shown up clearly by the differences in the absorption of ultra-violet light by different parts.

The whole optical system of the ultra-violet microscope is of quartz, and the objective is corrected for use at one particular wavelength. Optically worked quartz slides and cover-glasses are used. The object is mounted in some fluid transparent to ultra-violet light, such as distilled water, glycerin, normal saline, or castor oil, and must not be stained or prepared in the usual way. For high-power objectives, an immersion fluid to take the place of cedar oil is provided by making up a mixture of glycerin and water of refractive index 1·447.

The illumination of the object must be as intense as possible, and must also be monochromatic radiation of the wavelength for which the objective has been designed. An arc with electrodes of magnesium or cadmium provides a suitable source, as the spectrum of each metal contains sharp lines which are easily isolated. The radiation is passed through a spectrometer with quartz lenses and prism, and light from the required line is reflected along the microscope's axis by a quartz prism.

Fig. 259 shows the Zeiss vertical camera for photomicrography. The camera is easily raised for purposes of observation and adjustment, and the fluorescent eyepiece, *A*, can be swung into position for focussing the object and centring the image. This eyepiece contains a small fluorescent screen, which is viewed through a high-power magnifier. This method of focussing, by finding the image in the fluorescent eyepiece and then placing the photographic plate at a calculated distance, can hardly be as satisfactory as focussing the image directly on to a screen occupying the same position as the plate. The technique described by Barnard and Welch (*Practical Photomicrography*, 1936 edition) overcomes this difficulty by focussing the image in monochromatic visible light using a glass objective, and then changing over to the ultra-violet source and quartz objective.

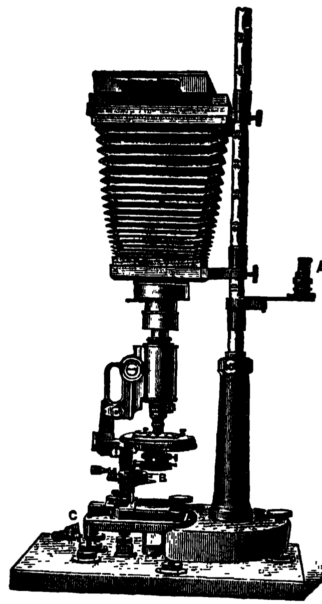


FIG. 259.
Photomicrographic camera.

The most recent developments of photomicrography seek to obtain high resolving power, not by the use of shorter wavelengths than the ultra-violet which quartz transmits, but by using beams of electrons which are focussed by magnetic fields arranged to act as objective and ocular.

Ultra-violet spectrographs.—The type of instrument used for work between 4000 Å.U. and 1850 Å.U. is shown in Fig. 260. The

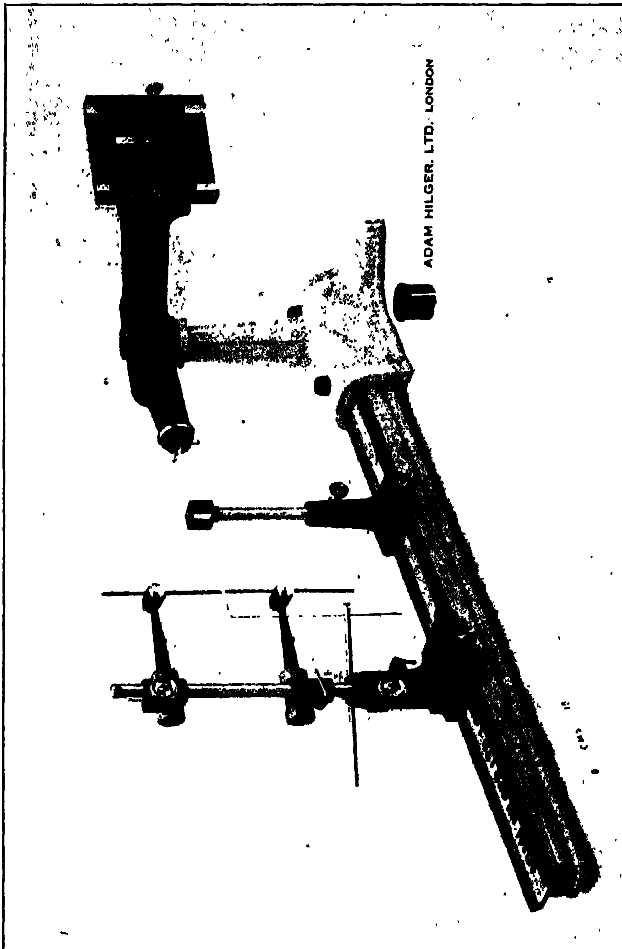


FIG. 260.—Ultra-violet spectrograph with arc and condensing lens.
(*Courtesy of Adam Hilger, Ltd.*)

optical arrangement is that of Fig. 208, p. 248. The lenses and prism are of quartz. It is usual to make the 60° prism out of two right-angled 30° prisms, one of " left-handed " and the other of

"right-handed" quartz, and hold the two together by a thin film of glycerin ; this is called a **Cornu prism.** On account of the great dispersion of quartz in this region the photographic plate is inclined considerably to the principal axis of the second lens.

The quartz lenses in this spectrograph have focal lengths of about 20 cm. for sodium light. The overall size of the instrument is thus something like that of the ordinary laboratory spectroscope. Now the separation between two lines on the plate is $f\phi$, where f is the focal length of the objective lens and ϕ the angular dispersion between them. Messrs. Adam Hilger make a spectrograph (in which $f=170$ cm.) giving 8·5 times as large a separation on the plate. If the ordinary optical arrangement were used, the instrument would also be 8·5 times as large, so a

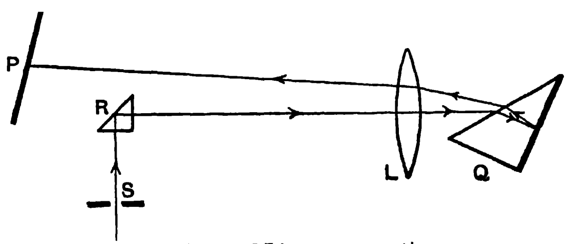


FIG. 261.—Plan of Littrow mounting.

much more compact form, known as the **Littrow mounting,** is employed (Fig. 261).

Radiation from the slit S is reflected by the right-angled quartz prism R to the lens L, from which it emerges as a parallel beam. The quartz prism Q has a refracting angle of 30°, and is coated at the back with a tin-mercury amalgam or some other alloy which reflects well in the ultra-violet. This reflects the radiation back through the prism ; the dispersed beam passes back through L, which forms an image of the spectrum on the plate P. Different regions of the spectrum are received at P by rotating Q. A small blackened screen is placed between R and L, in a suitable position to absorb reflections from the lens surfaces and so avoid fogging. The whole instrument is enclosed in a light-tight box, mounted on a single girder about 2 metres long.

Large spectrographs also employ concave gratings. Here again, much space is saved by a departure from the classical

Rowland arrangement, known as the **Eagle mounting.** The general scheme is best described as a combination of Littrow and Rowland mountings, and can be understood from Fig. 261 without further diagrams. The grating occupies the position of LQ, and both P and the virtual image of S in R are on its Rowland circle. The grating can be tilted about a horizontal axis to throw the spectrum up clear of R, and about a vertical axis to secure different parts of the spectrum or different orders of spectra on the plate.

Apart from the saving of space, the Eagle mounting has several advantages over Rowland's. No darkened room is needed, the

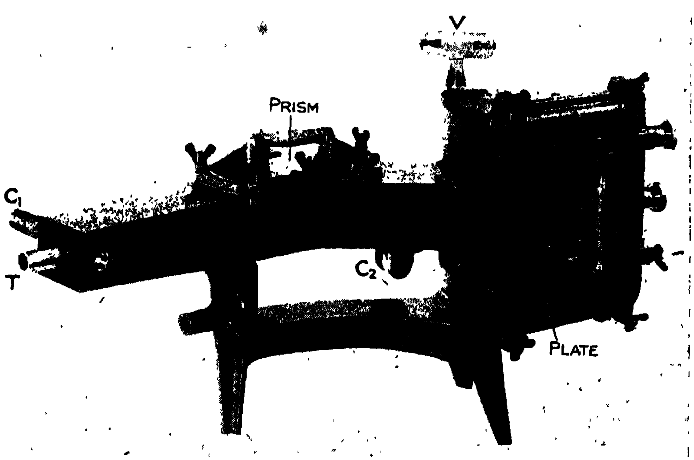


Fig. 262.—Fluorite vacuum spectrograph.
(*Courtesy of Adam Hilger, Ltd.*)

arrangement is very rigid, and it is easier to secure uniformity of temperature. Spectra on either side of the normal may be used (a great advantage, as in the ultra-violet the spectra are often of unequal quality, and the better spectra of different orders may not all be on the same side). Higher orders can be obtained, giving greater dispersion ; this is useful because the dispersion in a given order is much less for ultra-violet than for visible light.

Schumann explored the region between 1850 Å.U. and 1200 Å.U. (known generally as the **Schumann region**) using fluorite lenses and prism. The system was placed within a stout enclosure which

could be evacuated, and all manipulations were made from outside. When the source was a hydrogen discharge tube, this was mounted just outside the slit and communicated directly with the spectrograph, which was filled with hydrogen at the same pressure of a few millimetres of mercury. For other gases the discharge tube was separated from the spectrograph by a thin plate of fluorite.

Fig. 262 shows a modern fluorite vacuum spectrograph. The cones at C_1 and C_2 are vacuum-pump connections, and the windowless discharge-tube source is connected to T in front of the main slit. There is a second slit beyond C_1, which enables the discharge tube to be maintained at a different pressure from the enclosure, for at low pressures a gas passes very slowly through small openings. The discharge tube V serves to illuminate a wavelength scale in the instrument, which is recorded on the plate.

Lyman used a concave grating for the region between 1200 Å.U. and 510 Å.U., and his evacuated spectrograph took the form of a long tube. At one end of the tube was the grating, and at the other was the photographic plate and two slits. The slits and the plate were all on the " Rowland circle " for the grating, and the plate was bent by its holder to conform to this circle. On p. 298 it was stated, for the purpose of the *simple* discussion of reflecting gratings, that the lines acted as obstacles, and reflection from the spaces between the lines gave the diffracted light which interfered. This simple picture is not a *true* description of such a grating, for the whole of the original surface is removed by the ruling point and the diffracted light comes from the sides of the grooves thus traced. The relative intensities of the spectra of different orders depend greatly on the actual shape of each groove, and by suitable choice of the diamond point used for ruling it is possible to concentrate a great deal of light into the spectrum of one particular order.

Lyman worked with the brightest spectrum of a carefully chosen grating. The two slits enabled two different regions of the spectrum to be photographed on the same plate. The displacement due to the distances between the slits was calculated, and unknown wavelengths in the shorter-wavelength spectrum were determined from their positions on the plate relative to the

known lines of the other region. Fig. 263 shows a vacuum grating spectrograph similar in type to Lyman's, but with one slit, which is mounted on a side tube attached at as small an angle as possible to the main tube.

The chief difficulties in extreme ultra-violet spectroscopy have been the poor reflecting power of the gratings in the region of very short wavelengths and the small dispersion obtained. Prof. M. Siegbahn of Upsala has designed and ruled special gratings to be used at *nearly grazing incidence*, by means of which both these difficulties are overcome. He photographed the spectrum from a vacuum spark down to about 100 Å.U., and the technique is applicable to even shorter wavelengths.

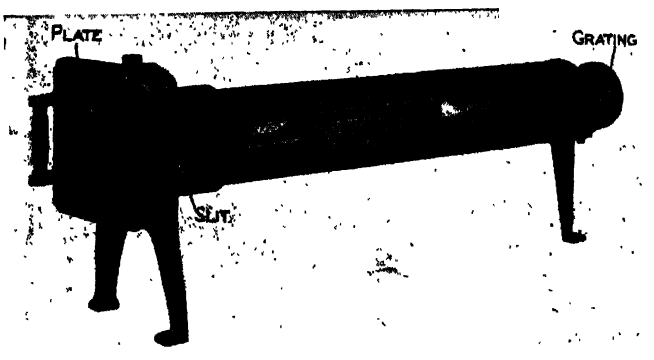


FIG. 263.—Vacuum grating spectrograph.
(*Courtesy of Adam Hilger, Ltd.*)

Instruments of the type shown in Fig. 260 are used in modern spectrum analysis for the recognition of the heavy metals, chiefly for the detection of small quantities of impurities. The substance examined may form one electrode of an arc, or may be placed on one of a pair of pure carbon or graphite arc electrodes. The spectrum may be photographed on the same plate as a comparison spectrum containing the lines of the suspected impurities. Quantitatively, the actual proportions of the impurities present can be estimated with precision by measuring the densities of the lines on the plate with a delicate instrument known as a microphotometer.

Biological effects of ultra-violet radiation.—The biologically active rays lie between 2000 Å.U. and 3000 Å.U., the most active being round about 3000 Å.U., which is near the limit of the sun's spectrum. Sunlight is actually the most beneficial source of the rays, and for most purposes artificial sources are valuable solely as its substitutes. Rays shorter than 2000 Å.U. kill bacteria, but are too strongly absorbed by all media to be used for sterilising materials in bulk. Using a very powerful quartz mercury-arc, mounted axially within an annular chamber, it is possible to sterilise liquids flowing through the chamber fairly slowly.

The human epidermis consists of living cells coated with a horny outer layer of dead cells. The biologically active rays do not penetrate below the epidermis ; rays of shorter wavelength are stopped by the horny layer, while rays of longer wavelength penetrate it completely and are converted into heat. The active rays, however, penetrate the horny layer and kill the outer living cells, thereby thickening the layer of dead cells. A pigment is produced in the lower living cells which absorbs the remaining radiation, causing an inflammation which is helpful in the treatment of some skin affections. Vitamin D, necessary for the prevention of rickets, is formed by the irradiation of ergosterol, a substance present in the skin and originally derived from the food, by rays of about 3000 Å.U. Other effects on the body include the oxidation of fats, increase in the rate at which calcium and other minerals are taken into the system, and probably stimulation of the ductless glands.

Fluorescence.—This name is given to the emission of radiation as the result of the absorption of radiation of shorter wavelength. It is commonly observed in oils, uranium glass, and fluorescein, where blue visible light is absorbed and a yellowish-green emitted. Many substances fluoresce with a characteristic visible light in ultra-violet radiation. Observations of the nature and intensity of the fluorescent radiation enable tests for many purposes to be made accurately and very quickly.

The radiation from a mercury-arc is first passed through a filter which will transmit only ultra-violet radiation. Fig. 263*a* shows (1) the portion of the spectrum of the quartz mercury-arc and (2) the portion of the spectrum transmitted by the filter.

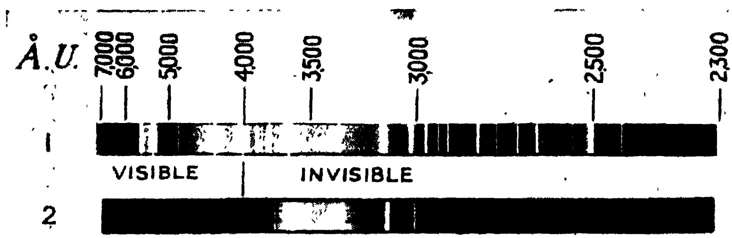


FIG. 263*a*. *(Courtesy of Hanovia, Ltd.)*
1. Spectrum of quartz mercury-arc. 2. Radiation transmitted by filter.

The following interesting applications are taken from among the many described in the pamphlet, *Fluorescence and Rapid Testing*, issued by Hanovia, Ltd., of Slough.

1. *Foodstuffs.*—The shells of new-laid eggs give a rose fluorescence ; stale eggs, blue or violet. Butter, margarine, lard, and other fats can be readily distinguished. Change in the fluorescent colour of cheese enables its ripening to be followed. Different flours, and different kinds of honey can be identified.

2. *Police work.*—Identity of fluorescence has been used to establish that samples of material are from identical sources. Substances used for secret writing, such as milk and saliva, can be detected. Finger prints can be photographed after dusting with finely powdered anthracene. Different inks used in forged documents have been distinguished. Blood, after treatment with reagents, gives a characteristic fluorescence.

3. *Medicine and bacteriology.*—Ringworm gives a green fluorescence by which it can be diagnosed, and diseased tissues (in dissections) can be detected. Bacterial cultures, similar in daylight, can be differentiated in ultra-violet radiation.

The testing of fluorescent chemicals is an obvious use. The applications of this method in museum work, mineralogy, and many industries, are also described in the pamphlet referred to, which gives a brief account of the technique. With many materials which deteriorate in sunlight it is possible to accelerate the change by exposure to intense ultra-violet radiation and to follow its course by observing changes in fluorescence.

No special lenses or plates are needed for photography, as the fluorescent light is, of course, *visible* and not ultra-violet. But it is necessary to use a filter before the lens to cut off scattered ultra-violet, and the exposures must be long.

X-rays.—This name is given to the region of the spectrum between about 10 Å.U. and 0·1 Å.U., produced by the electrical means described below. Modern technique has led to the production of even shorter wavelengths, so these figures should not be taken as representing definite limits. X-rays affect a photographic plate, excite fluorescence in many substances, cause metals to emit electrons, and ionize gases. They can traverse considerable thicknesses of air and light substances, but comparatively thin layers of heavy substances, such as lead, suffice to stop them. Serious and deep-seated burns may be caused by these rays, so that either the apparatus or operator, or both, should be enclosed in material which is opaque to them. The penetrating power depends on the wavelength ; X-rays of short wavelength are very penetrating and are called **hard,** while those of long wavelength are less penetrating and are called **soft.**

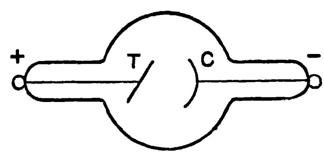


FIG. 264.—Early X-ray tube.

Sources of X-rays.—The pressure in discharge tubes used for studying the emission spectra of gases is about 1-5 mm. of mercury. When tubes of this kind are evacuated to about 0·01 mm., little light is given out by the gas, but the walls of the tube glow with a green fluorescence. At this pressure the negative electrode, called the **cathode,** is emitting a stream of "cathode rays," or "cathode particles." Their path through the gas is shown by faint blue streamers. It has been established that these particles are **electrons,** or atoms of negative electricity, of mass $0{\cdot}899 \times 10^{-27}$ gm. and negative charge $4{\cdot}774 \times 10^{-10}$ E.S.U. The cathode particles leave the cathode at right angles and travel in straight lines (apart from a slight tendency to diverge, as they repel one another—a tendency most marked when the pressure in the tube is very low) to the walls of the tube, instead of proceeding to the positive electrode, called the **anode.**

Röntgen in 1895 first observed that the walls of a discharge tube emitted radiation under the impact of cathode particles. The earliest X-ray tubes were designed as in Fig. 264 ; the concave aluminium cathode C focussed the cathode particles on a platinum

target T, called the **anticathode**, which also served as anode. The part of the target to which the cathode rays converge acts almost as a *point source* for the X-rays. The original idea of the target was to provide a surface which would not be melted by the heat generated when the concentrated beam of cathode rays was stopped ; it was soon observed that the properties of the X-rays emitted were determined to some extent by the nature of the target, and that heavy metals were most efficient for producing hard and intense radiation. Tungsten, which is less dense than platinum but has a much higher melting point and is also a better conductor of heat, is generally used for the target in commercial tubes, though copper is frequently used when it is desired to obtain soft X-rays.

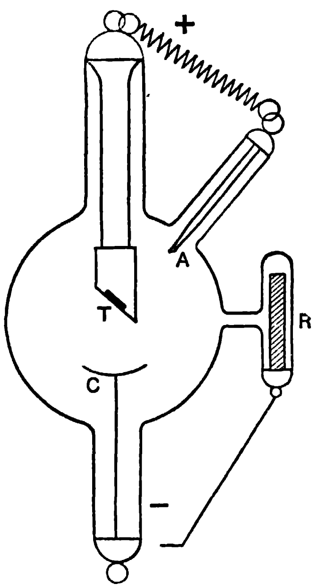


Fig. 265.—Gas-filled X-ray tube with regulator.

Fig. 265 shows a later development of the same type of tube. The target T is of tungsten surrounded by a massive block of copper, and a separate anode A is provided to steady the discharge. The **regulator** R contains asbestos on which gas has been occluded, and some gas may be liberated by connecting R to C and working the tube. This is needed because the pressure of the gas in the tube becomes lower during use. Both the hardness and the intensity of the radiation depend on the voltage used across the tube; if the pressure of the gas is high, a comparatively low voltage suffices to run it and the radiation is soft and of small intensity; if the pressure is low, the voltage required is high and hard rays are obtainable in greater intensity. Several other devices have been used for admitting small quantities of gas into the tube when necessary.

The **Coolidge tube** (Fig. 266), introduced in 1916, is highly evacuated. No gas is needed to supply ions for the maintenance of the

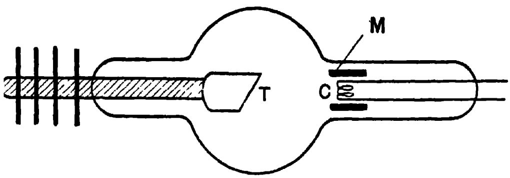


FIG. 266.—Coolidge tube.

discharge, for the cathode C is a heated filament carrying a current of a few amperes and giving off electrons by what is called "**thermionic emission**"; its behaviour is exactly the same as that of the filament of a wireless valve. No separate anode is used. A cylinder of molybdenum, M, serves to focus the stream of electrons on the target T, which may be cooled by radiator fins as shown, or by circulating a current of water inside it. The current across the tube, and hence the intensity of the X-rays, depends on the temperature of the filament, while the voltage across the tube determines the hardness of the radiation; hardness and intensity can thus be varied independently. Tubes of this type can be made much smaller than those containing gas, and very compact ones with metal envelopes are in common use.

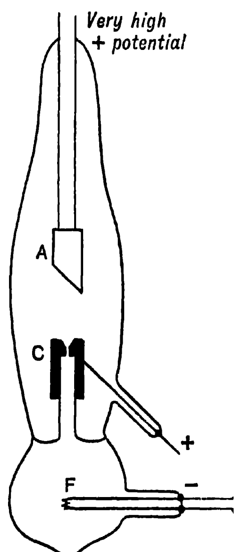


FIG. 267.—Lilienfeldt tube.

In the **Lilienfeldt tube** (Fig. 267), electrons from the heated filament F are driven to the perforated cathode C. The current through the tube is regulated by altering the voltage between F and C, instead of altering the temperature of F, and the voltage between C and the anticathode A controls the hardness of the rays.

The softest X-rays require about 1500 volts for their excitation.

Optical properties of X-rays.—Early attempts at diffraction experiments suggested that the wavelength of X-rays was of the order of 1 Å.U. No surface could be made sufficiently smooth to reflect radiations of this wavelength regularly. It has been calculated that the refractive index of glass should be slightly less than unity by about four parts in a million. *The fact that regular reflection and refraction are not observed is due entirely to the very short wavelength of X-rays.* When the rays strike a solid substance, **secondary X-rays** are given out. A large proportion of the secondary radiation is diffusely reflected or scattered—an effect which would be expected for light of very short wavelength ; there is also definite evidence that it is polarized. In addition, the secondary X-rays contain radiations characteristic of the solid used. Barkla isolated these as homogeneous beams, using screens of metal to cut down the scattered " background " ; the more penetrating radiation he called **K-radiation,** the less penetrating **L-radiation.** These K and L radiations correspond to beams of *monochromatic light.* The characteristic radiations given as secondary X-rays by a metal can also be distinguished in the radiation from an X-ray tube with an anticathode of that metal.

The diffraction gratings commonly used in schools have about 14,500 lines to the inch. The lines are thus about 17,000 Å.U. apart, a separation of *between two and three wavelengths* of the visible radiations for which they are used. To secure diffraction effects of the same kind with X-rays, the rulings would have to be spaced only a few Ångstrom units apart. The suggestion that *the regular arrangement of atoms or molecules* in the **cleavage planes** of a crystal might provide spacings of this order was made by v. Laue in 1912, and tested by Friedrich and Knipping, with apparatus shown diagrammatically in Fig. 268. A beam of X-rays passed through a series of apertures in lead screens and fell on a thin crystal plate C ; the photographic plate P received the transmitted beam. Fig. 269 shows the pattern obtained with a crystal of zinc blende. This is very similar in its symmetry to the kind of pattern obtained when a beam of light passes through two diffraction gratings set with their rulings at right angles, or

when a small distant source of light is viewed through very fine wire gauze. Friedrich and Knipping thus showed that crystals

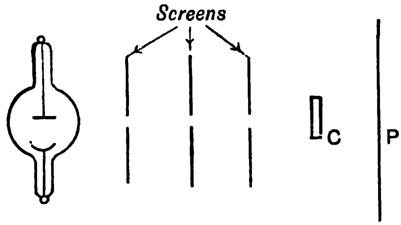


FIG. 268.

could be used as three-dimensional diffraction gratings for X-rays. Patterns such as would be obtained by rotating Fig. 266 about the direction of the original X-ray beam are given

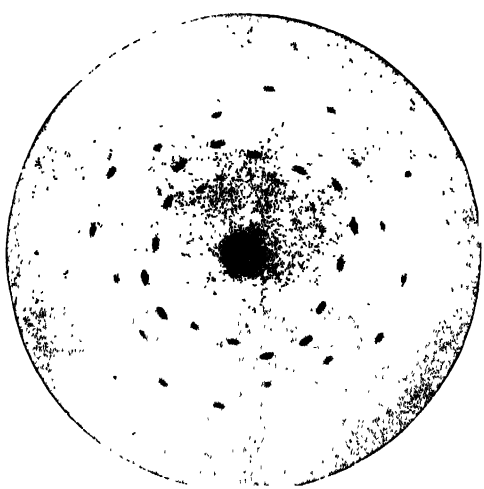


FIG. 269.—X-ray diffraction pattern for zinc blende.

if C, instead of being a whole crystal, is a tube containing a crystalline power.

Prof. von Laue believed X-rays to be electromagnetic waves, but in 1912 the possibility that they were corpuscular had not

been finally excluded. Prof. W. L. Bragg, then a student at Cambridge, soon showed that the pattern of Fig. 269 was due to the symmetry of the zinc blende crystal, and that very simple crystal structures could be deduced from such patterns; his father, Sir W. H. Bragg, developed the X-ray spectrometer. Both investigations confirmed the wave-diffraction explanation of the pattern.

The glancing angle.—A beam of homogeneous X-rays incident at a small angle θ on the natural face of a crystal will, for certain values of this angle, called the **glancing angle**, give an intense diffracted beam also making an angle θ with the surface. It is just as if regular reflection were occurring for only a few angles of incidence; the atoms or molecules in the parallel cleavage planes scatter the incident radiation in all directions, and for one of these

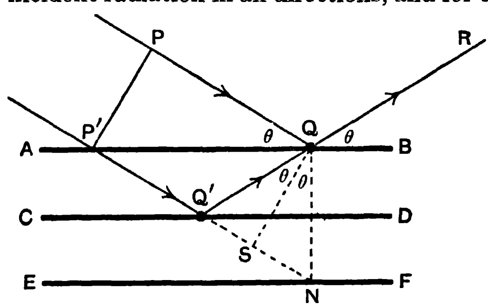


FIG. 270.

angles of incidence there is one direction in which the beams scattered from all the cleavage planes are in phase.

Let AB, CD, EF (Fig. 270) be three parallel cleavage planes separated at equal perpendicular distances d. Let PQ, $P'Q'$ be two rays from a parallel beam of homogeneous X-rays incident at a glancing angle θ. If the path difference $(P'Q' + QQ' - PQ)$ is a whole number of wavelengths, the rays scattered from the planes AB and CD reinforce one another in the direction QR, and so will the rays scattered from all parallel planes. Let $P'Q'$ meet EF in N. It is easily proved that QN is perpendicular to the planes. Now $QQ' = Q'N$, and if S is the midpoint of $Q'N$, it is clear that $PQ = P'S$. So the path difference $= P'N - P'S = SN = QN \sin\theta = 2d \sin\theta$. If λ is the wavelength of the homo-

geneous X-rays, the condition for reinforcement is $n\lambda = 2d \sin \theta$. If $n=1$, $\lambda = 2d \sin \theta$; if $n=2$, $2\lambda = 2d \sin \theta$, and so on. The crystal should thus act like an ordinary diffraction grating and give several spectra, called spectra of the first, second, third . . . orders, for corresponding values of n.

If only d can be determined for one crystal, the way is clear for absolute measurements of X-ray wavelengths less than $2d$, using the equation $n\lambda = 2d \sin \theta$; and once these wavelengths have been determined, the value of d can be found for any crystal. The fundamental step is the absolute determination of d for one crystal; **rock salt** was the crystal first calibrated.

Rock salt.—Sir W. H. Bragg and his son, Prof. W. L. Bragg, solved the problem of determining d for rock salt, working with homogeneous X-rays of constant but unknown wavelength. As both d and λ were unknown, use of the equation $n\lambda = 2d \sin \theta$ alone did not suffice; but this enabled the nature of the crystal structure to be found, and when this had been done the value of d was easily calculated from the density of rock salt and other known constants.

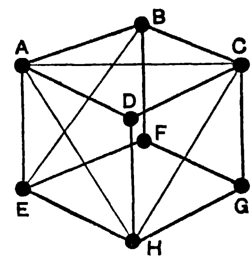


FIG. 271.

Rock salt is a cubic crystal. The molecules of sodium chloride, or atoms of sodium and chlorine, should thus be arranged in space in a lattice pattern made up of cubes, and d should bear some relation to the side of the fundamental cube units of which the lattice is composed.

There are *three* sets of cleavage planes in a lattice arrangement built up of cubes, and these can be seen in the simple cube of Fig. 271. They are:

a face of the cube, such as $ABCD$, called the (100) plane;

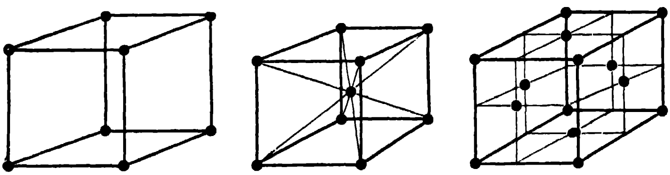
a section, such as $BCHE$, called the (110) plane;

a section, such as CHA, called the (111) plane.

The distances between successive planes in the three cases are known as d_{100}, d_{110}, and d_{111} respectively.

The atoms or molecules forming a cubic lattice may be arranged *in one of three possible ways.* These are :

(i) a simple cube lattice (Fig. 272*a*) ;
(ii) a cube centred lattice (Fig. 272*b*) ;
(iii) a face centred lattice (Fig. 272*c*).



(*a*) Simple cube lattice. (*b*) cube centred lattice. (*c*) face centred lattice.
FIG. 272.

The ratios $1/d_{100} : 1/d_{110} : 1/d_{111}$ are different for each arrangement, being

$1 : \sqrt{2} : \sqrt{3}$ for a simple cube lattice,
$1 : 1/\sqrt{2} : \sqrt{3}$ for a cube centred lattice,
$1 : \sqrt{2} : \sqrt{3}/2$ for a face centred lattice.

Now although λ was not known, these ratios could be determined *from relative measurements,* and it was shown that the structure of rock salt is that of the simple cubic lattice.

When reflections from the (111) planes were being examined, it was found that the spectra of alternate orders (that is, for alternate values of n) were weaker than the others. **The conclusion from this was that alternate (111) planes were composed exclusively of either chlorine atoms or sodium atoms,** for chlorine atoms, being heavier than sodium atoms, should scatter X-rays more strongly. The unit of which the cubic lattice is built was thus shown to be a cube with *atoms of sodium and chlorine* at its corners—as in Fig. 273, where the black dots represent sodium atoms and the light dots chlorine atoms. It would be more precise to refer to them as sodium ions and chlorine ions.

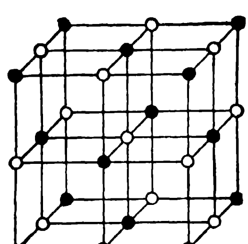


FIG. 273.—Space lattice of rock salt.

Now that the structure of the lattice had been definitely

ascertained, the actual value of d_{100} was calculated as follows. A cube of side d_{100} and volume $(d_{100})^3$ has 4 atoms of sodium and 4 chlorine atoms at its corners—a total mass equal to that of 4 molecules of NaCl. But these corners are each shared with seven other cubes when this cube is in its place in the lattice, so that the mass allotted to the cube of side d_{100} is actually $\frac{1}{8}$ of the masses at its corners, or half the mass of a single NaCl molecule. Now the density of a substance is mass/volume, so that volume = mass/density.

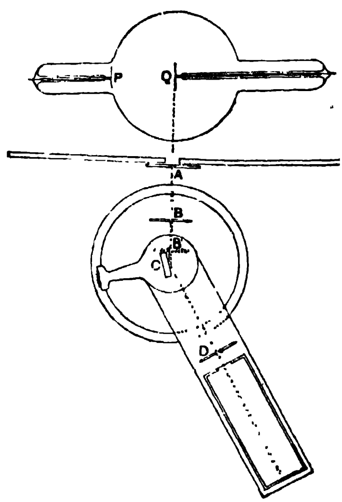


FIG. 274.—X-ray spectrometer. *A* and *B* are slits, and the ionization chamber is below *D*.

Hence $(d_{100})^3 = \frac{1}{2}$ mass of NaCl molecule/density of rock salt. Both of these quantities were known, so that d_{100} could at once be calculated. The value obtained was $2{\cdot}814 \times 10^{-8}$ cm., or 2·814 Å.U.

X-ray spectrometers.—In the original Bragg X-ray spectrometer a narrow, nearly parallel, beam of rays is transmitted through two slits in thick lead screens and falls on the crystal *C* (Fig. 274). An **ionization chamber,** containing some heavy gas, is mounted on an arm like that of the telescope of an ordinary spectrometer and receives the radiation reflected from the

crystal. Electrical arrangements, which need not be discussed here, are made for measuring the current which passes through the ionization chamber when the gas within is made conducting by the reception of X-rays.

For photography, a single slit S (Fig. 275) is used. This transmits a diverging beam of rays from different parts of the anticathode of the source. O is the centre of the spectrometer table, on which the crystal is mounted, and the photographic plate is set at P, somewhere on the circle of centre O and radius OS. When rays reflected at the glancing angle θ are received at P, it is clear that the angle $S'OP = 2\theta$, and that if the crystal is set with its face bisecting this angle the ray SO is diffracted along OP. If the crystal is now rotated so that its face cuts the circle drawn through S, O, and P in A, then the ray SA will be diffracted along AP, for the angles SAO and SPO are in the same segment of a circle and so are equal. This arrangement has the effect of focussing all diffracted rays of the same wavelength to the same part of the plate ; there is no need for accurate setting of the crystal. Indeed, the best way of working is to rotate the crystal slowly, thus eliminating the effect of blemishes which may be present at any one part.

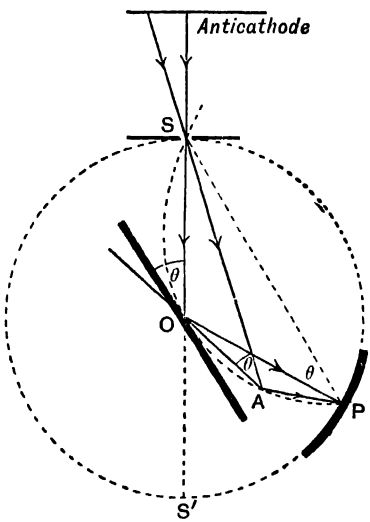


FIG. 275.—Rotating crystal method.

The softer X-rays are appreciably absorbed by air ; and Siegbahn used an evacuated spectrograph for studying the region above 1 Å.U. The single slit, crystal, and plate were arranged exactly as in Fig. 275, and the crystal and plate turned by external handles. The slit was covered with thin metal foil, which would withstand the atmospheric pressure without obstructing

the X-rays. More recently Siegbahn designed an X-ray vacuum spectrograph for work on the softer rays, using special ruled gratings at grazing incidence; the X-ray tube communicated directly with the spectrograph.

X-ray spectra.—Two types of radiation can be distinguished in the emission from an X-ray tube. The first is a **continuous spectrum**, which is definitely limited on the short-wavelength side; this limit depends on the voltage across the tube, from which it can be calculated. The formula connecting the shortest wavelength, λ Å.U., and the voltage, V volts, is $\lambda = 12{,}300/V$.

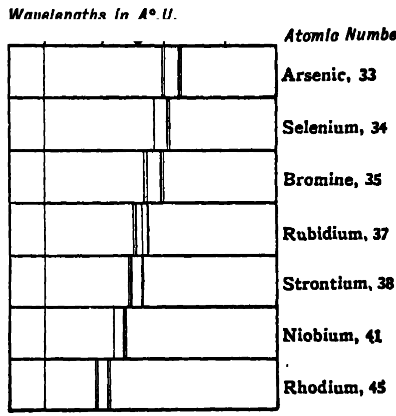


FIG. 276.—*K* series X-ray spectra.

Superimposed on the continuous background is a **line spectrum**, which is characteristic of the material of the anticathode. For heavy elements there are three series, called the *K*, *L*, and *M* series (the *K* series being the hardest of the three and the *M* series the softest). Each series consists of only a few lines. The explanation given on p. 256 for the emission of visible line spectra applies equally well to X-ray line spectra; but the electrons concerned with the emission are those in the three shells nearest the nucleus of the atom, now called the *K*, *L*, *M* shells after the spectra for which they are responsible.

The line spectra of all elements appear very similar, but the actual wavelengths of corresponding lines vary with the **atomic number**. This number represents the position which the element occupies in the periodic table, and also gives the number of electrons surrounding the nucleus of the neutral atom. The higher the atomic number, the shorter the wavelengths of the lines. Fig. 276 shows the regular decrease in wavelength of the *K* lines of various elements as the atomic number increases.

Moseley discovered the important relationship between the frequency, ν, of a given member of the series and the atomic number, N, of the element producing it. He used the photographic rotating crystal method. His crystal was a large and perfect specimen of potassium ferrocyanide, for which d was 8·454 Å.U., and he was able to work with the third order spectrum which gave larger values of θ, and hence greater accuracy of measurement. For any given line, followed through the series of elements he studied, the graph of $\sqrt{\nu}$ against N was a straight line, showing that the relationship was of the form $\sqrt{\nu} = AN + B$, where A and B are constants for the chosen line. Moseley's experiments were the first to show that *the atomic number is a constant of fundamental importance for each element far more important than the atomic weight, which had been the basis of the periodic classification of the elements.* The immense significance of Moseley's work is discussed fully in modern text-books of chemistry.

Absorption spectra are photographed by placing the absorbing material in the path of the X-ray beam. It is found that heavy elements absorb more strongly than light ones. The absorption by a given substance increases as the wavelength increases in general, but sudden discontinuities with a great decrease in absorption occur in the neighbourhood of the wavelengths of the shortest K, L, and M lines. These discontinuities are called **absorption limits**, or absorption edges.

γ-rays.—One of the products of the disintegration of the nucleus of a radioactive atom is a radiation of very short wavelength, known as γ-radiation. The wavelength depends on the transition which gives rise to it. Actually the softest γ-rays have a wavelength longer than that of the hardest X-rays, and Rutherford and Andrade determined the wavelengths of the radiations from radium B and radium C, using the glancing-angle method with rock salt (and also a somewhat different crystal-diffraction method), and showed that these wavelengths ranged from between about 0·1 Å.U. and 1·3 Å.U.—well within the short X-ray region. The properties of the rays are those to be expected of very hard X-rays.

Radioactive preparations also emit two corpuscular radi-

ations: α-particles, which are positively charged helium atoms, and β-particles, which are fast-moving electrons. The γ-rays are, however, easily isolated as they are able to penetrate thicknesses of metal which suffice to absorb the other radiations, and as they are undeviated by a magnetic field.

Uses of X-rays and γ-rays.—As light substances are fairly transparent to hard X-rays, while heavy substances are much more opaque, the shadow cast on a fluorescent screen or a photographic plate by any part of the human body will show the bones standing out as dark shadows on a less dark background. X-rays are used in medicine for the location of fractures and foreign bodies, for the treatment of malignant growths, and for other purposes.

With improvements in apparatus and technique, short-exposure radiography of any part of the human body is now possible. Less and less dense parts can be distinguished, until now the soft tissues and even the extent of tumour growths in them can be depicted. Modern fluorescent screens enable digestive movements, heart beats, or lung movements to be watched or cinematographed.

In addition to the use of X-rays for the localization of foreign objects in the human body, which is now a routine matter in hospitals, the rays have proved to be of great value in the treatment of malignant disease. Certain superficial forms of cancer, such as rodent ulcer, can be healed either by low-voltage X-rays or by the β- or γ-rays of radium. Small quantities of radium or low-voltage X-rays are, however, not capable of destroying cancer cells at a depth ; for such cases use is made of X-rays, which, excited at very high voltages, tend to approach γ-rays in quality, or of large quantities of radium placed at a distance remote from the skin in an attempt to obtain the large depth doses possible with X-rays.

At St. Bartholomew's Hospital, London, a remarkable installation of high-voltage X-ray apparatus for radiotherapy has been established, and was described in *Nature* of December 26, 1936 This installation is designed to give a beam of greater penetrating power, higher intensity, and shorter mean wavelength than any so far employed in the treatment of cancer. The X-ray tube is

30 feet long and weighs 10 tons. It may be operated at any voltage from 250,000 volts up to 1,000,000 volts, and is therefore adaptable for use in the treatment of a variety of cases.

Both X-rays and γ-rays have their own particular advantages in radiation therapy. The article in *Nature* referred to above describes the latest development on the high-voltage X-ray side. Another article, on radium beam therapy and high-voltage X-rays, appeared in the issue of *Nature* for January 9, 1937. In this the conclusion is reached that " the true advantage of radium treatment would therefore seem to depend on the property that a great part of the electromagnetic waves proceeding from radium are shorter in wavelength than those that emerge from any X-ray tube at present in use."

It is possible in the future that other sources of penetrating radiation will become available in sufficient quantity for use in radiation therapy. Beams of sufficient intensity from neutrons have already been proved to have strong biological effects, and γ-rays from artificial radioactive substances have already been obtained far in excess of anything which radium emits.

Both X-rays and γ-rays find many applications in the industries and arts—as, for example, in the examination for flaws and other defects in metal castings and forgings, welds, and assembled components. γ-rays are used in the radiography of metal specimens too thick for X-rays to be effective. In the National Gallery, London, an X-ray outfit has been installed for the examination of pictures and other objects of art.

Modern methods of X-ray crystallography have enabled the structures of many quite complicated minerals, and of some simple organic crystals, to be worked out. The following brief account indicates some of the problems to which this technique has been applied with success, and gives some idea of the extremely important results which its use may bring in the future. A general account of this work is given in an article by Sir William Bragg, which appeared in *Nature* of May 22 and 29, 1937.

At Manchester, the structures of such complex crystals as the zeolites and silicates have been completely investigated, and recent work there has been on the structures of alloys, and the effect of heat treatment upon these structures,

The gradual deterioration of metals under repeated stress, known as " fatigue ", has been shown to be due to a progressive breaking down of the crystalline structure.

It has recently been found that apparently non-crystalline organic substances with large and complicated molecules may possess a regular structure of a crystalline type. The proteins, substances of great importance in animal life, now form almost a branch of crystallography of their own. For example, it has been shown that keratin, the protein present in our hair and nails, has a chain-like structure, and the spacing of the atoms in this chain has been determined. X-ray crystallography is steadily increasing our knowledge of the structure of the proteins, and it may soon be possible to show how the biological behaviour of these substances depends on their inner structure.

INDEX

ANSWERS TO EXAMPLES

Chapter II (p. 28)

2. (*a*) 1·25 ft.-candles; (*b*) 0 213 ft.-candle. **3.** 0 68. **4.** 1·156 : 1.
5. 90·3 %. **6.** 1·054 : 1. **8.** 63° 36′.
12. *P*, 3 33 ft.-candles; *Q*, 1·18 ft.-candles; *R*, 0 298 ft.-candle.
13. Normal to circle is at 81° 52′ to the line. **14.** 125 : 27.
15. 2 63 metres. **16.** 0·32 ft.-candle.

Chapter IV (p. 66)

1. 1·2 inches.
2. Shift of object is $4\frac{1}{6}$ cm. and focal length is 25 cm. **6.** 2 radians.

Chapter V (p. 86)

2. 1 471. **9.** 6 cm. above the bottom.
10. 12 cm. above the pole.
12. The image shifts by 0·556 cm. and magnification changes from 0·50 to 0·5556.
13. 1 600.

Chapter VI (p. 105)

1. 2° 30′; 1·63. **2.** 3° 58′; 2′. **3.** 1·414.
4. 22° 22′. **6.** 50° 30′ for red, 54° 48′ for violet.
8. 3°; 1° 8′. **9.** 69° 42′. **10.** 12° 40′; 4′ 43″.

Chapter VII (p. 122)

2. Taking the refractive index of the glass as 1 5, (*a*) virtual and 20 cm. from the face nearer the observer; (*b*) real and 2 5 cm. from the second surface.

3. The image formed by refraction only is 2 cm. behind the face, and that formed after reflection is 10 cm. from the same face; both are virtual.

4. Angle of incidence 60°, angle of deviation 136°.

5. (*a*) 1·505; (*b*) 7·52 cm. The four significant figures would be obtainable if a vernier microscope reading to $\frac{1}{20}$ mm. were used, though the no-parallax setting on its crosswire would hardly justify their retention. An ordinary spherometer reading to $\frac{1}{50}$ mm. would do the job more accurately and with only *one* measurement. , Boys' method is elegant and direct, though less accurate than the spherometer, but would not do for such a thick lens.

Chapter IX (p. 153)

1. At the 53·0 mark ; 0·8. **2.** 48 cm.

3. $12\frac{6}{7}$ cm. above the surface ; about 25 cm. above this.

4. (*a*) 20·5 cm. ; (*b*) 12·84 cm. ; (*c*) 1·626. **5.** $4\frac{16}{21}$ cm.

6. Focal length 6″ ; radius of convex mirror 22″, of concave 14″.

7. Focal length 12 cm. ; 32 cm. long. **8.** (–) 30″ ; concave.

9. 1·633. **10.** Virtual, 8·8 cm. in front of concave lens. Nearly $-\frac{1}{25}$.

11. (*a*) 6″ inside tank, 2″ long ; (*b*) 8″ inside, $2\frac{2}{3}$″ long. **12.** 20·63 cm.

13. 24·24 cm. **14.** (–) 19·3 cm. **15.** (–) 20 cm. **16.** (–) 30 cm.

17. $14\frac{2}{7}$ cm.

18. (*a*) (–) 7·5 cm. (virtual image) ; (*b*) (+) 15 cm. (real image).

19. (+) 90 cm. Converging, so he is long- (or far-) sighted. 1·57.

Chapter X (p. 169)

1. 0 0327. **3.** 0·43 cm.

5. Converging, (+) 15 cm. ; diverging, (–) 30 cm.

6. Converging, (+) 37·5 cm. ; diverging, (–) 60 cm.

Chapter XI (p. 178)

1. 0·0332 inch. **2.** 3 : 1. **4.** 9 inches.

6. $7\frac{1}{2}$ ins. ; 0·83 inch. **7.** 0·0997 cm., say 1 mm.

Chapter XII (p. 189)

1. 6. **2.** Diverging, (–) 6 in. focal length.

3. (*a*) Diverging, (–) 300 cm. focal length ; range from infinity to 60 cm.
(*b*) Converging, (+) 50 cm. focal length ; range $42\frac{6}{7}$ cm. to 25 cm.

4. 5·3 cm.

5. Diverging, (–) 90 cm. focal length ; 18 cm.

7. Converging, (+) 30 in. focal length.

Chapter XIII (p. 217)

2. $25\frac{1}{6}$ cm. **3.** $17\frac{17}{22}$ cm.

4. Object distance 4 cm., magnification (–) 5. **5.** $35\frac{1}{3}$.

6. $15\frac{21}{65}$ inches ; angular magnification is $15\frac{3}{5}$. **7.** (+) 0·288 inch.

9. 14 inches for normal adjustment ; $13\frac{2}{3}$ inches if the final image is formed at the normal near point 10 inches from his eye ; shift is $\frac{12}{19}$ inch.

10. $8\frac{1}{3}$.

11. (*a*) 22·4 ; (*b*) 4·88 cm. diameter ; (*c*) separate the lenses by a distance of $(63 + \pi/20)$ cm. and get a real image on a plate at a distance of $(90/\pi + 3)$ cm.

14. 0·5°. **16.** 2.

Chapter XIV (p. 233)

2. 1 foot below the the surface. **6.** $2 \cdot 5 \times 10^6$ cm./sec.

7. Receding at 6×10^6 cm /sec.

Chapter XV (p. 245)

1. $V + v/V - v$. **2.** 2500, taking c as 3×10^{10} cm./sec.

3. $9 \cdot 501 \times 10^8$ feet per second, or 18×10^4 miles per second.

Chapter XVI (p. 264)

6. (*a*) 49° 15′ ; (*b*) 1° 31′ ; (*c*) about 2 6 cm. **9.** 22° 30′.

Chapter XVII (p. 279)

4. 0·20 cm. ; 6×10^{-5} cm. **8.** 0·094 cm.

9. The path difference is 98·88 kilometres. This distance contains 250·3 complete wavelengths at 395 metres and 244·2 wavelengths at 405 metres. If there is no phase change at reflection, maximum intensity will occur for those wavelengths which divide the path difference exactly by 250, 249, 248, 247, 246, 245 ; there will thus be 6 maxima with intervening minima.

12. 1·000224.

Chapter XVIII (p. 302)

2. 5701.